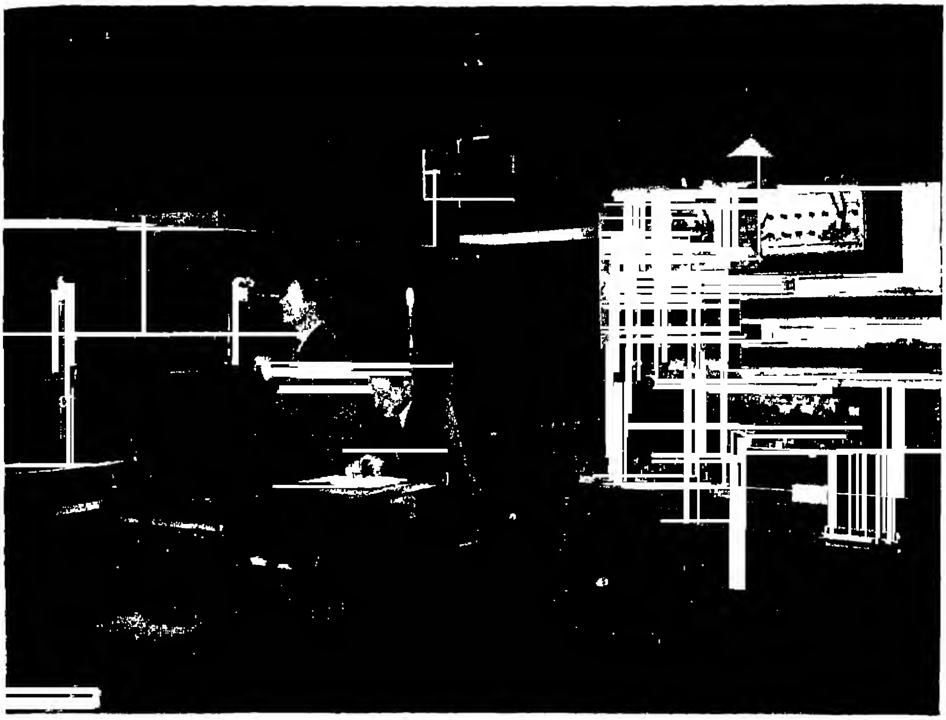
PHOTOMETRY



THE BENCH ROOM IN THE PHOTOMETRIC LABORATORY

Frontispiece

PHOTOMETRY

BY

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ILLUSTRATED WITH DIAGRAMS BY

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TO

MY FATHER AND MOTHER

THIS BOOK IS DEDICATED

PORTIA. How far that little candle throws his beams!
So shines a good deed in a naughty world.

NERISSA. When the moon shone, we did not see the candle.

PORTIA. So doth the greater glory dim the less. . .

(*Merchant of Venice*, Act V, sc. 1.)

PREFACE

Not infrequently during the past few years the author has been asked to recommend a book dealing with some branch of practical photometry. Although there are several good books on the subject in existence, notably Dr Liebenthal's wonderfully comprehensive "Praktische Photometrie," none of those which have appeared recently deals exclusively with photometry, but all treat it rather as an adjunct to the study of illumination It follows that the descriptions of instruments and methods given in such books are necessarily brief, and the past fifteen years have been so fruitful in the development of this branch of technical physics that the present volume may not seem altogether superfluous

In illustration of what has been said concerning the recent progress of photometry, it may be mentioned that nearly all the illumination photometers so widely used to-day have been designed within the last fifteen years, while the study of physical methods of photometry, undoubtedly a branch of the subject which is destined in the near future to find many technical applications, has taken place almost entirely within that same period

Because it has always been a cherished belief of the author that the practice of a branch of applied science cannot properly be carried on without some grasp of the theory underlying it, no apology is offered for the insertion of the two chapters in which an attempt has been made to give briefly and in outline some account of our present knowledge of the nature of light and the physiological phenomena connected with the sensation of vision

At the same time, it is hoped that the general treatment will be found to be "practical" in the true sense of the word. As Professor Stine has said in the preface to his book "Photometrical Measurements" —

"To those who desire to follow the practice of photometry, and lack an adequate knowledge of general physics, this work may appear too scientific for a manual, and be too insistent on details which apparently have little significance. To such the writer would state that photometry is not a simple and well-defined subject Bare directions will not suffice, but the practician must bring to the task a judgment trained for instrumental manipulation and an appreciation of the many modifying influences that the results which he obtains may possess any value"

The author has endeavoured to describe, in such detail as the limitations of space permit, those instruments and methods which he has used at the National Physical Laboratory, those which he has seen used elsewhere, or those which his personal experience has led him to regard as of value in photometric work The description of obsolete or obsolescent apparatus has been reduced to a few lines, and in many cases it has been omitted altogether, only mention by name and a reference to the original description having been inserted for the sake of completeness.

It has been found necessary strictly to limit the scope of this book to a description of those matters, both theoretical and practical, which affect the measurement of light flux, candle-power, illumination, *etc*, and to exclude any treatment of the use to which such measurements may be put after they have been made Thus, for example, in the sections dealing with illumination, only the methods of measuring that quantity have been described and no attempt has been made to outline the principles which should be followed in designing an installation to produce a given degree of illumination under certain specified conditions Similarly, the effect of different factors (*e.g*, voltage, gas pressure, *etc*) on the candle-power of light sources has been dealt with only to the small extent necessary to ensure that measurements of the candle-power of such sources may be free from errors due to lack of proper control of such factors. Again, no general discussion has been given of the effect of shades and reflectors on the distribution of light from a source On the other hand, a description will be found in Chapter VII of the methods of photometric measurement by means of which the effect of such appliances may be quantitatively determined in any given case

The principles governing the solution of such problems of practical application as those just mentioned belong properly to that branch of technics known as "illuminating engineering," and for a description of these principles and the method of applying them, some book on illumination (*e g*, one of those enumerated at the end of Chapter XII) should be consulted

Since it is evidently impossible in the space of a single volume to give specialised treatment to any branch of the subject, an attempt has been made to provide as complete a bibliography as possible of the whole subject of photometry by means of numbered references to the original literature These references are automatically classified by the position in which they occur in the book That this bibliography is really complete is more than the author may dare to hope, and he will be most grateful for information as to noteworthy omissions, as well as for corrections to the errors from which he may not presume to think he has escaped entirely

The principal journals dealing with gas and electrical engineering, illumination, physics and general science published in Great Britain, the United States of America, France, Germany and Italy have all been thoroughly searched from the first year of publication, and every paper included in the relevant sections of the International Catalogue of Scientific Literature published by the Royal Society has been referred to Where a paper has been reprinted, either in full or in lengthy abstract, in journals other than that in which it originally appeared, references to those journals have been placed in the notes after the reference to the original paper This has the disadvantage of extending the space devoted to the notes, but it is often useful where the only library readily available to the reader is a specialised one and the number of periodicals correspondingly limited. An abstract in a familiar language is, moreover, often of considerable value in giving an idea of the contents of a paper published in a less familiar tongue.

The abstract journals such as *Science Abstracts*, the *Beiblätter* (*Physikalische Berichte*), *Fortschritte d Phys*, *Chem Soc J* (Abstract

Section), *Chemical Abstracts*, *Chem Centralblatt*, *Photographic Abstracts*, *Abs Bull of the Eastman-Kodak Research Laboratory*, *Ophthalmic Year Book*, *Physiological Abstracts, etc* , although they have often given the clue to many valuable papers, have not, in general, been referred to in the notes

With only six exceptions every reference given in the notes has been actually consulted in order to verify its correctness

It will be noticed that throughout the book wave-numbers have been used in preference to wave-lengths. Although in current practice (except in the theory of radiation) the use of wave-lengths is more common, the fundamental character of the wave-number makes it desirable, in the author's opinion, that a change should be made To obviate any necessity for conversion from one system to the other, the values in the letterpress have been given in terms of both wave-length and wave-number, while the graphs have been arranged so that it is possible to use them with equal facility on either system

There remains, finally, the pleasant duty of thanking all those who have so kindly helped in what has proved to be a very congenial task In particular the author would like to take this opportunity of thanking his colleagues in the Photometry Division of the National Physical Laboratory for the kindness with which they have always unhesitatingly placed at his disposal the results of their many years of practical experience in photometry

While the Patent Office, British Museum, Science Museum and National Physical Laboratory Libraries have furnished the means for consulting most of the literature of the subject, the author is also much indebted to the librarians of the Royal Society, the Royal Astronomical Society, the Chemical Society, the Natural History Museum, the Institution of Electrical Engineers, the Royal College of Surgeons, the Royal Society of Medicine, the Royal Photographic Society, University College, London, the London University Library and the Radcliffe Museum, Oxford, as well as to Dr J. Kerr and Mr L Gaster, for much valuable assistance in obtaining access to original papers The editor of the *Journal des Usines à Gaz* also kindly furnished the references to some early papers in that journal.

For permission to reproduce Fig 30 thanks are due to the Royal Society Fig 244 is reproduced by kind permission of Messrs P J and J M Waldram, and the Illuminating Engineering Society

Much of the usefulness of any book of this character depends upon the accuracy and clearness of the diagrams with which it is illustrated It will therefore readily be understood what a debt of gratitude the author owes to Mr Lewis for the immense amount of care which he has bestowed on the drawings which appear in the following pages Thanks are also due to Mr F J C Brookes, of the National Physical Laboratory, for the trouble he has taken in making the photographs from which the plates have been prepared and to Miss D. Lipscombe for several drawings of Laboratory apparatus, notably Fig 67 showing the photometer bench

JOHN W T WALSH

TEDDINGTON,
St Peter's Day, 1925

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LIST OF ILLUSTRATIONS

LIST OF ABBREVIATIONS

THE following is a list of the abbreviations employed in this book to designate those books and periodicals which occur most frequently. The periodicals are, in general, those which contain most of the papers published on the subject of photometry and certain cognate matters Those marked with an asterisk are specially rich in photometric literature. Those marked with an obelisk have ceased publication

BOOKS

Abbreviations	*Full Title.*
Colour Vision	W de W Abney "Researches in Colour Vision and the Trichromatic Theory" (Longmans, 1913)
Colour Vision	J H Parsons "Introduction to the Study of Colour Vision" (Camb Univ. Press, 1924)
Dict Appl Phys	R. T. Glazebrook (editor) "Dictionary of Applied Physics" (Macmillan, 1923–24)
Encyc Brit. .	"Encyclopædia Britannica" (11th edition, unless otherwise specified)
Illumination, etc	A P Trotter "Illumination, Its Distribution and Measurement" (Macmillan, 1911)
Lehrbuch d Phot	F Uppenborn-B Monasch "Lehrbuch der Photometrie" (Oldenbourg, Munich and Berlin, 1912)
Photométrie .	A Palaz. "Photométrie Industrielle." (Carré, Paris, 1892) (Refs to pages in Eng trans. given in brackets)
Phys Handw .	"Physikalisches Handwörterbuch" (Springer, Berlin, 1924)
Physiol d Menschen	W Nagel "Handbuch der Physiologie des Menschen," Vol 3 (Vieweg, Brunswick, 1905)
Physiol. Optik .	H von Helmholtz "Handbuch der Physiologischen Optik" (3rd edition). (Voss, Hamburg, 1909–11) (Refs to Eng trans. in brackets)
Prakt Phot	E Liebenthal "Praktische Photometrie" (Vieweg, Brunswick, 1907.)

PERIODICALS

Abbreviations.	*Full Title*
Am Acad , Proc.	Proceedings of the American Academy of Arts and Sciences Easton, Pa , U S A
Am Chem Soc , J.	Journal of the American Chemical Society. Easton, Pa , U S A
*Am Gas Light J.	American Gas Light Journal New York
Am I E E , J (Trans , Proc)	Journal (Transactions, Proceedings) of the American Institute of Electrical Engineers. New York
Am J Sci	American Journal of Science (Silliman's Journal) New Haven, Conn., U S A
Am. Phil Soc , Proc	Proceedings of the American Philosophical Society Philadelphia, U S A.
Amsterdam Acad., Proc	Proceedings of the Royal Academy of Amsterdam (Eng trans. of the Koninklijke Akademie van Wetenschappen te Amsterdam) Amsterdam

*Ann. Chim Phys.	Annales de Chimie et de Physique Paris (Continued from 1914 as Ann de Phys and Ann de Chim)
Ann. d Chem	Annalen der Chemie und Pharmacie. (See Liebig's Ann.)
Ann de Chim	Annales de Chimie Paris
Ann de Phys	Annales de Physique Paris
Ann d Phys	Annalen der Physik Leipzig (1900 onwards for earlier vols see Gilb Ann , Pogg Ann , or Wied Ann)
Arch. des Sci .	Archives des Sciences Physiques et Naturelles (Bibliothèque Universelle) Geneva
Assn Franç , C r	Association Française pour l'avancement des Sciences, Comptes rendus des séances
Astron Nachr	Astronomische Nachrichten Kiel
Astrophys J	Astrophysical Journal Chicago
B A Report .	Report of the British Association for the Advancement of Science London
Berlin Ber. (Abh) .	Sitzungsberichte (Abhandlungen) der Preussischen Akademie der Wissenschaft zu Berlin (Phys - Math Klasse) Berlin
Bibl Univ .	Bibliothèque Universelle (see Arch des Sci)
Bureau of Standards, Bull. (Circ , Technol Paper)	Bulletin (now Scientific Papers) (Circular, Technologic Paper) of the Bureau of Standards, Washington, D C
Central-Ztg f Opt u Mech	Central-Zeitung für Optik und Mechanik, Elektrotechnik u. verwandte Berufszweige. Berlin
†Centralbl f Elektrot .	Centralblatt fur Elektrotechnik Münich (Continuation of Zeits. f angewandte Elektricitätslehre · now included in E T Z)
Chem. Soc , J (Trans)	Journal (Transactions) of the Chemical Society London
Chem Ztg.	Chemiker-Zeitung Cöthen
*C I.E , Proc	Receuil des Travaux de la Commission Internationale de l'Eclairage (before 1913 the Comm Int de Photométrie)
C R.	Comptes Rendus Hebdomadaires des séances de l'Académie des Sciences Paris
†Deut Phys Gesell , Verh	Verhandlungen der Deutschen Physikalischen Gesellschaft (now Z f Phys) Brunswick
Drudes Ann	See Ann d Phys
*†Ecl El.	L'Eclairage Electrique Paris (Now merged in Rev Gen de l'El , see also Lum El)
Electrician	Electrician London
Elettrot	L'Elettrotecnica Milan
El Rev	The Electrical Review London (Continuation of Telegraphic Journal, 1872–1891)
†El Rev (N Y)	The Electrical Review New York (Now included in El Rev and W Elect)
El Rev and W Elect	Electrical Review and Western Electrician Chicago (Includes El Rev. (N Y) from 1908)
El World	Electrical World New York
E u M.	Elektrotechnik und Maschinenbau Vienna (Continuation of Z f Elektrot)
*E T Z .	Elektrotechnische Zeitschrift Berlin
Frank Inst , J	Journal of the Franklin Institute Philadelphia
*Gas J	Gas Journal London (Formerly J of Gas Lighting)
Gen El Rev	General Electric Review Schenectady, N Y
†Gilb Ann.	Gilbert's Annalen der Physik (First series of Ann d Phys , 1799–1824)
Harvard Coll. Obs , Ann (Bull , or Circ)	Annals (Bulletin or Circular) of the Harvard College Observatory Cambridge, Mass , U S A.
*Illum Eng .	The Illuminating Engineer London

*†Illum. Eng (N.Y.) .	The Illuminating Engineer. New York (Continued (1912) as Good Lighting, which ceased publication in 1913)
*Illum Eng Soc N.Y., Trans	Transactions of the Illuminating Engineering Society of New York.
Inst Civ Eng, Proc .	Proceedings of the Institution of Civil Engineers. London
Inst El Eng, J .	Journal of the Institution of Electrical Engineers (formerly Society of Telegraph Engineers) London
Inst. Gas Eng, Trans	Transactions of the (Incorporated) Institution of Gas Engineers. London.
Inst. Mech Eng., J.	Journal of the Institution of Mechanical Engineers. London
J de Phys	Journal de Physique Paris. (Now J. de Physique et le Radium)
*J of Gas Lighting .	Journal of Gas Lighting London (Now the Gas Journal)
*J G.W	Journal fur Gasbeleuchtung und verwandte Beleuchtungsarten, sowie fur Wasserversorgung. Munich and Berlin (Now Das Gas- u Wasserfach)
J Sci Insts	Journal of Scientific Instruments. London
Kodak Publ	Abridged Scientific Publications from the Research Laboratory of the Eastman Kodak Company Rochester, N Y
Leipzig Ber (Abh	Berichte über die Verhandlungen (Abhandlungen) der (Königlich) Sächsischen Akademie (Gesellschaft) der Wissenschaften (Math -Phys Klasse) Leipzig
*Licht u Lampe.	Licht und Lampe Berlin
Liebig's Ann	Liebig's Annalen der Chemie Leipzig. (Continuation of Ann d Chem. u Pharmacie.)
*†Lum El . .	La Lumière Electrique Paris. (In the same series with Ecl. El)
†Mélanges phys chim	Mélanges physiques et chimiques tirés du Bulletin de l'Acad Imp des Sci. de St Pétersbourg.
Munchen Ber . .	Sitzungsberichte der (Königlich) Bayerischen Akademie der Wissenschaften (Math -Phys Klasse) Munich
Nat Acad Sci., Proc. (Mem).	Proceedings (Memoirs) of the National Academy of Sciences Washington
N Cimento	Il Nuovo Cimento. Pisa
Nela Bull	Abstract-Bulletin of Nela Research Laboratory Cleveland, Ohio.
N P L, Coll Res	Collected Researches of the National Physical Laboratory Teddington
Opt Soc Am., J	Journal of the Optical Society of America and Review of Scientific Instruments Menasha, Wis
Opt Soc, Trans	Transactions of the Optical Society. London.
Phil Mag	The London, Edinburgh and Dublin Philosophical Magazine and Journal of Science. London.
Phil Trans	Philosophical Transactions of the Royal Society (Series A is intended except where otherwise noted) London
Phot J	Photographic Journal London
Phys Rev	Physical Review New York
Phys Soc, Proc	Proceedings of the Physical Society of London
Phys Z . .	Physikalische Zeitschrift Leipzig.
†Pogg Ann	Poggendorff's Annalen der Physik (Second series of the Ann d Phys 1824–1877)
Rev Gén de l'El	Revue Générale de l'Electricité Paris.
Roy Astron Soc, M N	Monthly Notices of the Royal Astronomical Society London.
Roy Soc Edinburgh, Proc (Trans)	Proceedings (Transactions) of the Royal Society of Edinburgh

Roy Soc, Proc	Proceedings of the Royal Society. (Series A is intended unless otherwise indicated) London
Silliman's Journal	See Am J Sci
Soc Belge Elect, Bull	Bulletin de la Société Belge d'Electriciens Brussels.
Soc Chem Ind, J	Journal of the Society of Chemical Industry. London.
Soc Franç Elect, Bull	Bulletin de la Société Française des Electriciens.
Soc Int Elect, Bull	Bulletin de la Société Internationale des Electriciens. Paris
†Soc Telegraph Eng, J	See Inst El. Eng., J
†Telegraphic J. .	See El Rev
Washington Acad Sci, J	Journal of the Washington Academy of Sciences.
†Wied Ann .	Wiedemann's Annalen der Physik (Third series of the Ann d Phys 1877–1899.)
Wien Ber .	Sitzungsberichte (Math -Naturwiss Klasse) der (kaiserlichen) Akademie der Wissenschaften in Wien Vienna
Zentralblatt	See Centralblatt
*†Z f Bel .	Zeitschrift fur Beleuchtungs-Wesen, Heiz- und Luftungs-Technik Berlin
†Z f Elektrot .	Zeitschrift für Elektrotechnik Vienna. (Continued as E u M)
Z f Phys .	Zeitschrift fur Physik. Brunswick (Continuation of Deut Phys Gesell, Verh from 1920)
Z f I .	Zeitschrift fur Instrumentenkunde. Berlin
*†Z f angewandte Elektrizitätslehre	Continued as Centralbl f Elektrot and amalgamated (1890) with E T Z
Z phys. Chem	Zeitschrift fur physikalische Chemie Leipzig.
Z techn. Phys	Zeitschrift fur technische Physik Leipzig.
Z wiss Phot	Zeitschrift fur wissenschaftliche Photographie, Photophysik und Photochemie Leipzig.

The following abbreviations have been used in referring to the titles of periodicals not included in the above list.—

Abh .	Abhandlungen
Acad, Accad., Akad	Academy, Académie, Accademia, Akademie
Am .	America(n)
Ann .	Annals, Annales, Annalen
Assn	Association, Associazione
Ber . .	Berichte
Brit	Britain, British
Bull	Bulletin
Chem .	Chemical, Chemistry, Chemie.
Chim	Chimie
Com	Committee
C r .	Comptes Rendus
Deut . .	Deutsche
Ecl .	Eclairage
El .	Electric(al), Electro-, etc, Electrique
Elektrot . .	Elektrotechnische, Elektrotechnik
Eng . .	Engineer(s), Engineering
Exp	Experimental
Franç. . .	Français(e)
Ges	Gesammte
Gen	General
Illum.	Illumination, Illuminating
Ing	Ingenieur, Ingegnere
Inst, Ist . . .	Institution, Institute, Istituto.
Int	International
J. . .	Journal, Jornal
Mem . .	Memoirs, Mémoires, Memorie.

Obs	Observatory, Observatoire.
Opt	Optics, Optical, Optique, Optik.
Phil	Philosophical.
Photog.	Photography(ic), Photographie(ique), Photographische
Phys	Physics, Physica, Physique, Physik, Physical, etc.
Physiol.	Physiology, Physiological, etc.
Proc.	Proceedings.
Psychol	Psychology, Psychological, etc.
R .	Reale
Rend	Rendiconti.
Rep	Repertorium
Rev.	Review, Revue.
Roy.	Royal(e).
Sci	Science(s), Scientific, Scienza(e), Ciencia(s)
Soc.	Society, Société, Società.
Trans.	Transactions.
Verh.	Verhandlungen
Wiss	Wissenschaft(liche).
Z.	Zeitschrift.
Ztg .	Zeitung.

PHOTOMETRY

CHAPTER I

HISTORICAL NOTE

The First Photometers.—Almost exactly two centuries ago, in 1729, Pierre Bouguer (1698–1758), Professor of Hydrography at Havre, laid the foundation of the science of photometry by his description, in an "Essai d'optique sur la gradation de la lumière" ([1]), of the earliest known form of apparatus designed for the comparison of the luminous intensities of two sources of light. The instrument he described is shown in Fig 1, which is taken from the fuller description contained in Bouguer's "Traité d'optique sur la gradation de la lumière," published posthumously by the astronomer, the Abbé de la Caille, at Paris in 1760

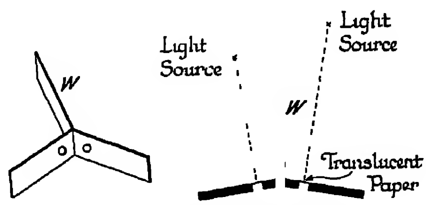

FIG 1 —Bouguer's Photometer

In the same year, 1760, there appeared at Augsburg the work of the versatile scientist and mathematician, Johann Heinrich Lambert (1728–1777), "Photometria, sive de mensura et gradibus luminis colorum, et umbræ" ([2]), which contained the enunciation of the fundamental laws of photometry, *viz*, the law of addition of illuminations, the inverse square law (already employed by Bouguer), the cosine law of illumination, the cosine law of emission, *etc* Lambert also described a form of shadow photometer ([3]) which is identical in principle with that later used and perfected by Count Rumford, the great American *savant*, philanthropist and statesman, Sir Benjamin Thompson, who in 1792 and 1793 wrote to Sir Joseph Banks letters in which he described "A Method of Measuring the comparative Intensities of the Light emitted by Luminous Bodies" These papers were communicated to the Royal Society ([4]) and contained a very detailed account of the photometer shown in Fig 2 It has been well pointed out by A P Trotter ([5]) that the ordinary text-book description of the Rumford photometer is most misleading Instead of one circular rod, as generally shown, Rumford employed

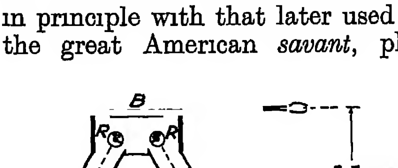

FIG. 2 —Rumford's Photometer.

two, R, R, of the form shown on the right in Fig 2 These were turned about their vertical axes until the shadows cast by the two lamps L_1 and L_2 were just in contact at the centre of the opaque paper screen B This was viewed by an observer, E, situated between the tables carrying the lamps, and equality of brightness was obtained by moving the lamps by means of cords operated by handles at H_1, H_2 (6)

The marked distinction which is generally drawn between the photometer of Bouguer and that of Lambert or Rumford seems to lack justification Both photometers, like those which have superseded them, depend on the comparison of the brightness of the two parts of a surface which are respectively illuminated by the two sources to be compared The dividing wall, W, of the Bouguer form may be regarded as equivalent to the shadow-forming object, R, of the Lambert form, since the function of both is to prevent the light given by one of the sources from reaching that part of the comparison surface which is illuminated by the other source The chief difference lies in the use of transmitted or reflected light Thus the Foucault photometer (7), which was used by Dumas and Regnault in their study of the lighting of Paris, has sometimes been referred to as a modification of the Lambert-Rumford form, whereas it would be described more correctly as a Bouguer instrument (Liebenthal "Prakt Phot," p 161, Trotter, "Illumination, *etc*," p 80, *etc*). It is remarkable that the instrument officially adopted in this

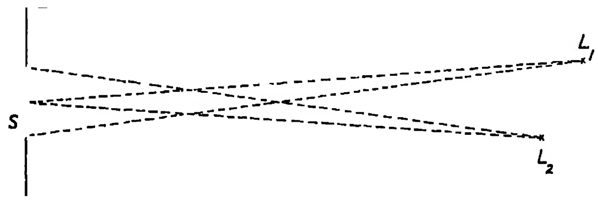

FIG 3—The Gas Referees' Photometer

country for gas testing until a few years ago was a simple modification of the original Bouguer photometer (8) Its general construction will be clear from Fig 3 S is a translucent surface and L_1, L_2 the lamps to be compared

From Rumford's time until the invention of the Ritchie wedge but little progress was made in the science of candle-power measurement W. A Lampadius in 1815 used an instrument in which the criterion of intensity was the number of sheets of a semi-transparent material, such as horn, which had to be placed before the eye to cause the source to disappear (9). Such an instrument, which may be called an "extinction photometer," had already been employed before Bouguer's time by François Marie (10), and although possessing little real claim to the title of photometer, it has repeatedly been revived in various forms (11), and is still in use for astronomical purposes where the ordinary methods of photometry are not always suitable (see p 426).

The Ritchie Wedge.—A distinct advance in photometry was made by William Ritchie, who, after many attempts to adapt the

differential thermometer to the comparison of light sources, devised the form of photometer head which is generally known by his name ([12]). In its first form, shown in Fig 4, this photometer was a modification of Bouguer's Two pieces of mirror, *CF*, *FD*, reflected the light from the sources to the translucent paper *EG* ([13]) Ritchie, however, also used the same form of head without the translucent paper, pasting opaque matt white paper over the mirrors, and comparing the brightness of these two surfaces directly. The prism, *CFD*, which is generally known as the Ritchie "wedge," has formed the basis of many instruments, each designed, generally, with the object of producing as sharp and fine a division as possible between the surfaces, for this was early recognised as being an important feature in accurate photometry ([14]) The forms of Ritchie wedge devised by Sir J Conroy ([15]), S. P Thompson and C. C. Starling ([16]), L Weber ([17]), A P Trotter ([18]), and Q. Majorana ([19]) are shown in Fig. 5, from which the particular features of each will be seen immediately In Yvon's form ([20]) the sources are arranged so that the light is incident normally at both surfaces of the wedge.

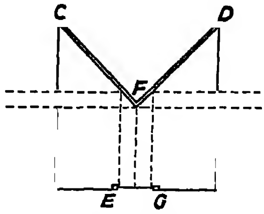

FIG 4 —Ritchie's Photometer.

Polarisation Photometers.—The work of Malus and Arago in the first years of the nineteenth century on the laws governing the intensity of polarised light made possible a new form of photometer

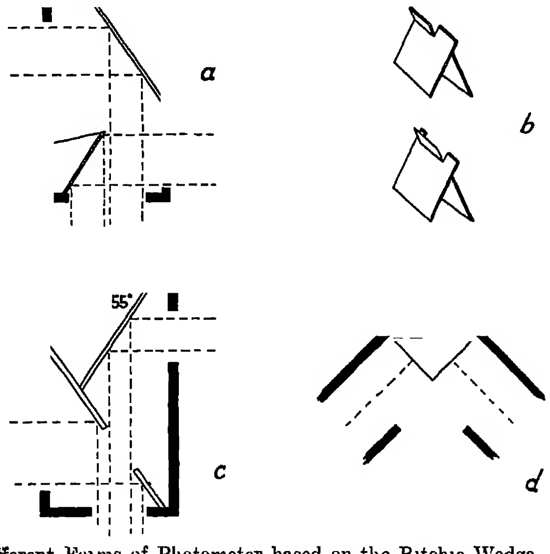

FIG 5 —Different Forms of Photometer based on the Ritchie Wedge (*a*) Conroy (*b*) Thompson-Starling (*c*) Trotter and Weber (Dachphotometer) (*d*) Yvon

which was independent of the inverse square law. The first polarisation photometer was that of D F. J Arago ([21]), in which he made use of double refraction prisms for both polariser and analyser. Later instruments of the same type were those of A Beer ([22]),

F. de la Provostaye and P Desains (23), F. Bernard (24), H. W. Dove (25), E. Becquerel (26), D. Salomons (27) and the modern instrument of F. F. Martens (28) Reflection from glass (see p. 31) was used as the means of polarising the light from one or both sources by J. Jamin (29), J Babinet (30), J. C. F. Zöllner (31), W. Crookes (32), and H Wild (33) The use of polarisation as a means for varying the intensity is very common in spectrophotometry, owing to the optical difficulties introduced by altering the distance between the photometer head and the source of light.

The Bunsen Photometer.—In the design of instruments depending on the inverse square law, little progress was made after the work of Ritchie until 1843, when R Bunsen first described the famous photometer head known by his name This photometer was originally designed for use in an exhaustive investigation on the chemical action of light which Bunsen was then carrying out in collaboration with Sir H E Roscoe (34) The Bunsen photometer, which is still frequently employed and which is, in fact, capable of exceedingly accurate work when carefully constructed and properly used, is fully described in Chapter VI of this book The chief modifications of it, the Joly block and the Lummer-Brodhun cube, are also described in detail in the same chapter The latter was first used by W Swan (35) and described by him some thirty years before its introduction by Lummer and Brodhun These workers, however, are undoubtedly responsible for the use of the contrast principle which has enabled still greater accuracy to be obtained in modern photometry

Standards of Light : The Candle.—The gradual improvement in the instruments available for photometric measurement naturally resulted in a growing dissatisfaction with the hitherto accepted standard of luminous intensity, the candle, for it was soon found that this standard was not reproducible to the accuracy of measurement even when the composition, form and rate of burning were carefully specified (36). Although once officially adopted in several countries and by various testing bodies (37), candles have now been entirely superseded as practical standards of light For instance, the British Parliamentary candle, defined in the Metropolitan Gas Act of 1860, was discarded in favour of the pentane lamp by the Metropolitan Gas Referees in their *Notification* for 1898.

The Carcel Lamp.—The first standard to supersede the candle was, however, the Carcel lamp, a modification of the Argand (38), in which a clockwork pump was used to supply the wick with colza oil at a given rate (see Fig 6) The standard luminous intensity was that given when the rate of oil consumption was 42 gm per hour Small departures from this standard rate were allowed for by a simple proportional rule, the actual consumption being conveniently measured by using the lamp on a form of balance and noting the time in which it lost 10 gm. in weight This lamp, devised by Carcel in 1800 (39), was used by Dumas and Regnault in their photometric work (40) but the difficulties attending its use are very great, and different observers are unable to obtain results either consistent among themselves (41) or in agreement with one another to the same degree of accuracy as that attainable with other flame standards.

The actual candle-power under the standard conditions has been variously estimated at from 9 4 to 10 international candles (42).

Kerosene lamps in various forms have been proposed as standards or as sub-standards at different times (43), but have never been found to be wholly satisfactory.

A coal-gas flame of given dimensions was proposed by H Giroud (44), but this was inferior as regards constancy to the light given by a specified area of the brightest part of a gas flame. This form of standard, originally proposed by W W. Fiddes (45) and later by S Elster (46), was developed by Methven, who found that the brightness was constant when the flame was burning under specified conditions in an Argand burner (47) After much experiment this standard also was found to be unsatisfactory (48) and subsequent modifications did not improve it sufficiently to make it suitable for use as a standard of light (49)

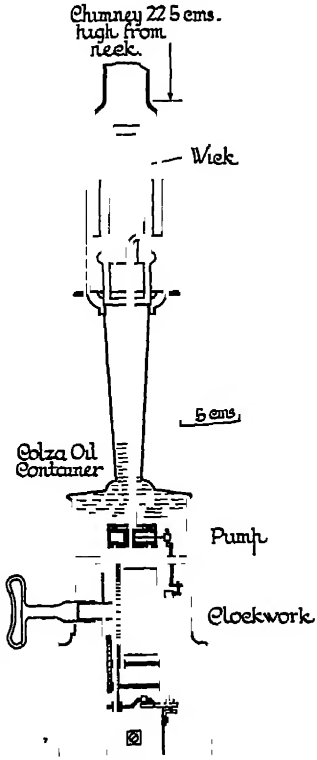

FIG 6 —The Carcel Lamp

The Pentane Lamp.—The only practical flame standards are those in which a volatile hydrocarbon of definitely known chemical composition is burnt in a lamp of carefully specified dimensions. The first lamp of this kind was made in 1877 by A. G Vernon Harcourt (50). It burnt a mixture of pentane vapour (C_5H_{12}) and air from a wickless burner, and had a luminous intensity of one candle After undergoing several modifications (51), including forms in which a wick was used to convey liquid pentane into the burner tube, although the wick did not enter the flame or even approach the top of the tube (52), the lamp was completely redesigned in a larger form so as to have a luminous intensity of ten candles (53), and this, as has been said already, was adopted in 1898 as the official standard for gas testing in London This lamp is described in Chapter V Lamps burning pentane were devised also by W J. Dibdin [ten candles] (54) and by J Simmance [two candles] (55)

The Hefner Lamp.—Other hydrocarbons that have been used are (i) benzol, either alone (56) or mixed in definite proportions with ethyl ether (57), or with ethyl alcohol (58), (ii) naphthalene (59), (iii) acetylene (60), and (iv) amyl acetate ($C_7H_{14}O_2$). The last-named fuel is used in the lamp which was first devised in 1884 by F. von Hefner Alteneck (61), and which is still the official standard of candle-power in Germany having, in 1893, superseded (62) the *Vereinskerze* set up in 1868 by the Deutsche Verein von Gas- und Wasserfachmannern (63) It was also adopted as the custodian of the *bougie décimale* by the International Electrotechnical Congress at Geneva in 1896 (64). A description of the lamp is given in Chapter V.

Other Proposed Standards.—In spite of very careful specifications of constructional details and numerous elaborate determinations of the correction factors to be applied on account of atmospheric conditions, none of the flame standards has proved adequate to the needs of modern photometric practice It has repeatedly been proposed to construct some form of standard depending upon the radiation given by a specified area of some surface at a given temperature Several such standards are described in Chapter V ([65]), but while some of these show promise of leading eventually to the desired end, none can be said at present to be any more satisfactory than the flame standards

The same is true of the absolute physical standards which have been proposed, *i e.*, standards defined as a given amount of energy in the form of radiation having a certain spectral distribution ([66]), for these cannot be of practical value until much more progress has been made in what may be termed broadly " physical " photometry

A standard which cannot well be classed with any of the others that have been proposed is that furnished by a definite area of a discharge tube containing helium gas ([67]) The light given by such a tube has, however, a discontinuous spectrum, and this standard has therefore never been developed.

The Electric Lamp as Standard.—The proposal to use an electric lamp as a standard has frequently been made ([68]), but as it is not at present possible even to specify an electric lamp to the extreme accuracy required for a standard of light, much less to manufacture it, it is impossible to obtain a true standard in this way

It is, however, possible to base the unit on the known candle-powers of one or more individual lamps, and this is the position of the international candle at the present time, the International Commission on Illumination having, in 1921, adopted the unit based on comparisons of certain electric incandescent lamps in the chief national standardising laboratories of the world ([69]) This unit will be dealt with further in Chapter V

Other Problems.—Side by side with the development of accurate means for measuring the candle-power of sources of light, and the search for a convenient standard reproducible at least to the accuracy of measurement, other cognate problems have had to be studied as they arose The comparison of sources giving lights of different colours was, from the first, found to be an operation of special difficulty, and the means which have been adopted for overcoming this difficulty are described in Chapters VIII and IX of this book

The gradual improvement which has taken place in the methods of physical photometry, leading to the production of the " instrumentum thermometro analogus " of Lambert, will be found briefly described in the chapter specially devoted to this branch of the subject, so that it need not be further mentioned here

The measurement of illumination, as distinct from the measurement of luminous intensity, is a branch of photometry of comparatively recent growth The first illumination photometer was constructed by Sir W H Preece in 1883 ([70]), and since that time the instruments produced for this special class of measurement have been exceedingly numerous The rapid development of the lighting art, and its transference from the domain of pure empiricism to that

of scientific method, which has been a marked feature of the last decade of engineering progress, have tended to emphasise more and more the importance of this branch of photometric practice.

The study of illumination (as distinct from the methods of producing it) and the rapid development of means for redistributing the light given by a source, have in the last few years brought about a radical change in the rating of illuminants. It is the total light output, rather than the candle-power in a single direction or group of directions, which is chiefly of interest to the illumination engineer, and the result has been a rapid development of methods of measuring luminous flux. The use of integrating photometers, generally of the Ulbricht sphere type, is now the rule rather than the exception in photometric laboratories, and Chapter VII is therefore devoted to this branch of photometry

The Future.—Progress in the future lies mainly in two directions : (i) the production of instruments and standards of greater precision for use in the laboratory where accurate measurement is called for, and (ii) the simplification of photometric apparatus, particularly the portable photometer, without too much sacrifice of accuracy, so that the measurement of illumination may become as simple and as common an operation as the measurement of temperature or length, and require as little special training or technical experience. Considerable progress has already been made in the latter direction, but one of the principal difficulties, that of a convenient source of supply of electric current for the comparison lamp, is still unsolved. Both the alternatives at present available, *viz* , a lead accumulator or a dry cell with indicating instrument and rheostat are obviously unsatisfactory, but no attempt to use a self-luminous material as a comparison surface has met with any success up to the present ([71])

In precision photometry, unless some visual criterion still more sensitive than that of the contrast field can be discovered, progress must necessarily lie in the use of physical methods. Already these have been shown to be capable of detecting differences of illumination quite inappreciable to the human eye ([72]), and it seems likely that the principal field of usefulness of the physical photometer, at any rate in the immediate future, will be as a detector of minute differences rather than as a measurer of integral illumination, thus playing a part analogous to that of the galvanometer in the measurement of electrical resistance It must, further, be recognised that physical measurement can, strictly, be applied only to the comparison of lights of identical spectral composition The comparison of lights of different colours depends on the relative sensitivity of the eye to equal amounts of energy in different parts of the spectrum, and not only is there no known physical instrument which can conveniently be made to imitate the eye in this respect, but, which is still more embarrassing, the eyes of normal-sighted persons differ from one another quite appreciably in their colour sensitivity curve It follows that heterochromatic comparisons must be based ultimately on a *convention* This done, physical photometry may be called in to give, possibly, the same degree of accuracy as is attainable by its use in homochromatic comparisons

There is no definite limit which can be set to this accuracy, but

it has to be remembered that the precision of the other measurements involved must be correspondingly increased. Thus, for example, in order to attain a certainty of 1 part in 10,000 in measuring the candle-power of an electric lamp by means of a photometer based on the inverse square law, the voltage must be measurable to 1 part in 40,000, and the effective position of the light source must be known within 0 1 mm if the lamp be at a distance of 2 metres from the photometer It is probable, therefore, that progress in the direction of increased precision in photometric measurement will be slow. The accuracy at present attainable is ample for commercial purposes, but barely sufficient for scientific research. Experience in other branches of measurement (*e g* , that of power, whether electrical or thermal) shows, however, that what is sufficient to-day may lag seriously behind even commercial requirements in ten or twenty years' time. Progress, therefore, is essential Increased precision must be attained so that, in all that concerns the production and utilisation of light, progress may not be hindered nor development retarded ([73])

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(62) Centralblatt f d Deutsche Reich, 20, *1893*, p 124

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(69) C I E , Proc , Paris, *1921*, p 40 , Illum Eng , 14, *1921*, p 169 , Inst Gas Eng , Trans , *1920–21*, p 627 , Gas J , 156, *1921*, p 226 , Rev Gen de l'El , 10, *1921*, p 82.

(70) Roy Soc , Proc , 36, *1884*, p 270

(71) See note (45), p 374

(72) See notes (61), p 337, and (66), p 338

(73) "All the natural sciences aim, then, at becoming exact sciences and become exact through the making, correlation and reduction of measurements. Any branch of natural science without measurements is not above the qualitative stage The number and degree of precision of the measurements in a branch of science is a gage of the extent to which that branch has become exact " (A E Kennelly. Illum Eng. Soc N Y Trans , 6, *1911*, p 580)

CHAPTER II

RADIATION

The Nature of Light.—Photometry is, as its name implies, the measurement of light, and it is clearly desirable that a book professing to deal with any branch of metrics should begin with a brief account of the nature of that which is to be measured. This chapter will therefore be devoted to a general description of the theories at present held regarding the nature and properties of radiation in general, and in particular that form of it, termed "luminous radiation," which is capable of producing in the human eye the sensation of vision It will also deal briefly with the principal phenomena accompanying the emission, propagation or reception of this radiation, especially those made use of in the various methods of photometry

Light, like radiant heat, Rontgen rays (X-rays), and the electromagnetic waves used in wireless telegraphy, is a form of radiant energy, that is to say it is that form of energy which is transmitted through space in straight lines, and without the intervention of matter, at a constant velocity of about $2\,9986 \times 10^{10}$ cm per sec ([1]) This velocity, which is apparently one of those invariable fundamental quantities which have been termed "constants of nature," will be referred to throughout this chapter as c

Until about twenty years ago it was thought that a complete explanation of the phenomena of radiation was to be found in the wave theory of light It has now been shown, however, that this theory alone cannot satisfactorily account for all the facts, and an additional hypothesis has therefore been put forward At present this hypothesis, termed the quantum theory, is in a more or less experimental stage as, while it affords a satisfactory explanation of some of the phenomena which could not be explained on the old theory, it does not appear to fit in with certain of the known facts for which the simple wave theory was perfectly adequate. Nevertheless, its success in certain problems of radiation has been so striking that it must be considered briefly in a later portion of this chapter, though for historic reasons, as well as for the sake of convenience, the wave theory in its original form will be considered first.

The Wave Theory of Light.—About the year 1678 the Dutch philosopher Huyghens first propounded a theory of light in which it was supposed that a luminous body acted as a source of disturbance in a hypothetical all-pervading medium called the (luminiferous) ether. This disturbance was imagined to travel through the ether in the form of waves, which, on reaching the eye, produced the sensation of vision. On this theory light waves travel in space with the velocity c and carry energy from the body which produces them to that by which they are absorbed ([2]).

The essential characteristic of a wave motion is that, by means of a periodic disturbance transmitted continuously from one portion

of a medium to the next portion in the line of propagation, energy is carried from one place to another without any motion of translation on the part of the medium or of any portion of it The familiar example of a sheet of water, one end of which is agitated by a regular up-and-down movement of a piece of wood, will serve to illustrate these fundamental characteristics of a wave system Ripples will be formed by the wood and will travel across the water with a certain velocity of propagation which is almost independent of the size of the ripples That the actual particles of water have no motion, other than that of a simple vertical oscillation, is shown by the fact that a floating cork merely bobs up and down and is not carried along with the ripples That energy is transmitted through the water by means of the ripples is shown by the fact that a piece of wood floating at the far end of the water will be caused to oscillate in synchronism with the movement of the wood causing the disturbance

Effect of Frequency and Amplitude of Waves.—In the last paragraph the general characteristics of a wave motion were described. It now remains to consider in what ways different kinds of waves may be distinguished one from another, and at once there are two prominent characteristics which claim attention. These are (*a*) the amplitude, or extent of the oscillation , and (*b*) the frequency, or number of oscillations executed per second. With regard to the first, it is at once apparent that the energy conveyed by any given wave will depend on the amplitude of that wave, and it may be said at once that in the case of the ether waves the energy transmitted is proportional to the square of some vector which, in the absence of any definite knowledge of the method of propagation of energy waves in the ether, may be termed for convenience the amplitude of vibration For, in the simplest example of oscillation, the case of a particle executing free vibrations in simple harmonic motion, $x = a \sin \omega(t-\theta)$ where x is the displacement, $2\pi/\omega$ the period (periodic time), and a the amplitude Hence the total energy, which is equal to the kinetic energy $(\frac{1}{2}m\dot{x}^2)$ when $x = 0$, is $\frac{1}{2}m\omega^2a^2$

With regard to the second characteristic, in the case of ether waves the frequency determines the kind of effect which they will have on our senses In fact, the different kinds of radiant energy enumerated on p 16 above are carried by ether waves which differ only in frequency If f be the number of vibrations per second (3), then, when f is below about 10^{12} per second the waves are quite incapable of affecting our senses at all These are the waves used in wireless telegraphy When f lies between about 10^{12} and $4 5 \times 10^{14}$ the waves are capable of producing on our bodies the sensation of heat, but they cannot affect our eyes in such a way as to produce the sensation we call vision, though it is possible that the night vision of nocturnal beasts is due in part to the sensitivity of their eyes to waves for which f lies below the limit of sensitivity of the human eye To waves for which f is above the limit of $4 0 \times 10^{14}$ and does not exceed about $7{\cdot}5 \times 10^{14}$ the human eye is variously sensitive (4), and the effect produced on it differs both in kind and degree according to the value of f

The variation in *degree* will be more fully considered in the next chapter, which deals with the behaviour of the eye as a receptor of luminous radiation. It is sufficient for our present purpose to say

that the eye is not equally sensitive to light waves of all frequencies ; in fact, the degree of its response to a given amount of radiant energy, even when this is conveyed by waves within the limits of frequency stated above, depends on the absolute frequency of the radiation or, what is the same thing, on the colour of the light. For the variation in the *kind* of effect produced on the eye by waves of different frequencies gives rise to the sensation which we term " colour " Waves for which $f = 5 \times 10^{14}$ give the sensation of red, those of rather higher frequency form what appears to us as yellow light, and so on through green and blue to violet, which is the sensation produced by light waves of the highest frequencies which the eye is capable of appreciating at all. Waves of still higher frequency (the ultra-violet) cannot affect the eye in such a way as to give the sensation of light though they may affect it profoundly in other ways These waves are, however, capable of producing chemical changes in a photographic emulsion and are termed " actinic " Waves of still higher frequency, from 3×10^{17} to 2×10^{19}, are produced by special means, and are the Rontgen or X-rays which, on account of their power of penetrating the less dense forms of matter, are used for photographing, on plates specially sensitive to waves of this frequency, the forms of denser objects concealed within a medium which, although less dense, is nevertheless opaque to luminous radiation

Wave-Length and Wave Number.—It has long been customary to express the periodicity of radiation in terms of " wave-length " (λ) rather than " frequency " (f) The former quantity is the distance (in the line of propagation of the radiation) which separates consecutive points undergoing the same " displacement " ([5]) For example, in the case of ripples on water it is the distance between consecutive points having the same absolute displacement and the same direction of motion, such as A and B (Fig 7) From this definition it follows that for light *in vacuo* $\lambda f = c$ Since, as will be seen later, the velocity of light varies according to the medium through which it is propagated, and since the frequency of vibration cannot alter, λ must depend on the medium It follows that the frequency is more fundamentally characteristic of a particular kind of radiation than is its wave-length and it will therefore be used generally throughout this book

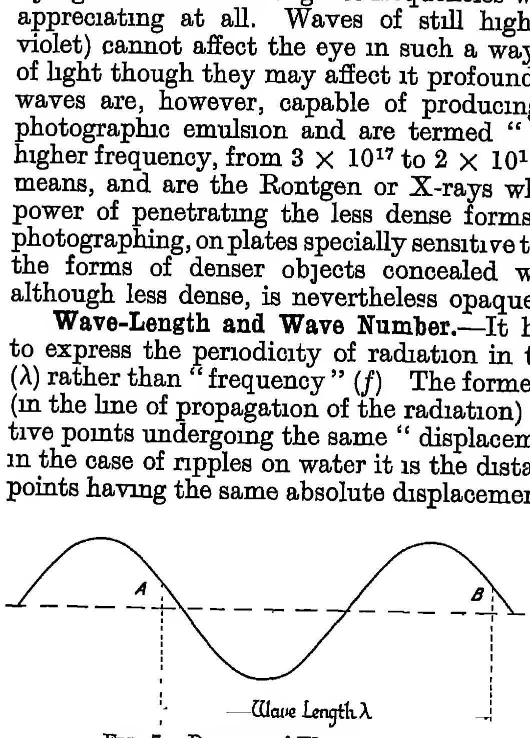

FIG 7—Diagram of Wave-motion

It so happens that the characteristic of light which can be measured most conveniently and accurately is its wave-length, and, in fact, the accuracy of this measurement ([6]) far surpasses that of the velocity of light which is known to an accuracy of only about 1 part in 10,000 Hence the use of values of frequency calculated from the relation $f = c/\lambda$ has very little advantage over the use of the simple reciprocal of λ This reciprocal is termed the wave-number ν It is directly proportional to the frequency and can therefore be used in what follows exactly as frequency would be used It is, moreover,

independent of any change which may be brought about by a more accurate determination of c

Applications of the Wave Theory: The Inverse Square Law and Cosine Law.—From the outline of the theory given above its agreement with the principal observed facts concerning radiation is readily demonstrated For instance, the two fundamental laws of photometry, the inverse square law and the cosine law, follow at once if a luminous point be considered as the source of a system of spherical waves diverging from it as centre. For the area of any such wave as it travels outwards from the source must increase as the square of its radius, and since its energy must be regarded as uniformly distributed over its surface, the surface density of this energy must vary inversely as the square of the radius of the wave, that is, of the distance from the source Similarly, since the direction of motion is always perpendicular to the wave front, it follows that an elementary area can only receive energy in proportion to its area projected in that direction (Fig 8), that is, the surface density of the energy received by any such area is proportional to $\cos\theta$ where θ is the angle between the normal to the surface and the direction of propagation of the incident wave.

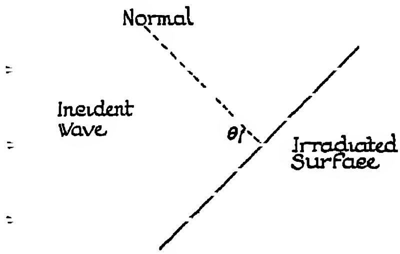

FIG 8 —The Cosine Law of Irradiation or Illumination

Reflection and Refraction on the Wave Theory. — The well-known laws of reflection and refraction are also in agreement with the wave theory For (Fig 9) let AA' be the trace of the surface of separation of two media, and AB the trace of a plane wave surface incident at it.

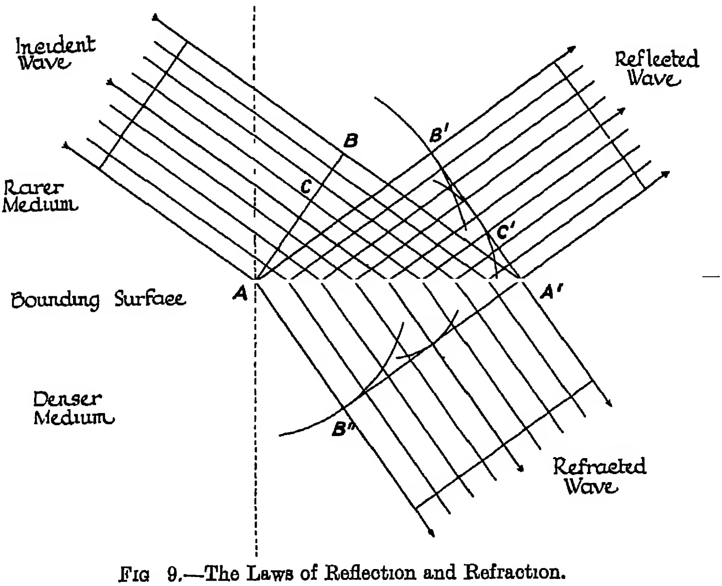

FIG 9.—The Laws of Reflection and Refraction.

(This may, for convenience, be regarded as a portion of a spherical wave emanating from a very distant source) Let it be supposed that the plane of the paper is perpendicular to both these surfaces Then each successive portion of the separation surface AA', as soon as the incident wave surface reaches it, is assumed to become the origin of two new waves, one in the upper medium (the reflected wave) and the other in the lower medium (the refracted wave) [7] If BA' be drawn perpendicular to AB, BA' is the direction of propagation of the incident wave, and if AB' and $A'B'$ be drawn equal respectively to $A'B$ and AB, then it is clear that when the reflected wave originating at A has just reached B' along the path AB', the original wave from B will just have reached A' Similarly, the time taken from any point C on AB to a corresponding point C' on $A'B'$ will be found to be the same Hence $A'B'$ must be the trace of the reflected wave surface at the instant the original wave reaches the point A'

This new wave surface is the plane envelope of spheres having their centres at A, A' and all intermediate points, and their radii equal to the distances of these points from the line $A'B'$. It will, therefore, be perpendicular to the plane of the paper Hence it follows that the incident and reflected rays (perpendiculars to the wave fronts) are in the same plane with each other and with the normal to the surface of separation, and further, that they make equal angles with this normal on opposite sides of it.

Now let a point B'' be taken below AA', such that (i) $AB'' = BA'/n$, and (ii) $A'B''$ is perpendicular to AB''. Then, if it be supposed that the velocity of propagation of the wave in the lower medium is $^1/n$ times that in the upper medium, it follows, as in the last paragraph, that $A'B''$ is the trace of the refracted wave surface at the instant the wave reaches A' Again it follows that the incident and refracted rays are in the same plane with each other and with the normal to the surface, and that in this case the sines of the angles made with this normal are in the ratio of n to 1 or, if i and r be the angles of incidence and refraction, $\sin i/\sin r = n$

An important case arises when light passes from an optically denser to a rarer medium so that $n < 1$ If, in this case, i be greater than $\sin^{-1}n$ the equation of refraction gives r an impossible value. The light, in fact, does not emerge at all, but is reflected at the bounding surface according to the ordinary law of reflection This phenomenon is known as "total reflection" and is much used in optical instruments For instance, in the constant deviation prism shown in Fig 14 (p 24) the light at B is totally reflected at the glass-air surface because $i = 45°$, whereas n for glass to air is 0 67, so that $\sin^{-1}n = 42°$

It will be noticed that a new property has now been ascribed to the mechanism of wave propagation, *viz.*, that the velocity of propagation is inversely proportional to the refractive index of the medium, as measured by the deviation suffered by a ray of light on entering that medium This assumption has been fully verified by direct experiment [8] and so forms one of the great triumphs of the wave theory of radiation

Lenses and Prisms.—The deviation suffered by light on passing from one medium to another is one of the most important of optical

phenomena forming, as it does, the basis of the action of all lenses and prisms Let ABC (Fig. 10) represent a right section through a

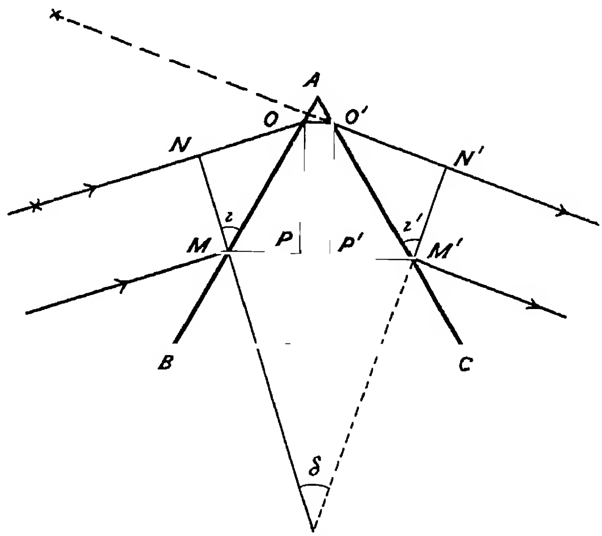

Fig 10 —The Passage of Light through a Prism

triangular glass prism, and let MN represent the front of a plane wave perpendicular to the paper and travelling in the direction NO By the previous paragraph the position of the wave front inside the glass will be parallel to PO where $NO = n\ MP$ On emergence into air again the reverse action takes place and the final wave front is parallel to $M'N'$ where $O'N' = n\ P'M'$ The angle through which the wave front has been turned by the action of the prism, *i e.*, the "deviation," is δ It is clearly equal to $i + i' - A$, where A is the "refracting" angle of the prism, and i, i' represent respectively NMO and $N'M'O'$ the angles of incidence and emergence of the light Since $\sin i = n \sin POM$, and $\sin i' = n \sin P'O'M'$, while

$$POM + P'O'M' = A$$

i e,

$$\sin^{-1}\left(\frac{\sin i}{n}\right) + \sin^{-1}\left(\frac{\sin i'}{n}\right) = A$$

it follows that δ can be found when either i or i' is known For the special case $i = i'$, δ is a minimum, and is equal to $2i - A$, where $2 \sin^{-1}\left(\frac{\sin i}{n}\right) = A$, *i e*, $\sin i = n \sin \frac{A}{2}$, so that

$$\delta = 2 \sin^{-1}\left(n \sin \frac{A}{2}\right) - A.$$

It will be clear that an object seen through a prism will appear to be shifted from its real position and will seem to lie on the backward continuation of the emergent ray It follows that if an object be seen through a prism of the type shown in Fig. 11 it will appear

to be doubled, for to an eye at E it will seem as if the object lay on the backward continuation of each of the lines EM, EM'. This form of prism, known as a Fresnel biprism is used in some photometers (see, for example, p 159)

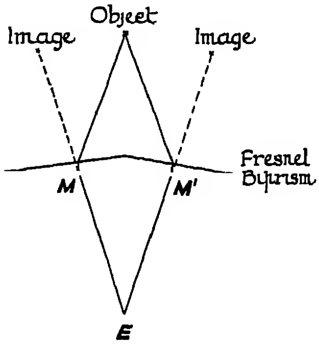

FIG 11 —The Action of a Fresnel Biprism

The effect of a lens can most easily be explained by reference to Fig 12. For, suppose AMB to be the spherical surface, radius r_1, of a refracting medium of index n Then, if a wave diverge from a point P, at the instant it reaches A it will have the form $AN'B$, instead of the form ANB which it would have had if unrefracted, MN being equal to nMN' If $PN = u$, the new radius of curvature of the wave s is given by the equations $r_1DM = uDN = sDN' = \frac{1}{2}AD^2$ if AD be assumed small with respect to u and r_1 (9) On eliminating DM and DN by the aid of the relationship $MN = nMN'$, it is found that

$$\frac{1}{u} - \frac{n}{s} = \frac{n-1}{r_1} \quad . \quad . \quad \text{(i.)}$$

If now the light emerge again through a surface of radius r_2 in the opposite direction, the radius of curvature v of the emergent wave is similarly found to be $\frac{n}{s} - \frac{1}{v} = \frac{n-1}{r_2}$ . . . . (ii)

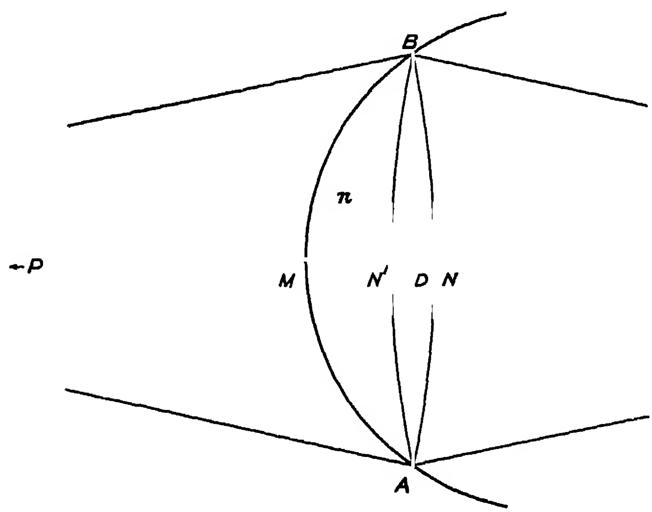

FIG 12 —Refraction at a Spherical Surface

Hence $$\frac{1}{u} - \frac{1}{v} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad . \quad \text{(iii)}$$

This equation shows at once that light radiated from a point at distance u from a double convex lens having surfaces of radii r_1 and

r_2 and refractive index n, is brought to a point again at a distance v on the opposite side of the lens Thus, for various values of u the image can be kept stationary either by moving the lens (change of v) or altering its curvature (change of r_1 or r_2) or both For a fixed lens, such as those used in optical apparatus, the reciprocal of the constant $(n - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$ is called the focal length f and it is clearly the distance from the lens of the image of a distant point for which $u = \infty$. The position of this image is called the "principal focus" of the lens

A special case of some importance in photometry is the apparent shift in the position of an object seen through a plate of glass or other refractive medium of thickness d In this case $r_1 = r_2 = \infty$, and the equations for finding the relation between u and v become

$$\frac{1}{u} - \frac{n}{s} = 0 \text{ and } \frac{n}{s+d} - \frac{1}{v+d} = 0,$$

so that $u - v = d\left(1 - \frac{1}{n}\right)$. Thus the effective distance of a light source from a surface is shortened by the insertion of a plane sheet

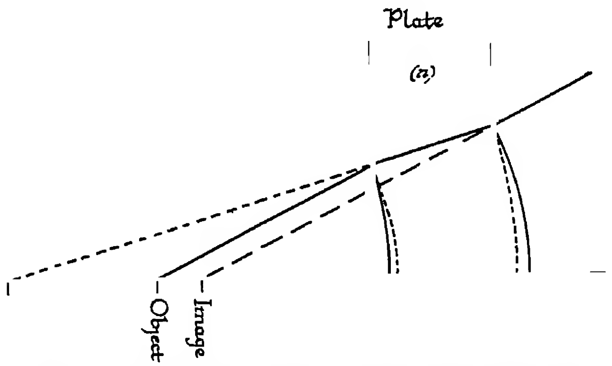

Fig 13 —Apparent Shift of an Object seen through a Plate

of refractive medium, the amount of shortening being equal to the thickness of the sheet multiplied by $(n - 1)/n$ (see Fig 13)

Unless the requirement mentioned above, that the "aperture" (AB, Fig. 12) should be small compared with u and v, is adequately met, the rays from one point do not converge accurately to a single second point, but intersect one another somewhere along a short portion of the line forming the axis of the system (PMN is the axis in Fig 12) This departure of the rays from the stigmatic condition is termed "spherical aberration," and is a serious defect when a clearly-defined image is required It can only be corrected by the use of non-spherical refracting surfaces, or by using a combination of lenses [10]

It should be noticed that a paraxial ray of light (i e , a ray whose inclination to the axis of the optical system is small) passes undeviated through the *centre* of a thin lens, since to a first approxi-

mation the portions of the lens surfaces at which the light enters and leaves may be regarded as parallel, and therefore the deviation at entrance is compensated by the equal and opposite deviation at emergence.

It follows from this fact that the linear dimensions of an object and its image formed by a lens are in the ratio u v, so that the areas are in the ratio $u^2 \cdot v^2$.

Reflection at a spherical mirror may clearly be treated in a similar manner to refraction by a lens, putting $r_2 = r_1$, and $n = -1$

Dispersion.—In the last three paragraphs it was tacitly assumed that the light was *homogeneous*, that is, that it consisted entirely of waves of a single frequency. Most light ordinarily met with, however, is composite, and may be regarded as a mixture, in varying proportions, of waves of all frequencies to which the eye is capable of responding [11]

It can readily be demonstrated by experiment that the ratio $\sin i/\sin r$, or n, generally termed the index of refraction of the second medium with reference to the first, is not the same for waves of all frequencies In passing from air to glass, for instance, n increases as the frequency increases, so that it follows that the velocity in glass must be less for violet light than for red light, for in space (and, very nearly, in air) the speed of propagation is the same for waves of all frequencies, otherwise a new star, or an occulted star, would gradually change colour after its first appearance [12]

An important application of this variation of velocity in glass is to the resolution of a composite light into its various components by means of a glass prism A convenient form of prism used for this purpose in the spectrometer is that shown in section in Fig 14, where the composite light entering at A is spread out by reason of the different refractive indices of the glass for the waves of different frequencies After total reflection at B and refraction at C, the light emerges in the form of a coloured band, known as a "spectrum," which extends from red to violet if all colours are present in the original beam

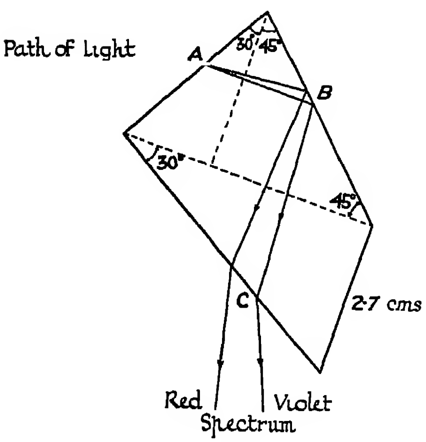

FIG 14 —The Constant Deviation Prism

This variation of refractive index with frequency is frequently made use of in optical and photometric (especially spectrophotometric) apparatus, but allowance for it has also to be made occasionally when analysis of the light is not desired Clearly, no spreading can take place when the light is incident normally to the surface, for then $i = r = 0$.

From the presence of the quantity n in formula (iii.) of p. 22, it will be clear that the position of the image formed by a lens depends

on the frequency of the light, and in the case of composite light the image of a point source is spread out into a line of images ranging in colour from violet (nearest the lens) to red. This spreading is termed "chromatic aberration," and can only be corrected by the use of a combination of lenses.

Interference.—Another phenomenon which can be accounted for satisfactorily on the wave theory of light is interference. This phenomenon may, in brief, be looked upon as the effect of the superposition of two or more waves so as to produce a resultant of different amplitude It is clear that, referring again to the illustration of the ripples on water, if two sets of ripples of the same amplitude and frequency be arranged to meet, so that at any instant the crests of either set alone would occupy exactly the same positions as the troughs of the other set alone, the result of the superposition of the two sets will be complete extinction, and if two sets of ripples cross, regions of undisturbed water will be seen at the positions where such extinction takes place (13). In other positions, however, where crest coincides with crest, and trough with trough, the ripples will be approximately doubled in amplitude By suitable means, one of which will be described below, similar effects can be detected in the case of light waves Both extinction and reinforcement must be regarded as manifestations of "interference," and perhaps the most satisfactory definition of this term for the purpose of this chapter is that given by Schuster, as follows (14). "If the observed illumination of a surface by two or more pencils of light is not equal to the sum of the illumination of the separate pencils, we say that the pencils have interfered with each other and class the phenomenon as one of 'interference'"

The Interference Grating.—One of the most frequently employed pieces of apparatus involving interference is that known as a "grating." This consists, in its ordinary form, of a glass plate on which have been ruled with a fine diamond point a large number of exceedingly fine lines at regular close intervals (Rowland's gratings

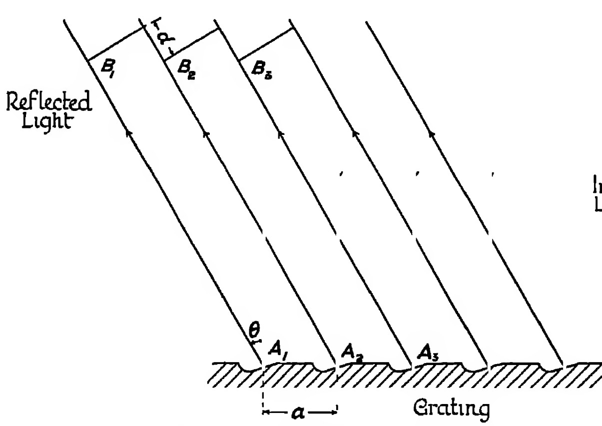

FIG 15—The Diffraction Grating

have about 14,400 lines per inch) The principle of its action will be clear from Fig 15, which represents a section through the grating,

it being understood that the whole effect depends on exact *similarity* between the individual lines or corrugations Let A_1, A_2, A_3 be the trace of the incident wave front at the instant it reaches the similar points A_1, A_2, A_3 . of the grating. Then A_1, A_2, A_3 become the centres of new waves, and, on account of the similarity at these points, all these waves will be equal in amplitude at equal distances from their centres Considering the direction A_1B_1, it is clear that, when the wave from A_1 has travelled as far as B_1, that from A_2 will have reached B_2, and so on The waves thus follow in regular succession at a distance of separation equal to the projection of B_1B_2 on B_1A_1, that is, d The value of d depends on the distance between the lines on the grating, and the direction of the reflected waves considered It is clearly equal to $a \sin \theta$, where a is the separation of the lines and θ the angle which the reflected light makes with the normal. If it so happen that the distance d is half the length of a wave (or an odd number of half wave-lengths), destructive interference will take place between each pair of waves, and at a considerable distance from the grating darkness will result If, however, in the direction of the reflected waves considered, d be equal to one or any whole number of wave-lengths, then all the sets of waves will reinforce one another and brightness will result in that direction, for every other set of similar points of the grating will behave like the set A_1, A_2, A_3 . .

This clearly provides a method for measuring the value of f, for if a and the smallest value of θ for maximum light be known, and λ is the wave-length, $f = c/\lambda = c/a \sin \theta$. It also provides a means for analysing a composite light, for since f is inversely proportional to $\sin \theta$, it is clear that the reflected light will be analysed in such a way that the maximum brightness for light of lower frequency (red) will appear nearest to the direction of the incident light, while that for the higher frequency waves (violet light) will appear nearly twice as far from this direction Thus a grating may be used to produce a spectrum in which the different colours of a composite light are arranged in the order of their frequencies according to a known law of spacing (equiangular, since $\sin \theta \simeq \theta$ when θ is small). In this respect its spectrum is superior to that of a prism in which the spacing is dependent upon the kind of glass employed (see Chapter IX., p. 276)

It is now possible to see why the effect of the elementary wave surfaces originating at all the points between A and A' in Fig 9 of p 19 could be considered as practically confined in the aggregate to the regions forming the envelope represented by the line $A'B'$, for, when all these waves are taken together, it is easy to see that at regions not on this envelope interference will take place, and a careful analysis shows that darkness will, in fact, result everywhere except on the "wave-front" $A'B'$ Lack of space prevents any detailed treatment of the subject in this book, and a treatise on the theory of light should be consulted ([15])

Interference Bands in Thin Films.—There is one phenomenon of interference which is made use of in some types of photometer (see p 184), *viz.*, the alternate light and dark bands produced when light passes through, or is reflected from, a thin layer of air between two glass prisms Let ABC (Fig 16) be a ray incident at such an

angle that BC is close to the direction of total reflection, then this ray is partly reflected and partly refracted at C, the refracted ray CD making with the interfaces of the prisms an angle r which is very nearly 90° The ray CD is partly transmitted along DG, and

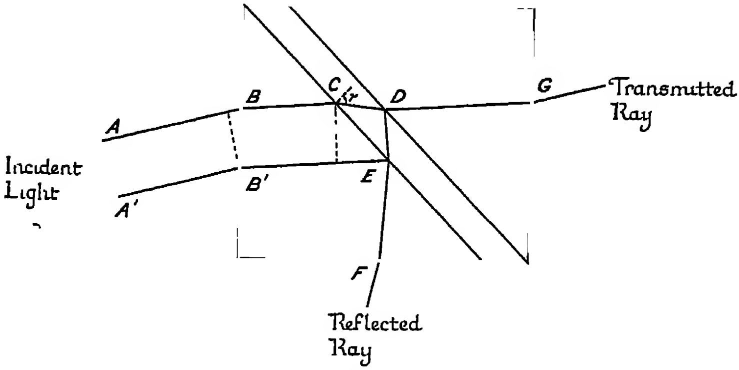

FIG 16 —Talbot's Bands

partly reflected along the path DEF. Now there is a ray $A'B'$, parallel to AB, which is refracted along the path $B'E$, and is partly reflected at E If the path difference between the rays reflected at D and at E be an odd multiple of a half wave-length, i e , if $2CD \cos^2 r = (m+\frac{1}{2})\lambda$, where m is an integer, then destructive interference will take place and the total intensity in the direction EF will be diminished It is clear that for another pair of rays very slightly inclined to AB and $A'B'$ the path difference will be m or $(m + 1)$ wave-lengths, so that the reflected parts of these rays will reinforce each other Thus a featureless surface seen by reflection in such a double prism will appear to be covered with alternate bright and dark bands. Like reasoning will show that an exactly similar process takes place in those portions of the light which are transmitted along DG, *etc* ([16])

Double Refraction.—In what has been written so far it has been assumed that the medium in which the waves of light are propagated is isotropic, that is, that it behaves in exactly the same way whatever be the path which these waves pursue within it In some crystals, notably a form of calcite termed Iceland spar, this is not the case, except for waves in which the light vector, which has so far been termed a vibration without any statement as to its physical nature, is in a certain direction ; for it is found that in these crystals the incident light gives rise to two elementary waves at every point of the bounding surface One of these waves is spherical, and obeys the ordinary laws of refraction, being propagated with the same velocity in all directions The other is, however, propagated with a velocity which depends on its direction in the crystal It is consequently not spherical, but ellipsoidal, and its minor axis is (for Iceland spar) of the same magnitude as the radius of the spherical wave, and is in the direction of a line of symmetry called the optic axis of the crystal ([17]). The two waves are shown in Fig 17, where AB is the trace of the wave front in air, the circle CD that of the

elementary "ordinary" spherical wave originating from A, and the ellipse $C'D$ that of the "extraordinary" ellipsoidal wave originating from the same point, AD, the optic axis of the crystal, is here taken as in the plane of the paper $A'C$ is now the ordinary wave front, and $A'C'$ the extraordinary wave front, in the crystal. Naturally, these two sets of waves give rise to two images, and the familiar double appearance of an object seen through a crystal of Iceland spar results

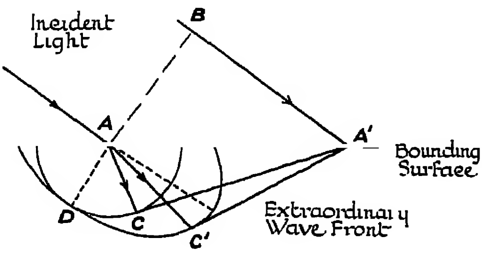

FIG 17—Double Refraction

It thus appears that a crystal of Iceland spar possesses the property of resolving a light wave into two components quite independently of frequency (since both images are practically uncoloured) In one of these components the waves follow the ordinary laws of refraction and are propagated with the same velocity in all directions. In the other component, the velocity of the wave is found to depend upon the direction of propagation. This phenomenon can be at once explained on the wave theory of light by assuming that the light vector or "vibration" is transverse, that is, perpendicular to the direction of propagation, for if this "vibration" be longitudinal there can be no possible dissymmetry of the wave about the direction of propagation, but with a spherical transverse wave it is at once apparent that dissymmetry exists, and that the vector at any point of the wave can be resolved into two components respectively in and perpendicular to any given plane containing the line of propagation ([18]). If this plane be supposed to be that containing the optic axis of the crystal at the centre of the wave (the plane of the paper in Fig 17), then it may well be that when the vector is perpendicular to the paper the velocity of propagation is the same, whatever be the angle between the optic axis and the direction of propagation (angle DAC), while when the vector is in this plane the velocity of propagation varies with this angle (angle DAC'). This supposition provides an adequate explanation of double refraction if the law of variation is assumed to be $c^{-2} = c_o^{-2} \cos^2 \theta + c_e^{-2} \sin^2 \theta$, where θ is the angle DAC', c_e is the maximum velocity of the extraordinary wave, and c_o the velocity of the ordinary wave c/c_e is known as the extraordinary index of refraction of the crystal, n_e

Polarised Light.—In ordinary light, as would naturally be expected, the light vectors are oriented without any regularity, and in the waves set up by any source they are distributed in all directions in the plane of the wave surface. Inside a doubly refracting crystal, however, matters are different, and each of the two wave surfaces contains only waves in which the vectors are in a single direction Such waves are said to be "plane-polarised," and as light of this nature is much used in some branches of photometry it is desirable to consider its properties in further detail

The Nicol Prism.—Unless special means be adopted to isolate one

of these sets of waves the two will naturally be mixed on emerging once more into the air, and the property of polarisation will be lost. The simplest method of isolation is that used in the Nicol prism,

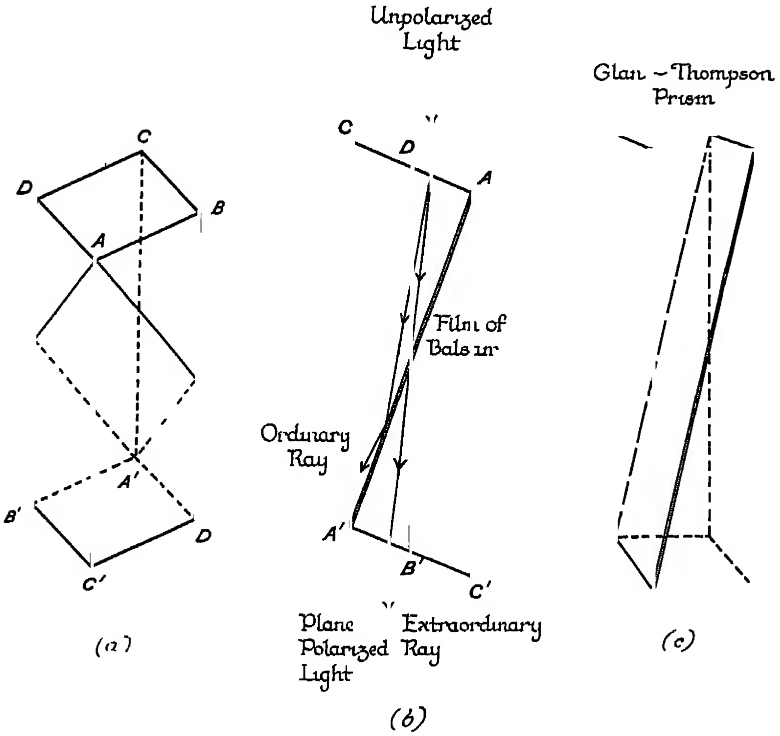

FIG 18.—The Nicol Polarising Prism

which consists of a rhomb of Iceland spar of the form shown in Fig 18*a*, where AB' is about three times AD. This is cut in half by a plane passing through AA' and parallel to BD, and the two pieces are then cemented together by Canada balsam. Now the refractive index of the balsam is greater than n_e and less than n_o, so that if the light be incident sufficiently obliquely on the balsam surface the ordinary ray will be stopped by total reflection (see p 20), while the extraordinary ray will be transmitted (Fig 18*b*). It results that light passing through the Nicol prism will be plane polarised.

If this light be transmitted through a second Nicol, its intensity will depend on the relative directions in space of the optic axes of the two prisms, for, clearly, if the second Nicol can only transmit light for which the vector is in the direction OA (Fig. 19), and if the light falling upon it be plane polarised so that the vector is in the direction OB and of amplitude a, then this light will be divided into two components, of which one will be stopped and the other transmitted. The latter portion will have its vector in the direction OA and, by the law of resolution of vectors, will have amplitude $a \cos \theta$, so that the intensity of the light passing through both

prisms varies as $\cos^2 \theta$, where θ is the angle between the optic axes

It is to be noticed that in the ordinary form of Nicol prism described above the light entering and leaving the prism is not normal to the bounding surfaces The results of this are the production of a certain amount of elliptic polarisation in the transmitted light and a lateral displacement of the emergent ray. The latter difficulty may be avoided by trimming the end faces of the prism so that they are perpendicular to the sides This form of Nicol is generally used in photometric instruments in which the prism has to be rotated [19] One form of prism in which the elliptic polarisation is avoided is the Glan-Thompson form shown in Fig 18c

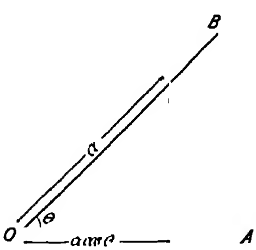

FIG 19.—The Law of Resolution of a Vector

The Wollaston Prism.—Another device used for separating the ordinary and extraordinary waves is known as Wollaston's prism, and consists of two prisms of calcspar, or quartz (another doubly refracting crystal, which differs from Iceland spar in that c_e is always less than c_o), cut with the same angle and cemented together as shown in Fig 20. The optic axis of the left-hand prism is parallel

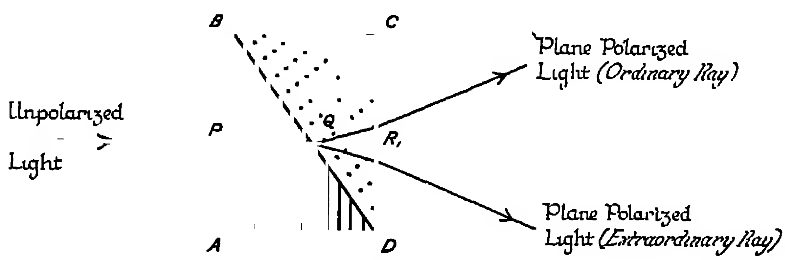

FIG. 20.—The Wollaston Prism.

to AB, while in the right-hand prism it is perpendicular to the paper. It results that if light be incident normally on the face AB, no separation of the waves takes place, the ordinary wave traverses the path PQ with velocity c_o, while the extraordinary wave traverses it with velocity c_e. On arrival at Q separation takes place and the ordinary wave traverses the second prism as an extraordinary wave, while the extraordinary becomes an ordinary wave If α denote the angle of the prism and δ the deviation of the extraordinary wave at Q, $\sin(\alpha + \delta)/\sin\alpha = c_e/c_o$, and if r_1 be the angle of emergence at R_1, $\sin \delta/\sin r_1 = c_e/c$, so that, since δ is small, $\sin r_1 = (n_o - n_e) \tan \alpha$ The angle between the two wave envelopes is, therefore, $2r_1$.

It is clear that if the two waves thus separated be caused to pass through a Nicol prism, the intensity of one on emergence will be $\sin^2 \theta$, and of the other $\cos^2 \theta$, since they are polarised in mutually perpendicular planes It results that the ratio of the intensities will vary as $\tan^2 \theta$.

It is to be noted that, since the waves suffer deviation in passing through a Wollaston prism, a certain amount of colour analysis takes place

In Rochon's prism this effect is avoided by having the prism ABD cut so that the optic axis is parallel to AD (see Fig 20). The ordinary ray then passes through BCD without deviation (and therefore without dispersion), the extraordinary ray being deviated as in the Wollaston prism

Polarisation by Reflection.—The light reflected from the surface of a glass plate is polarised to an extent depending on the angle of incidence (see Chapter IV, p 113) The polarisation should theoretically be complete when $\iota = \tan^{-1} n$, where n is the refractive index of the glass [20] Reflection at the polarising angle ι has been used for polarising a beam of light in some of the older forms of polarisation photometers (see p. 4)

The Electromagnetic Theory of Light.—In what has gone before, the adequacy of the wave theory of light to explain the results of experiment has been demonstrated, but so far no hypothesis has been suggested as to the nature of the vector involved in the propagation of the waves other than that this vector is perpendicular to the direction of propagation

In 1864 Maxwell proposed the electromagnetic theory, in which the light waves are assumed to be of the same nature as the electromagnetic waves set up by a rapidly oscillating electric current, such as that obtained in the spark discharge. The identical velocity of the two sets of waves was experimentally established, and the theory is now universally accepted, as it not only is in agreement with the known phenomena, particularly as regards polarisation, but it connects the waves of heat and light with those produced by electrical means, and it has enabled phenomena in the realm of electricity to be related to other phenomena, before apparently quite independent, in the realm of optics

Maxwell's hypothesis rests on his conception that, just as stress in an elastic solid produces a strain, that is, a displacement of matter, so in dielectrics the application of electric force produces a displacement of some unknown character, and this is identical in all its effects with an electric current The observed laws connecting an electric current and a magnetic field are then sufficient to enable him to deduce the three following equations connecting the components of current u, v, w with those of magnetic force α, β, γ —

$$4\pi u = d\gamma/dy - d\beta/dz, \quad \text{etc} \qquad \text{(i.)}$$

and three of the following form, where μ is the magnetic permeability and P, Q, R are the components of electric force —

$$-\mu\dot{\alpha} = dR/dy - dQ/dz, \quad \text{etc.} \qquad \text{(ii.)}$$

But for a non-conductor whose specific inductive capacity is K

$$u = (K/4\pi)dP/dt, \quad \text{etc}$$

Hence (i) and (ii) may be combined to give the following three equations .—

$$K\dot{P} = d\gamma/dy - d\beta/dz, \quad \text{etc.} \qquad \text{(iii.)}$$

Further, differentiating the three equations of (ii.) with respect to x, y and z respectively, and adding

$$\frac{d}{dt}\left(\frac{d\alpha}{dx} + \frac{d\beta}{dy} + \frac{d\gamma}{dz}\right) = 0$$

so that $\frac{d\alpha}{dx} + \frac{d\beta}{dy} + \frac{d\gamma}{dz}$ is a constant with respect to time, and this constant is zero, since that is its value for the undisturbed medium

Differentiating (ii.) with respect to time, and eliminating P, Q and R by means of (iii), the following equations therefore result :—

$$K\mu\ddot{\alpha} = \nabla^2\alpha, \text{ etc } (^{21}).$$

Similarly, the following equations may be obtained —

$$K\mu\ddot{P} = \nabla^2 P, \text{ etc}$$

These two sets of equations show that both the magnetic and electric forces are propagated with a velocity $1/\sqrt{K\mu}$ For *vacuo* and (very nearly) for all dielectrics $\mu = 1$ K *in vacuo* is equal to 1 on the electrostatic system of units, but $1/m^2$ in the electromagnetic system if m be the number of electrostatic units of quantity of electricity in one electromagnetic unit Hence the velocity of propagation is equal to this ratio m, which experiment shows to be approximately 3×10^{10} c.g s. units This is the same as the velocity of light within experimental error, and the remarkable verification of this theoretical deduction from Maxwell's hypothesis is one of the triumphs of the electromagnetic theory of light. It is to be noted that Maxwell's theory does not afford any explanation of the nature of light "It only expresses one unknown quantity (light) in terms of other unknown quantities (magnetic and electric disturbances), but magnetic and electric stresses are capable of experimental investigation, while the elastic properties of the medium through which, according to the older theory, light was propagated could only be surmised from the supposed analogy with the elastic properties of material media Hence it is not surprising that the electromagnetic equations more correctly represent the actual phenomena Whatever changes be introduced in future in our ideas of the nature of light, the one great achievement of Maxwell, the proof of the identity of luminous and electromagnetic disturbances, will never be overthrown " ([22])

The application of the theory to the phenomena of reflection, refraction, polarisation, etc , cannot be dealt with here. It will be found in any modern book on the theory of optics ([23])

Theory of the Light Radiating Mechanism.—For light waves the origin of the oscillatory current is considered to lie within the atom or molecule of the light-giving body The atom is pictured as consisting of a heavy nucleus positively charged, and surrounded by a number, greater or less, of negatively charged electrons, each carrying a unit charge equal to $4\ 77 \times 10^{-10}$ electrostatic units of electricity These electrons are normally bound to the nucleus, but in certain (radioactive) substances they escape according to a statistical law The ratio of their charge to their mass has been found to be $1{\cdot}769 \times 10^7$ E M.U gm $^{-1}$ ([24]) These electrons form the carriers of electricity in a conductor

Now it is clear that if a rapid oscillation of one of these electrons take place within the atom, since this electron possesses an electric charge, it is equivalent to an oscillating electric current, and so a train of electromagnetic waves will be originated. Further, if the frequency of the oscillation be suitable, the emitted waves will be light waves ([25]).

Since energy is thus emitted from the atom, fresh energy must be supplied to it from exterior sources, and the different means for the supply of this energy provide a convenient classification for the various kinds of light emission known. When the energy is drawn from the molecule or, possibly, the atom by chemical action, as in the case of the oxidation of phosphorus, or the chemical action which is probably the cause of the glow emitted by light-giving animals, such as the fire-fly, the emission is called *chemi-luminescence* or *phosphorescence* ([26]) When the energy is supplied by light which has been absorbed by a substance and stored up within its molecules as in a reservoir, the subsequent emission of the stored-up energy is called *photo-luminescence* or *fluorescence* When the energy is supplied by electrical discharge through a gas, the emission is called *electro-luminescence* These three classes will be considered in more detail later in this chapter At present it is necessary to consider in some detail the most important class of light emission, *viz*, that in which the energy supply is due to the impact between the molecules of the radiator consequent upon its temperature. This kind of emission is called *temperature radiation* or *thermo-luminescence.*

Pure Temperature Radiation.—It is first necessary to define what is generally known in the study of radiation as a " black body " or " complete radiator " ([27]) Every body in nature reflects some of the radiation which is incident upon it, and most bodies are selective in the extent to which they reflect radiation. Objects which do not themselves radiate in the visible spectrum are only seen by means of the light which they reflect to the eye after they have received it from some self-radiator. A truly black body is one that totally absorbs light of all frequencies, and reflects none of the radiation that falls upon it. Such a body must, from thermodynamical considerations which will now be described briefly, emit radiation in which the energy contained in any frequency range is connected with that frequency and with the temperature of the radiator according to a law which can be deduced from theoretical considerations.

Kirchhoff's Law.—To Balfour Stewart and G Kirchhoff is due the principle that the ratio of the radiation emitted to that absorbed by any body in thermal equilibrium depends only on the temperature, and that this ratio is equal to the emission of a black body (for which the absorption is perfect) at the same temperature. This is a consequence of the experimental fact that a number of bodies within an impervious enclosure which contains no source of heat will ultimately acquire the same temperature, but the law enunciated above goes further in that it applies to radiation of any given frequency, and not merely to the total radiation, for it is possible to imagine a hollow enclosure of uniform temperature in which a portion of the wall is composed of a body whose absorption factor for radiation of wave number ν is α_ν, and whose emission at the temperature is η_ν, while the remainder of the wall is perfectly black,

and therefore has absorption unity and emission E_ν. The absorption and emission of all parts of the wall must balance, or the temperature equilibrium will be disturbed. Hence, if A_ν be the energy in frequency ν received by all parts of the wall, for the non-black portion $\alpha_\nu A_\nu = \eta_\nu$, and for the black portion $A_\nu = E_\nu$. Hence $\alpha_\nu E_\nu = \eta_\nu$ (28).

Three deductions follow at once from this result. (i.) If α_ν be determined for any body, and the relation between E_ν and temperature be known, η_ν is found. (ii.) Since α_ν cannot be greater than unity, no body can radiate more at any frequency than can a black body at the same temperature. Hence a black body is often called a "complete, or total, radiator." (iii.) In a hollow enclosure of uniform temperature, the radiation proceeding by reflection and emission from any part of the inner surface is the same as that emitted by a black body at the same temperature. For suppose it receives A_ν, then it absorbs $\alpha_\nu A_\nu$, and since it emits η_ν and, when the temperature is constant, emission and absorption must be equal, it follows that $\eta_\nu = \alpha_\nu A_\nu$. Thus $A_\nu = E_\nu$, and, again, since it must return, by reflection and emission, as much as it receives, it follows that the emission is E_ν. Hence, since no surface is known which behaves exactly as a black body at all temperatures, the best approach to a complete radiator is a very small opening in a uniformly heated enclosure.

Other consequences of this law are (i.) good reflectors are bad absorbers, and consequently bad radiators; (ii.) transparent bodies, being bad absorbers, are also bad radiators. This is well shown by the bright appearance of a spot of opaque material on a piece of heated glass.

The Pressure of Radiation.—It is now easy to show on thermodynamic principles that the pressure exerted by a succession of plane waves incident normally at a perfectly black surface is equal to the energy contained per unit volume of the space in which these waves are travelling; for, considering a cylinder of unit cross-section whose axis is normal to the oncoming waves, if this cylinder be closed at one end by a black surface, the energy of the waves will be completely absorbed, and the amount so absorbed in time t will be Ect, where E is the energy contained per unit volume of the cylinder. If now the black surface be displaced a distance δx along the axis of the cylinder, then the work done by the movement of the surface is equal to the energy contained in the increased volume δx of the cylinder, i.e., to $E\delta x$, so that if p be the pressure upon the surface, $p\delta x = E\delta x$, or $p = E$.

As an example of the magnitude of p, it may be said that since the energy flow from the sun at the earth's surface (the solar constant) is $1{\cdot}3 \times 10^6$ ergs/sec. per sq. cm. (29), this energy is contained in 3×10^{10} c.c. Hence the value of E in this case is 4×10^{-5} ergs per c.c., i.e., $p = 4 \times 10^{-5}$ dynes per sq. cm., or 4×10^{-8} gm. weight (approx.)

The Stefan-Boltzmann Law.—From Kirchhoff's law and the result obtained above it is possible to deduce the law connecting the radiation from a black body and the absolute temperature T of the body; for, considering a closed cylinder of length x and unit cross-section, the walls of which are totally black and impervious

to radiation, if the energy per unit volume be E, the pressure on the end wall will be $\frac{1}{3}E$, since the contained energy must be due to waves travelling in *all* directions, and the average resolved component in one of three mutually perpendicular directions will, therefore, be $\frac{1}{3}E$ Now suppose the end of the cylinder to move inwards by a distance δx, and the energy density to be changed by an amount δE in consequence, then (*a*) the work done by the movement will be $\frac{1}{3}E\delta x$, and (*b*) the change in the amount of energy contained within the cylinder will be $Ex - (E - \delta E)(x - \delta x)$ Thus, by the principle of the conservation of energy,

$$\frac{1}{3}E\delta x + Ex - (E - \delta E)(x - \delta x) = 0$$

or $\frac{4}{3}E\delta x + x\delta E = 0$, or E varies as $x^{-\frac{4}{3}}$ Further, if the change of temperature produced by the adiabatic volume change δx be equal to δT, then by Carnot's principle

$$\frac{1}{3}E\delta x/Ex = -\delta T/T,$$

so that T varies as $x^{-\frac{1}{3}}$, *i e*, E varies as T^4 [30]

Since a black body absorbs all the radiation that it receives, it follows from Kirchhoff's law that the radiation in equilibrium with it is proportional to that which it emits from unit area in unit time. Hence the energy of the radiation emitted by a black body at a temperature T is equal to σT^4, where σ is a constant known as the Stefan-Boltzmann constant, after the names of the discoverers of the law, the first by experimental work, the second from theoretical principles. The value of σ is $5\ 709 \times 10^{-5}$ erg cm $^{-2}$ deg $^{-4}$ sec $^{-1}$ [31]

Wien's Displacement Law [32] —In the Stefan-Boltzmann law the energy is treated as a whole, and its partition among waves of different frequencies is not considered It now becomes necessary to find out in what manner this partition is affected by a change in T It can readily be proved that the effect of the compression δx above considered is to increase the frequencies of the radiation enclosed in the cylinder, for if a series of plane waves of length λ and frequency f strike a surface moving with opposing velocity u, $f\lambda = c$ initially, and after reflection $f' = f + 2u/\lambda$ [33] Hence $\delta f = 2u/\lambda = 2uf/c$ Now $u = -\dot{x}$, and the number of reflections per second in the cylinder is $\frac{1}{3}c/2x$, for again, since the waves within the cylinder are travelling in all directions, the average resolved component in one direction is $\frac{1}{3}c$ Hence the rate of increase of frequency is the increase per reflection multiplied by the number of reflections per second, *i e.*,

$$\dot{f} = (2fu/c)(c/6x) = -(f/3x)\dot{x}$$

$$\therefore df/dx = -f/3x \quad \text{or } f \text{ varies as } x^{-\frac{1}{3}}$$

Now let the energy per unit volume of the radiation in the range of frequency from f to $(f + \delta f)$ be denoted by $E_f\delta f$, where E_f may be called the energy density per unit range at frequency f. When the contraction δx takes place, $E_f\delta f$ varies as $x^{-\frac{4}{3}}$ (see above) But f, and therefore δf, varies as $x^{-\frac{1}{3}}$, so that E_f varies as x^{-1}, *i.e.*, as T^3

This shows that, comparing E_f in two full radiations at different temperatures, and taking *corresponding* frequencies given by $T/f =$ constant, E_f is proportional to the cube of the temperature This relation may then be written

$$E_f = T^3\phi(T/f), \text{ or } E_f = f^3\psi(T/f).$$

This is known as Wien's displacement law, and it leads readily to an expression for f_{max}, the frequency of maximum energy at any temperature, for, at any given temperature, when E_f is a maximum E_f/T^3 is a maximum, so that $\phi(T/f_{max})$, and therefore (T/f_{max}), is a constant independent of temperature.

It should be noted that since $f = c\nu$, the above expressions preserve the same form if written with ν substituted for f throughout The value of T/ν_{max}, which may be represented by A_ν, is found to be 0 5079 cm. degree (34).

The expressions in f or ν may be further transformed to the corresponding expressions in λ by using the transformation $f = c\nu = c/\lambda$, so that $\delta f = c\delta\nu = -(c/\lambda^2)\delta\lambda$ Since $E_\lambda\delta\lambda = E_f\delta f$, the expression for E_λ becomes $E_f(c/\lambda^2) = (c^4/\lambda^5)\psi(T\lambda/c)$, which may be written

$$E_\lambda = \lambda^{-5}F(\lambda T).$$

Here E_λ is the energy per unit range at wave-length λ Similarly, it may be shown that the constant $\lambda_{max}T = A_\lambda = 0\ 2885$ cm. degree (34).

The Quantum Theory.—In the above expression for the energy at any frequency the form of the function written as $F(T/\nu)$ is left quite undetermined. The further consideration of the form of this function depends fundamentally on the manner in which energy can be exchanged In the classical mechanics it was assumed that energy could be exchanged between molecules, *etc* , in any amount, and not necessarily in definite multiples of an indivisible unit or "quantum" On this assumption it can be shown (35) that the probability that any one molecule, regarded as a "seat of energy," will have energy lying between the values E and $E + \delta E$ (the probability being defined as the fraction of the total time for which its energy lies between these limits) is $(1/kT)e^{-E/kT}\delta E$, where k is the atomic gas constant, *i.e* , the ordinary gas constant R divided by the number of molecules in the gramme-molecule (36). It follows that the *average* energy in such a seat is $\int_0^\infty (E/kT)e^{-E/kT}dE$, or simply kT This is the theorem known as the "equipartition of energy," and the result just quoted can readily be shown to lead to the value $8\pi RT/\nu$ for $F(T/\nu)$ in Wien's formula, which thus becomes $E_\nu = 8\pi R\nu^2 T$. This is known as the Rayleigh-Jeans form

It will be at once apparent that if ν be very large, E_ν will also be very large, and in consequence it must be concluded that the energy density in the ether becomes infinite for waves of infinite frequency, a conclusion which is generally regarded as quite untenable. In consequence, Planck in 1901 (37) was led to propose a new hypothesis, in which he supposed that energy can only be communicated in multiples of some indivisible amount which varies directly as the frequency of the oscillation by means of which the

energy is transferred. If this hypothesis be granted, a seat of energy can only gain or lose by multiples of this quantity, and it is then found that the probability of such a seat having energy $p\epsilon$ is $e^{-p\epsilon/kT}/(1 - e^{-\epsilon/kT})$, and the average energy is now found to be $\epsilon/(e^{\epsilon/kT} - 1)$ This is the partition formula resulting from Planck's theory, and it will be recognised at once that since when ν is very large, $p\epsilon$ is also very large, the average energy at very high frequencies is very small, a result to be expected *a priori*, since the probability of exchange taking place at all is small if it can only be carried out by the transfer of a large quantity of energy The factor of proportionality between ϵ and ν is hc, so that $\epsilon = hc\nu$, and thus the average energy of a system of seats of energy will be not kT, but $hc\nu(e^{hc\nu/kT} - 1)$. The constant h is called Planck's elementary quantum, and has the value $6\ 554 \times 10^{-27}$ erg/sec. ([38]).

Wien's displacement law now becomes $E_\nu = 8\pi hc\nu^3/(e^{hc\nu/kT} - 1)$, which may also be written $E_\nu = C_1\nu^3/(e^{C_2\nu/T} - 1)$, where C_1 and C_2 are constants respectively equal to $8\pi hc$ and hc/k ([39]). The curve calculated by this formula for the energy distribution in the spectrum of a black body is in remarkable agreement with that found by experiment

This agreement is exhibited by the curve of Fig 21, which shows the energy emitted within unit wave-number interval by unit area

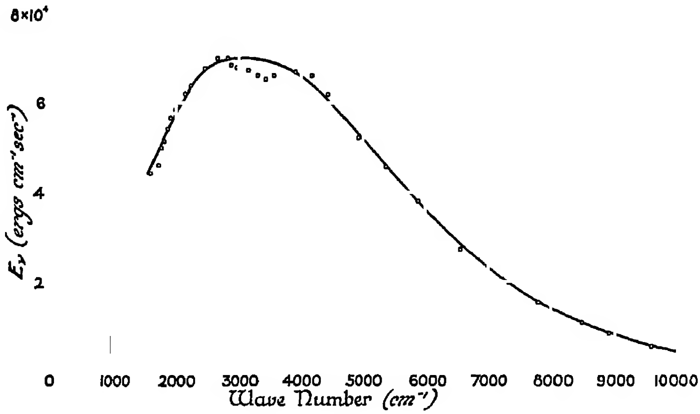

FIG 21 —Planck's Formula Circles indicate observations, full line represents the computed curve (Recomputed from Bureau of Standards, Bull, 13, *1916*, p 476)

of a black body radiator in unit time when the temperature of the radiator is 1,596° K The line gives the distribution calculated according to the Wien-Planck law, using the values $C_1 = 3\ 703 \times 10^{-5}$ erg/cm ^{2} sec.$^{-1}$, and $C_2 = 1\ 4330$ cm degrees, while the circles indicate observational points ([40])

The values of the constants clearly depend on the value of the black-body radiation found experimentally at some temperature measured by thermometric apparatus ([41]), and the above figures are based on the present accepted values of the melting points of gold and of palladium, *viz*, 1,336° K. and 1,829° K. ([42]), at which temperatures the total radiation of 1 sq. cm. of a black body is found to be 18·2 watts and 63·9 watts respectively.

"Grey" and Selective Radiation.—The black body, then, forms a basis for energy measurements at different regions of the spectrum of other incandescent bodies. It has been said already that no surface is perfectly black, and it follows that the radiation from such a body as, for example, a tungsten filament, may follow quite a different law from that found for the total radiator at the same temperature. It has already been pointed out (p. 34) that no body can emit more temperature radiation than a black body at the same temperature. A body whose emission, when compared with that of a black body at the same temperature, bears a constant ratio to it at all frequencies is known as a "grey body," and radiation having this characteristic distribution is known as "grey-body radiation." Any radiator for which the distribution curve is different from this is said to be "selective," and all known bodies radiating in the open are selective to a greater or less extent. The nearest approach to a black body is carbon.

The sun, as viewed from the earth's surface, may be regarded as a black body at a temperature of about 5,400° K., but the composition of sunlight is somewhat different from that of the light given by a black body, owing to unequal absorptions at different frequencies during its passage through our atmosphere.

For slightly selective bodies, such as platinum, iron or copper oxide, tungsten, carbon, *etc.*, it has been found that their emission can be sufficiently well expressed by means of the generalised formulæ $E = \sigma_1 T^{\alpha}$, and $E_{\nu} = c_1' \nu^{(\alpha-1)} e^{-c_2'\nu/T}$, where σ', α, c_1' and c_2' are constants of the selective body ([43]).

In the case of some selective radiators the energy distribution in the visible region of the spectrum approximates closely to that of a "grey" body, *i.e.*, for any frequency within this region the energy given by the selective radiator is a constant fraction of that given by a black body at some temperature ([44]). Tungsten behaves almost in this manner, having an energy distribution curve which is very closely the same as that of a grey body in the visible, while at lower frequencies its emissive power is smaller than would be the case if it were truly "grey" ([45]).

It is clear that the colour of the light given by such a selective radiator is sensibly identical with that of a black body at some definite temperature. This temperature is, therefore, termed the "colour temperature" of the selective radiator, and will be considered further in Chapter IX.

Complete Radiation and Selective Radiation.—The above description of the principal characteristics of temperature radiation may be summarised briefly as follows:—

(1) For a complete radiator the *emission at any frequency* increases continually with the temperature, but the higher frequencies increase most rapidly, so that the frequency of *maximum emission* shifts continually towards the blue as the temperature rises. The area of the curve within the limits of the visible spectrum is but a small portion of the whole, so that the energy radiated as light is only a small portion of the total energy emission. This proportion increases, however, as the temperature rises, until it reaches a maximum of about 50 per cent. at a temperature of slightly over 6,000° K.

(2) For a selective radiator the *emission at every frequency* is less than that of a total radiator at the same frequency, but the *distribution* is different, so that the proportion of the total emission which is within the limits of the visible spectrum may be higher than for a total radiator at the same temperature For some selective radiators the energy distribution in the *visible* part of the spectrum is approximately the same as that in the spectrum of a grey body

Electro-Luminescent Radiators.—It will have been observed that all that has been said above applies only to temperature radiation, but, as already mentioned, there are types of radiators which do not depend upon temperature to supply the energy which they emit in the form of ether waves, and to which, therefore, the above conclusions do not apply One of the most important of these types of radiator is that in which the source of the energy is electrical, and as this type is of considerable importance in light production, it will be considered briefly here

It has long been known that when an electric discharge is passed through certain gases contained in a tube at low pressure they glow with a light which is characteristic of the gas and of the conditions under which the discharge takes place. The well-known Geissler tubes are examples of this, and the mercury vapour lamp and the Moore tube are applications of the same principle to practical lighting. For these sources the energy distribution departs very markedly from that of a black body, and, in fact, it is found that the radiation is concentrated, to a greater or less extent according to circumstances, in the neighbourhood of certain definite frequencies, which are well-marked characteristics of the nature of the gas, insomuch that the gases present in a tube can generally be named at once from inspection of the energy distribution curve under electric discharge The striking fact of the invariability of these frequencies is the basis of spectrum analysis

The Rutherford-Bohr Atom Model.—In order to form a clear mental picture of the mechanism which is thought to underlie the phenomena of electro-luminescence, it is necessary briefly to survey the Rutherford-Bohr theory of atomic structure ([46]) It is supposed that the atom is composed of an inner central electric charge, concentrated on a nucleus, and a surrounding planetary system of electrons (elementary negative charges) rotating in certain orbits This conception of the structure of the atom, due to Rutherford, is the outcome of a study of the phenomena of radioactivity and the properties of the α- and β-particles ([47]) It was pointed out by Bohr ([48]), however, that such a planetary system of electrons is unstable unless Planck's quantum hypothesis be adopted, for otherwise the energy of rotation of these electrons would gradually be radiated, and the orbit would in consequence become smaller and smaller, so that the electron would finally be merged in the nucleus. Further, the alteration of orbit would cause a gradual change of frequency in the emitted radiation, and this is certainly contrary to observation Bohr therefore assumes that the electron can only move in an orbit such that its energy consists of an integral number of quanta, and that while moving in this orbit no energy is radiated Passage from one orbit to another, however, may take place owing to a "crisis" involving the emission or absorption of one or more

quanta $hc\nu$ of energy in the form of homogeneous radiation of wave number ν, where $c\nu$, the frequency, has a value intermediate between the frequencies of rotation of the electron in its original and final orbits

This theory has been applied with wonderful success to the case of the hydrogen atom where it is supposed that there is but one electron of charge $-e$ and mass m, rotating about a nucleus of charge $+e$, in a circular orbit of radius a with angular velocity ω Then, according to the ordinary laws of mechanics $e^2/a^2 = ma\omega^2$ and the kinetic energy W of the electron is necessarily equal to $\frac{1}{2}m\omega^2a^2$ Bohr assumes that the energy of an electron moving in an orbit of angular velocity ω is equal to some multiple of its frequency of oscillation $(\omega/2\pi)$ multiplied by $\frac{1}{2}h$ so that $W = \tau h\omega/4\pi$ where τ is an integer From the three equations just given it follows that

$$W = \frac{2\pi^2me^4}{\tau^2h^2}, \quad \omega = \frac{8\pi^3me^4}{\tau^3h^3}, \quad 2a = \frac{\tau^2h^2}{2\pi^2me^2}$$

Putting in the ordinarily accepted values for e, e/m and h it is found that $W/e = 13$ volts, $a = 0\ 53 \times 10^{-8}$ cm and $\omega/4\pi = 6\ 2 \times 10^{15}$ sec^{-1} These quantities are of the order of magnitude of the ionisation potential (49), atomic size, and optical frequencies respectively

Application to Line Spectra.—The triumph of the theory lies in the explanation which it gives of the relation found to exist between the numerical value of the frequencies at which the radiation from hydrogen gas at low pressure is emitted Balmer, in 1885 (50), pointed out that these frequencies could be arranged in series, so that to very considerable accuracy the frequencies in each series were expressed by the equation $\nu = N(1/n_1{}^2 - 1/n_2{}^2)$ where, for a well marked series in the hydrogen spectrum, n_1 has the value 2 and n_2 has the values 3, 4, 5 Clearly this expression follows at once if it be assumed that energy $hc\nu$ is emitted by the passage of an electron from an orbit in which τ has the value 3, 4, 5 to one in which $\tau = 2$. For $hc\nu = W_1 - W_2 = (2\pi^2me^4/h^2)\ (\tau_1{}^{-2} - \tau_2{}^{-2})$ so that the constant N, known as the Rydberg constant, becomes $2\pi^2me^4/h^3c$. The series of frequencies for which τ_1 has the value 1 and τ_2 the values 2, 3, 4 . . are in the ultra-violet, while those for which $\tau_1 = 3$ are in the infra-red. Both of these series are known to exist, though the former was not discovered until after its prediction by the theory Assuming the truth of Bohr's theory, the known value of N, *viz*, 1 0930 $\times$ 10^5 cm^{-1} (51) gives another method of determining Planck's constant, and the value thus found is in excellent agreement with that determined by quite different methods For less simple atoms than those of hydrogen the application of the theory becomes very difficult, but the above treatment will serve to show the nature of the experimental evidence which supports the Rutherford-Bohr theory of atomic structure.

The radiation emitted by a gas when an electric discharge passes through it may then be looked upon as due to disturbances in the electronic system of an atom by the free electrons taking part in the conduction of the current through the gas These electrons are constantly coming within or quitting the sphere of action of the gaseous atoms, and in so doing they cause the "crises" among the

electrons within the atom which result in the emission or absorption of radiation of a given frequency

The colour of the light given by such a process is clearly quite different from that due to temperature radiation, and the laws which govern the intensity at any frequency have not yet been reduced to any theory. It may be mentioned here that the range within which the radiation at any dominant frequency is confined in a spectrum such as that just described is generally extended slightly by increasing the pressure of the gas above a few millimetres of mercury ([52]) This form of spectrum, generally characteristic of elementary bodies in the gaseous state, is spoken of as a "line" spectrum because, when analysed by a prism or grating the light appears in a number of sharply defined lines For undissociated compounds the emission takes place in groups of frequencies so that the spectrum consists of a number of broad bands although each band is resolvable into a number of fine lines by a spectroscope of sufficiently high power This type of spectrum is termed a "band" spectrum The spectrum obtained from temperature radiation is, on the other hand, a "continuous" spectrum and may be regarded, for the sake of convenience, as a mixture of waves of all frequencies

Photo-Luminescence, or Fluorescence.—There is a radiation phenomenon which has received a certain degree of application to problems of practical illumination and which therefore deserves mention here, *viz*, the fluorescence exhibited by certain substances such as eosine, rhodamine, phenosafranine, *etc*, which, when illuminated strongly emit radiation of frequencies quite different from that of the incident radiation It has generally been assumed that the frequency of the emitted radiation is always less than that of the incident radiation (Stokes' law) but exceptions to this rule have been found, notably in the case of sodium vapour ([53])

Attempts have been made to explain fluorescence as an absorption of energy by the electrons which thereupon emit it in radiation of a different frequency But this does not explain why all absorbers should not exhibit fluorescence It seems probable that the absorbed radiation is stored up, either within the substance by chemical reaction or within the molecule by some mechanism at present unknown, and given out again almost instantaneously in the form of radiation characteristic of the material A fluorescent body may thus be regarded as a kind of catalyst for the transformation of energy from one frequency to another When the transformation is not instantaneous, as in the case of certain bodies such as calcium and zinc sulphides, barium platino-cyanide, *etc* the phenomenon is sometimes misleadingly termed phosphorescence. The last-named substance is used for X-ray screens on account of its power of transforming the short Rontgen waves into others lying within the visible spectrum.

Some bodies, notably zinc sulphide, exhibit the power of emitting luminous radiation under the action of the α- or β-particles from radioactive substances These are termed radio-luminescent, and are used, mixed with radium, for the illumination of watches and other objects which are required to be recognisable in the dark.

Receptors of Radiation.—The above sections of this chapter have dealt mainly with the phenomena of the propagation and emission

of radiation, and of the theories put forward to explain these phenomena In this section the reception of radiation will be dealt with

Naturally the most widely useful forms of energy receptor are those in which the energy is converted into heat and so measured. The bolometer and the thermopile are the two instruments principally employed for the purpose and in them the energy is absorbed by a black surface and the resulting rise in temperature causes an electric current to flow according to the well-known laws of thermo-electricity ([54]) This current, measured by a galvanometer, gives a measure of the energy incident at the surface Energy distribution is investigated by first obtaining a separation of frequencies by means of a prism or a grating, and then using a bolometer or thermopile of extremely small breadth so as to receive radiation from as restricted a range of frequency as possible This subject will be dealt with more fully in Chapter XI

For the spectral region which is chiefly important in photometry, *viz* , the visible range, the most important receptor is the eye Its behaviour will therefore be described in some detail in the next chapter

There is, however, a third and entirely different form of receptor, the photo-electric cell, which has received an increasing amount of attention in recent years from workers in photometry The phenomena underlying its action must therefore be noticed briefly.

The Photo-Electric Cell.—It was shown in 1888 by W Hallwachs ([55]) that a negatively charged body gradually lost its charge when illuminated by ultra-violet light, while a positively charged body remained unaffected This phenomenon was called the photo-electric effect It has since been found that some substances, notably the alkali metals, show the same effect under the action of light in the visible spectrum, and the effect has therefore been applied to the measurement of illumination The particular form of apparatus used in photometric work is termed, somewhat misleadingly, a photo-electric cell, and is described in Chapter XI (p. 326)

The phenomenon of photo-electricity results from the fact that a metallic surface receiving radiation of a sufficiently high frequency emits electrons whose velocity, it has been found, may have any value from zero to a certain maximum u (for light of a given frequency). It seems probable that all the electrons leave the atom to which they originally belonged with an initial velocity u, but that this velocity is reduced by an amount depending on the length of path they have to travel within the metal before they finally emerge from the surface It has been found that the number of electrons emitted is a function of the intensity of illumination only, and not at all of the frequency of the incident radiation, while on the other hand u depends only on the frequency of the incident radiation, and not at all on the intensity of the light ([56]) Further, there is a value for the frequency of the radiation which forms a minimum below which no photo-electric effect is produced at all With sodium, for example, if the frequency lie below that of the green light $\nu = 17{,}300$, no electrons are emitted ([57]). The value of u is given by an expression of the form $\frac{1}{2}mu^2 = kc\nu - w_0$ where k and w_0 are constants for the

metal, and it is found that the average value of k for a number of elements ranges from 4 9 to 5 7 $\times$ 10^{-27} erg/secs., while w_0 is approximately equal to e multiplied by V_0 the ionisation potential of the substance, so that w_0 is the work which has to be done on an electron to bring it out of the sphere of attraction of the atom Hence it seems reasonable to assume that the incidence on an atom of an amount of energy $kc\nu$ is required before the electron can be emitted. Now k is always slightly less than, but of the same order of magnitude as, Planck's constant h, so that again the conclusion seems inevitable that exchange of energy between matter and ether can only take place in multiples of the indivisible quantity $hc\nu$ This would explain at once the inability of light of less than the critical frequency to produce the photo-electric effect, for in this case a single quantity $hc\nu$ would be too small to liberate the electron, and the probability of the simultaneous incidence of two such quantities on the same atom is too small to be considered Further, as ν increases so does the quantity $hc\nu$, and since the whole of this quantity must be absorbed by the atom it is employed in increasing u

It should be mentioned that other explanations of the photo-electric effect have been proposed with varying success, but the explanation given by the quantum theory is now almost universally accepted ([58]). The chief difficulty which besets other theories is that a photo-electric effect can be observed when the energy reaching the surface is of the order of 10^{-7} ergs per sq cm. per sec (of the same order as that detectable by the human eye) It follows that the energy reaching a single atom is much less than 10^{-20} ergs, so that if the electron can absorb only the amount of energy falling on the molecule in which it is contained, a very long period must elapse from the time the light is admitted to the beginning of the effect No such period exists The effect takes place simultaneously with the admission of the light If, however, the light be imagined to be concentrated in quanta, the emission of a single electron may take place as soon as one such quantum has reached the surface, a time quite inappreciable by any means at our command, even for the feeblest illuminations.

The cause of the abnormal sensitivity of the alkali metals (and, to a much less extent, the metals of the alkali earths) has not so far received any theoretical explanation. The magnitude of this selective effect has been found by some workers to depend on the orientation of the plane of polarisation and on the angle of incidence of the light reaching the surface, but this has been contradicted by others ([59])

The sensitivity of a photo-electric cell, *i e*, the current due to a given illumination, is increased by filling the cell with an inert gas, such as argon, at a pressure of a few millimetres of mercury This increase is due to ionisation of the gas molecules by collision ([60]). The electrons set free from the metal surface travel towards the anode owing to the electric field, and if in so doing they collide with a molecule of gas, the force of the impact may be sufficient to set free an electron previously held bound within the molecule These released electrons immediately travel towards the anode, and thus increase the current passing through the cell.

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(3) A frequency of 10^{12} vibrations per second is sometimes termed a frequency of 1 fresnel.

(4) On the limit of the visible spectrum, see, *e g*, R Tigerstedt, Centralbl f d. gesamte Biologie (Abth II), 1, *1905*, pp 1 and 33. This paper contains a useful bibliography of the subject

(5) This term is here used generally to indicate the degree of departure from normal conditions which is brought about by the propagation of an "ether wave" through a point.

(6) The wave-length of a certain homogeneous light (Cd red) in air at 15° C. and 760 mm. pressure with $g = 980\,67$ is $6438\,4696 \times 10^{-10}$ metres. (Int. Solar Union Trans, 2, *1907*, p 20) (The radiation is assumed to be emitted in a gravitational field of the value stated.) International Critical Tables See note (41), below.

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(17) See, for example, C Huyghens, "Traité de la Lumière," chap V, or T Preston, "Theory of Light," § 178 Also A Guillet, J de Phys, 6, *1916*, p 129

(18) This resolution of a vector into two components may be looked upon as analogous to the resolution of momentum which takes place when two billiard balls in contact are simultaneously struck by a third ball

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(20) See T Preston, "Theory of Light," §§ 166 and 171.

(21) ∇^2 is written for the operator $-\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right)$.

(22) A Schuster and J W Nicholson "Theory of Optics," § 141 *et seq*

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(25) H Hertz has shown (Wied Ann, 36, *1889*, p 1, P Drude, "Lehrbuch d Optik," p 487 (530)) that if the distance between two equal and opposite electric charges e undergo a periodic change of amplitude a, then the electromagnetic energy emitted in unit time by both charges is $L = (16/3)\ \pi^4 ce^2a^2\nu^4$ where ν is the wave number E Wiedemann has found (Wied Ann, 37, *1889*, p 177) that the energy emitted per second in the two D-lines by 1 gm of sodium is $13\,45 \times 10^{10}$ ergs Now an atom of sodium is known to weigh 4×10^{-23} gm, and since sodium is univalent, each atom is connected with a single ion For the D lines $\nu = 17{,}000$ cm $^{-1}$ so that a is found to be of the order of 10^{-11} cm The diameter of a molecule as calculated from the kinetic theory is about 2×10^{-8} cm, so that it appears that the emission of light by sodium vapour may well be due to oscillation of the electrons within the sphere of action of the atom

(26) This term has been loosely applied to other forms of luminescence, such as photo-luminescence See, *e g*, W S Andrews, Gen El. Rev, 28, *1925*, p. 103.

(27) Also "hohlraum" or "cavity."

(28) A similar line of argument shows that E_ν cannot depend on the nature of the black body for so long as $a_\nu = 1$, $\eta_\nu = E_\nu$

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(30) T is here expressed on the absolute (thermodynamic) scale

(31) See note (41), below A Kussmann (Z f Phys, 25, *1924*, p 58) gives as the value of σ, 5 795 × 10^{-5}

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(33) For the number of waves reaching the moving surface in unit time is increased by u/λ owing to the advance of the surface, and since the reflected waves arise from a surface advancing with velocity u, the number reaching a stationary point per unit time is again increased in the ratio $c/(c-u)$, i e, $f/(f-u/\lambda)$

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The values adopted in this book are those given in the "International Critical Tables of Numerical Data of Physics, Chemistry and Technology," now in preparation under the direction of the National Research Council, Washington, U S A

It should be noticed that the constants C_1 and C_2 are connected with the constant σ of Stefan's law, and the constant A_ν ($= T/\nu_{max}$) by the following relationships —

(i) $\sigma T^4 = \int_0^\infty C_1\nu^{-3}(e^{c_2/\nu T} - 1)^{-1}\,d\nu$

The expression under the integral sign may be expanded by the binomial theorem and, when integrated term by term, it gives

$$\sigma T^4 = (6\,C_1T_4/C_2{}^4)\sum_1^\infty n^{-4} \text{ or } \sigma = C_1\pi^4/15\,C_2{}^4.$$

(ii) Since the equation $dE_\nu/d\nu = 0$ gives the value of ν_{max}, it follows that if a be put for C_2/A_ν, then $3(e^{-a} - 1) + a = 0$ which gives $a = 2\,8214$ Thus C_1 and C_2 may be calculated when σ and A_ν are known (In wave-lengths $5(e^{-\beta} - 1) + \beta = 0$, where $\beta = C_2/A_\lambda$ This gives $\beta = 4\,9651$) On the calculation of C_2, see J H Dellinger, Bureau of Standards, Bull 13, *1917*, p 535, and H M Roeser, *idem*, 14, *1918*, p 237

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(50) Wied Ann, 25, *1885*, p 80
(51) International Critical Tables, see note (41)
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(52) A A Michelson Astrophys J, 2, *1896*, p 251
(53) R W Wood and J H Moore Phil Mag, 6, *1903*, p 362
(54) See, for example, F B Pidduck, "Treatise on Electricity" (Camb Univ Press, *1916*), chap VI
(55) Wied Ann, 33, *1888*, p 301
(56) P Lenard Ann d Phys, 8, *1902*, p 149; R A Millikan, Phys Rev, 1, *1913*, p 73
(57) O W Richardson and K T Compton Phil Mag, 24, *1912*, p 575
(58) For an account of a photo-electric determination of "*h*," see R A Millikan, Phys Rev, 7, *1916*, p 355
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CHAPTER III

THE EYE AND VISION

WITH the, at present, comparatively unimportant exception of certain methods of physical photometry, all light measurement depends ultimately on the use of the human eye Even in physical photometry the aim is to obtain an instrument which will give results in accord with those which would be obtained visually, so that it may be said that the eye remains, and must remain, the ultimate judge of light, both qualitatively and quantitatively Certainly the spectroscope makes it possible to distinguish between light waves of neighbouring frequencies which are quite indistinguishable by visual means, and similarly a sensitive radiometer can detect differences of intensity which the eye cannot perceive , but for all ordinary photometric purposes the eye is the final arbiter, and it is, therefore, necessary in any treatment of the subject of photometry to give some description of the construction and mode of working of this organ of special sense (1)

The Structure of the Eye.—The eye is an ellipsoidal, nearly globular organ about 23 mm in diameter It will be seen from

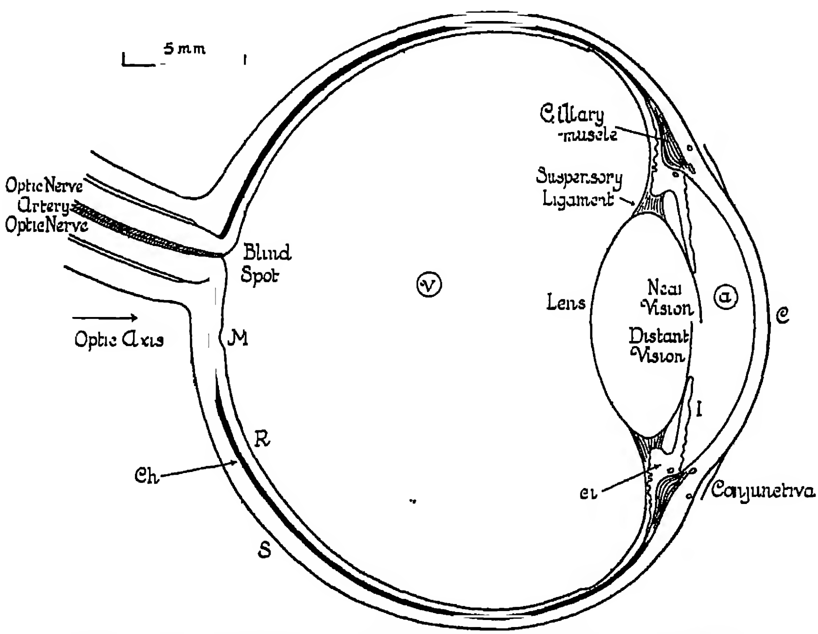

FIG 22 —Human Right Eye divided nearly Horizontally through the Middle

the section given in Fig. 22 that it is enclosed in a tough outer skin (*S*) termed the *sclera*, which is opaque to light This is replaced

at the front of the eye, however, by a transparent cartilaginous lamina termed the *cornea* (*C*), which is more convex than the sclera. Under the sclera is a second coat, termed the *choroid* (*Ch*), which is much more delicate and consists almost entirely of blood vessels and nerves. It is black, being covered with a layer of black pigment which prevents interference with vision due to stray light reflected from its surface. In front of the eye this coat is replaced by the *iris* (*I*), which forms an adjustable shutter capable of considerable alteration of aperture (from about 2 to 7 mm. diameter), according to the amount of light entering the eye. Its very rapid and absolutely automatic action in "stopping down" when a bright object is looked at, tends to prevent injury to the retina from sudden exposure to excessive radiation (see p. 55). The innermost coating of the eye, the *retina* (*R*), is an extremely delicate film, consisting chiefly of nervous fibres which spread out from the *optic nerve* over the whole of the anterior surface of the choroid.

The interior of the eye is filled with three transparent media. The first, which fills the space between the cornea and the crystalline lens, is nearly pure water, and is termed the *aqueous humour* (*a*). This has a refractive index of 1·336. Just behind the iris is a lens formed of a substance resembling very thick jelly or soft gristle, and having a refractive index of 1·45 at the centre, changing to 1·41 at the edge, so that the effective index is 1·437 ([2]). This lens is double convex, with the posterior surface of greater curvature than the anterior. It is suspended in its place by a set of little bands proceeding from the choroid coat, and known as the *ciliary processes* (*ci*). The whole of the space enclosed between the crystalline lens and the retina is filled with a thin jelly termed the *vitreous humour* (*v*), which has a refractive index of 1·338.

Accommodation.—When an object is looked at, waves of light proceeding from its various points enter the eye at the cornea and are refracted by the various media so that an image of the object is formed on the surface of the retina. Now the position of an image produced by any optical device depends upon the relative positions of the object and of the refracting elements of the device (see p. 22). Hence, in order to produce a sharp image on the retina for objects at different distances, the refracting elements of the eye must be adjusted to suit the position of the particular object viewed at any given instant. This adjustment, termed *accommodation*, is achieved almost involuntarily by means of the ciliary muscles, which produce a slight change in both the position and the curvature of the crystalline lens. The effect of these changes was described in the last chapter (p. 22).

When light enters the eye at the cornea, it undergoes refraction at the surface, according to equation (1), p. 22, with r_1 put equal to 7·8 mm. It undergoes further similar changes due to refraction at the surfaces separating the various media within the eye, and finally an image is formed on the retina if the crystalline lens be properly adjusted in position and curvature.

When the eye is at rest it is focussed for distant objects, and the retina is at the position of the principal focus of the refracting system, *viz.*, 15·9 mm. behind the posterior surface of the lens, while the radii of curvature of the front and back surfaces of the

lens are respectively 10 mm. and 6 mm When the eye is accommodated for seeing near objects, these radii are changed to 6 and 5 5 mm. respectively, while the front surface of the lens is shifted outward by about 0 4 mm. The limit of accommodation is generally for objects about 25 cm from the eye for comfortable vision, although nearer objects may be accommodated with a certain amount of strain. The power of accommodation generally decreases with age

The Mechanism of Vision.—So far a description has been given of the optical method by which objects seen form an image on the retina (3) The mechanism by which this image is conveyed to the brain so as to produce the impression of sight is still obscure. The retina consists of a large number of elements (of the order of five million in all) of two kinds, termed respectively the "*rods*" and the "*cones*" (4). These are shown diagrammatically in Fig 23, where a

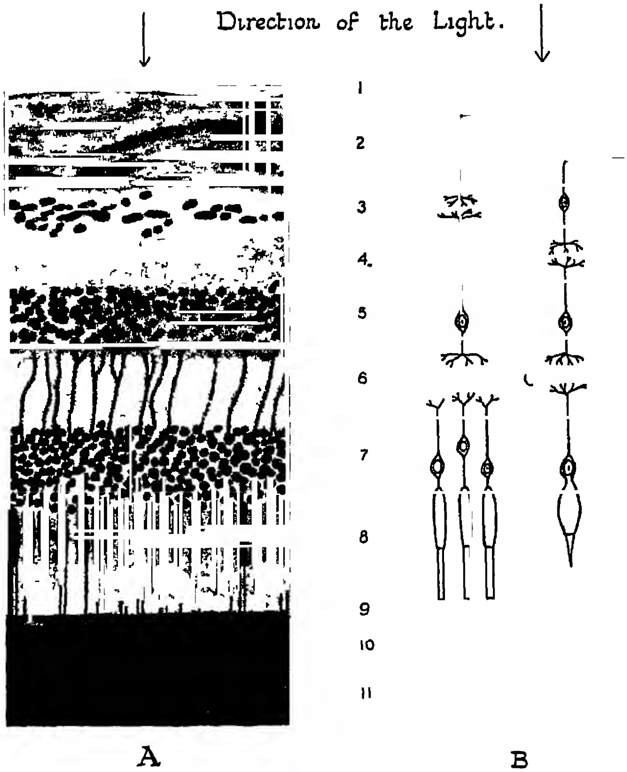

FIG 23 —Section through the Retina

A Semi-diagrammatic *B Schematic only*

1 Membrana limitans interna.
2 Nerve layer
3 Layer of ganglion cells
4 Inner reticular (molecular) layer
5 Inner nuclear layer
6. Outer reticular layer
7 Outer nuclear layer
8 Layer of rods and cones.
9 Pigmentum epithelium.
10 Choroidea.
11. Sclera

magnified section through a portion of the retinal layer is also given. When the retina is illuminated the cones become shorter and thicker, their tips receding from the normal unilluminated position. In and

around the rods and cones flows a peculiar photochemically sensitive fluid, which in the dark is of a deep reddish-purple colour, but which is rapidly bleached on exposure to light [5]. This fluid is called the visual purple, and there seems now to be no doubt that it is in some way connected with the phenomenon of vision. It has been found, for instance, that the rate at which it is bleached under radiation of different frequencies is proportional to the sensitivity of the human eye to light of those frequencies (see p. 73).

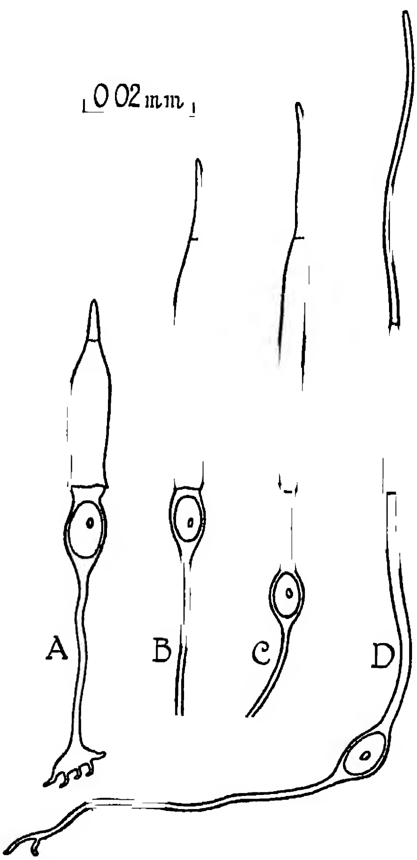

FIG. 24.—Cones from different parts of the Retina

A Equatorial Region
B Periphery of Macula
C Margin of Fovea
D Centre of Fovea

The rods and cones are not equally distributed over the surface of the retina. There is at a point about 6° below and inwards of the optical centre of the eye an area, some 2 to 2.5 mm. in diameter, over which there are very few rods at all, while the cones are very fine in structure and very closely packed (see Fig. 24). It is significant that it is at this region of the retina (termed the *macula lutea*, M in Fig. 22) that vision is most clear, and the muscles controlling the eyeballs always move these so that the image of an object, or part of an object, to which attention is being directed at any instant is formed in each eye on the very centre of the macula, where vision is at its sharpest. This centre, termed the *fovea centralis*, is 0.24 to 0.3 mm. in diameter, and consists entirely of cones of extremely fine structure, rods being completely absent over an area about 0.8 mm. in diameter in the centre of the macula. Over the other parts of the retina the proportion of rods to cones steadily increases with distance from the macula, until at the periphery they preponderate in a ratio of about 10 to 1. The diameter of the cones varies from 0.0015 to 0.0054 mm. at the fovea. This is approximately the same as the image size of the smallest detail of an object which the optical

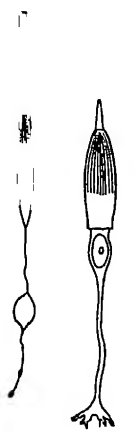

FIG. 25.—Rod and Cone from the Extra-Foveal Part of the Retina (Greeff)

system of the eye is capable of resolving, for, on account of interference of the light waves, two points cannot be distinguished if their angular separation at the eye is less than $1 \cdot 2/\nu D$, where D is the diameter of the pupil. Since the refractive index of the eye is 1 34, this angle becomes $1 \cdot 2/1 \cdot 34\nu D$, and at a distance of about 22 mm this produces images which are about 0·003 mm apart, for $\nu = 16{,}400$ and $D = 0 \cdot 4$ cm It has been stated that two points cannot be resolved unless their images on the retina are separated by at least one unaffected cone (6). Many theories have been put forward to explain the phenomena of vision, but as most of these find their most searching test in the explanation they afford of the phenomena of colour vision, consideration of them will be postponed until after these phenomena have been described.

From the descriptions of photometric apparatus to be found in later chapters of this book it will be seen that the eye is called upon to perform various tasks, *viz* (*a*) to perceive small differences in the brightness of adjacent surfaces of either the same or different colours, (*b*) to perceive small differences of colour between equally bright adjacent surfaces, and (*c*) to perceive the flicker due to rapid alternations of either equally bright but differently coloured surfaces, or identically coloured but unequally bright surfaces. The phenomena which do not involve colour differences will be dealt with first, and of these sensitivity to small differences of brightness is the most important

Contrast Sensitivity.—In 1858 G T Fechner enunciated the law (7) that as the stimulus to the eye increased in geometric progression, the resulting sensation increased in arithmetic progression According to this law the difference of brightness just perceptible, δB, is proportional to B, and the ratio $\delta B/B$ is termed *Fechner's fraction* The researches of A Konig and E Brodhun (8) have shown that, while this ratio is fairly constant over a very wide range, it ceases to hold for values of B below about thirty candles per sq. metre (9) It should be mentioned here that as the illumination of the retinal image varies as the size of the pupil, which is inconstant and not under control, it is usual in sensitivity measurements to reduce all the results to those which would be obtained with a standard pupillary aperture of 1 sq. mm (*) For normal brightnesses it must be remembered that the actual illumination of the retinal image is probably at least ten times as great as this, while for very low brightnesses it may be thirty to forty times as great.

In Fig 26 are given curves showing the relation between $\delta B/B$ and B for white light and for light of various frequencies (For the sake of convenience, values of $\log_{10} B$ instead of B have been taken as abscissæ) It will be seen that for values of B above about 30 photons the Fechner fraction falls very slowly to a practically constant value of about 1 8 per cent With a normal pupillary aperture of 10 sq. mm this critical brightness would be three candles per sq metre This is a normal illumination for photometric work, and it is recognised that if lower illuminations have to be used the

* The name "photon" has been given to the unit of retinal illumination, *i e*, the illumination of the retina when the brightness of the object looked at is one candle per square metre and the aperture of the pupil is 1 sq mm (10).

sensitivity is reduced (11). At low intensities the contrast sensitivity for red light is much less than that for white or blue light.

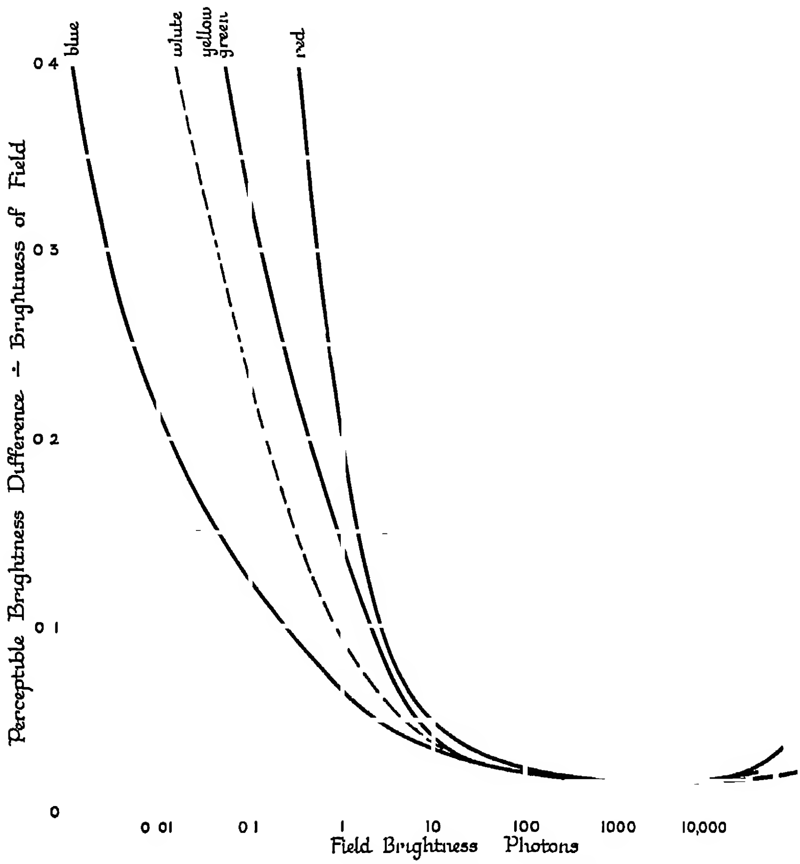

FIG. 26 —Variation of Contrast Sensitivity with Brightness

Various more or less empirical expressions have been proposed to fit the contrast sensitivity curve at all intensities (12) None of these, however, is really satisfactory

The contrast sensitivity of the eye is reduced, especially at low values of field brightness, by reduction in the angular size of the field of view (13)

Since equality of contrast indicates equality of *ratio* of brightness, it has been suggested that when photometric observations are reduced the mean taken should be the geometric (antilog of mean log) instead of the arithmetic (14) The difference is generally too small, however, to be appreciable.

Visual Acuity.—The ability to distinguish detail is very closely connected with the brightness of the object viewed, and the contrast which its details present to the eye Individuals, as is to be expected, vary widely in this respect, but it has been found that in the case of black type printed on white paper there is always a limiting brightness above which no marked increase in visual acuity is perceptible.

The curves of Fig 27 show the results obtained by Dow for red and green lights (15) The ordinates represent, on an arbitrary scale, the

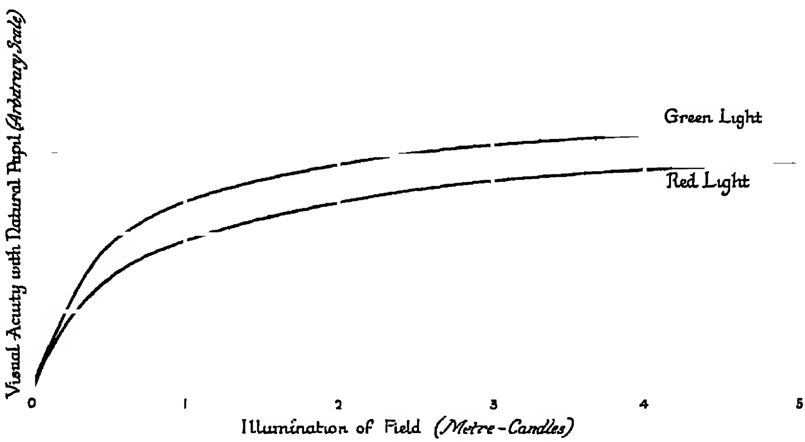

FIG. 27.—Visual Acuity.

fineness of detail which can be recognised with the illuminations represented by the abscissæ. These results agree well with those obtained in other experiments in which either speed of reading, the recognition of black type on white or coloured paper, or the discrimination of threads in a woven material, was used as the acuity criterion (16)

The dependence of visual acuity on colour and brightness may be described generally by the statement that decrease in the Fechner fraction results, *ceteris paribus*, in an increase of visual acuity. It appears, however, that acuity is higher with a monochromatic than with a composite light (17) The subject is complicated by the increase of acuity brought about by pupillary contraction (see below)

Forms of illuminometer for rough photometric work have been based on the ability to distinguish detail as a criterion (see p 236 and note (32), p 373).

Dark Adaptation of the Eye : Pupillary Diameter.—The enormous range of sensitivity possessed by the eye can readily be appreciated from the fact that the ratio of the brightness of objects seen by direct sunlight to that of the same objects seen by starlight on a clear moonless night is at least 10 million to 1. So perfect is the adaptation over a very extensive part of this range that alterations of brightness in a ratio of as much as 100 to 1 are scarcely noticed if the change be not too sudden, for a book may be read quite as comfortably in a room which is well lighted by artificial means as under the open sky at noon on a fine day in winter when the brightness of the pages is at least 200 times as great (18) This power of adapting itself to different conditions of lighting makes the eye useless for the direct measurement of luminous energy, and, as has been said already, photometry depends on the eye solely for the determination of the *equality* of two adjacent fields as regards either brightness or contrast (19).

The process of adaptation, although so thorough, takes some little time for its completion, and the inability to see detail when the eye is brought suddenly from dark to bright surroundings, or *vice versâ*, is evidence of this The contraction or opening of the iris, it is true, takes place very rapidly, but this motion is limited to a comparatively small range, the minimum light admitted being at least one-twentieth of the maximum It has been said, in fact, that the contraction of the iris under bright light is not so much for protection as for the improvement of the image on the retina, since it is a well-known consequence of the optics of image formation by lenses that the smaller the aperture through which the light is admitted, the more perfect the image formation through the elimination of the defect known as spherical aberration (see p 23) ([20]) Naturally a

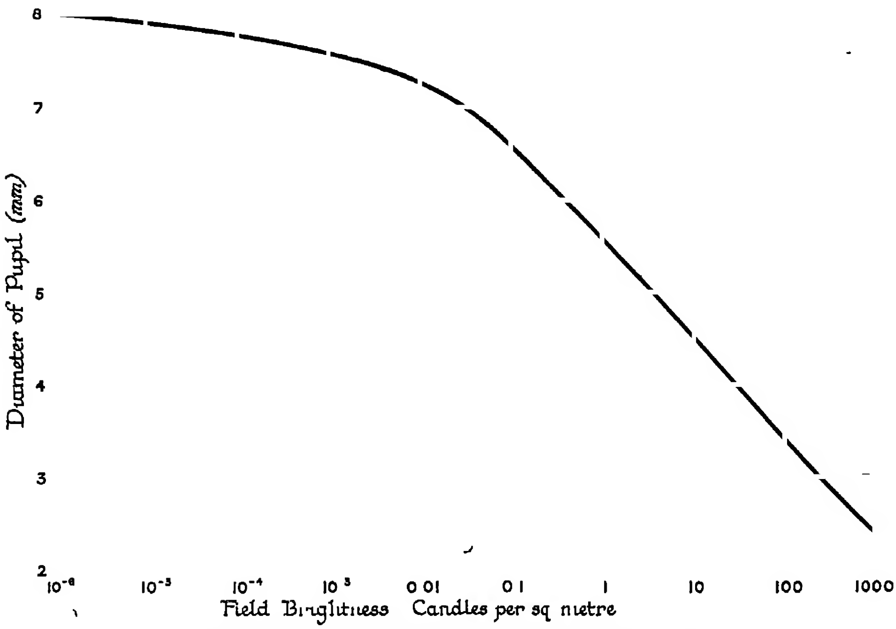

FIG 28—Variation of Pupillary Diameter with Field Brightness

limit is set to improvement in this direction by the fact that the brightness of the image varies as the area of the aperture, and it is therefore only at comparatively high intensities that the iris is able, by stopping down, to improve the definition of the retinal image without decreasing its brightness below that required for easy vision In support of this conclusion it may be mentioned that the iris also contracts when the eye is looking at very close objects (within about 30 cm), a condition under which the effect of spherical aberration is more marked The variation of pupillary diameter with brightness of the object looked at is shown in Fig. 28 ([21]) This variation has been made the basis of a suggested method of absolute photometry ([22])

Since the range of adaptation is so much wider than can be accounted for by changes of pupillary diameter alone, it follows that

the greater part of the process must be located in the retina, which in some way alters its sensitivity to suit the order of brightness of the images formed on it The mechanism by which this is accomplished is not yet understood, but it is a phenomenon of extreme importance in photometry, especially when surfaces of exceptionally low luminosity are being compared ([23]) The retinal sensitivity limit (defined as the reciprocal of the minimum observable, or threshold, brightness in photons) is, for the dark-adapted eye, about 7,000 For the same eye immediately after continued exposure to 2,000 photons the sensitivity limit is reduced to about 0 02 ([24]) Adaptation takes

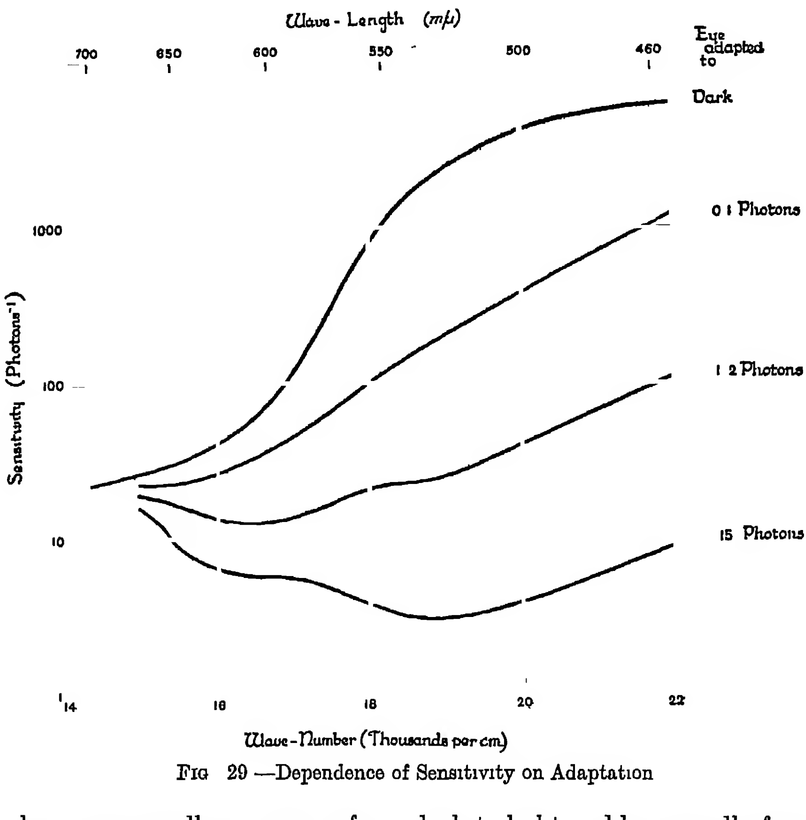

FIG 29 —Dependence of Sensitivity on Adaptation

place very rapidly in passing from dark to light and less rapidly from light to dark In fact, the comparatively slow increase of sensitivity of the eye suddenly brought into a dark room is a matter of common observation It has been found that a period of at least ten minutes is required for dark adaptation after the eye has been accustomed to normal daylight conditions, while full adaptation takes at least an hour after exposure to very intense illuminations ([25])

The increase of sensitivity varies at different parts of the spectrum, and the curves of Fig 29 show the relative sensitivity limits (photons^{-1}) throughout the visible spectrum of the eye adapted to various degrees of brightness ([26]) The minimum illumination of the retina required to excite the sensation of (colourless) vision in the dark adapted eye has been studied by many workers ([27]) (see p. 67)

Glare : Effect of Lateral Illumination.—The temporary loss of sensitivity of the eye due to simultaneous or recent vision of an object, the brightness of which much exceeds that of the object looked at, is known as glare. It has been defined as "embarrassment of the eyes or vision associated with strong light sensation" ([28]), and is probably due to the fact that "the existence of (relatively) excessively bright areas within the field of vision tends to shift retinal adaptation toward that required for the brighter area. Whatever the process of adaptation, it is probable that one part of the retina is affected to some extent by the adaptation of another part" ([29]). Thus, lack of sensitivity to contrast in a darker part of the field of view is the consequence of a general raising of the adaptation level to something approaching that demanded by the adjacent brighter parts ([30]).

There is but little quantitative work available on the subject ([31]), for one of the difficulties in defining or measuring glare is the fact that it does not seem to be simply related to visual acuity. Thus Cobb ([32]) finds that, while for objects of relatively low brightness the presence of a surrounding field of relatively high brightness has the effect of lowering both the contrast sensitivity and the visual acuity, yet in the case where the surrounding field is slightly brighter than the test object, both visual acuity and contrast sensitivity are better than for a similar object surrounded by a dark background ([33]). Surroundings of a brightness equal to or less than that of the test object appear to give the same results as dark surroundings.

Cobb also investigated the effect of a bright source of light in the field of view of an eye looking at a test object ([34]). He found that the visibility of an object was reduced by an amount which increased with the brightness ratio of the source and object, and with diminution of their angular separation ([35]), except when the test object was very bright, in which case the presence of the lateral illumination produced an increase of visual acuity. Cobb also concluded that the retinal image of the lateral light did not cause any interference with vision (at least with a separation of 15° or over), but that this interference was due to light emanating from the bright source and scattered over the retina by reflection or diffusion in the eye media. This scattered light may be a sufficiently small fraction of that transmitted to cause no diminution in the clearness of an object looked at, but in the case of a very bright source it may seriously interfere with the sharpness of the image of a much less brilliant object ([36]).

There seems, then, to be no close parallelism between the impairment of contrast sensitivity or of visual acuity on the one hand, and the visual disturbance and discomfort generally described as "glare" on the other. It seems that "the unpleasant feeling of dazzling and the disturbance of vision produced by dazzling are totally different things, and need by no means necessarily occur to the same extent at any given time" ([37]). Nutting has found that to an eye adapted to a field brightness of F candles per square metre the brightness of field which is just sufficient to produce discomfort is given by $G = 3,600\,F^{0.32}$ ([38]).

In photometry, no doubt, it is the disturbance of vision which it is necessary to avoid as much as possible. At the same time it is

well known that anything which tends to the discomfort, not of the eye alone, but of any organ of the body, induces fatigue and a consequent impairment of the capacity of the eye for accurate work, so that from this point of view any condition which can be described as glaring, even to a slight extent, must be avoided if the highest degree of photometric precision is to be achieved This point will be further dealt with in a later chapter, when the psychology of photometric measurement is under consideration It is to be noticed that a highly polished surface, by directly reflecting light from a source to the eye of an observer, may cause glare in the same way as a self-luminous source ([39])

Veiling Glare.—Another so-called form of glare is the loss of sensitivity due to the interposition, between the eye and the object looked at, of a "veil" of luminous material, as, for example, a very slightly diffusing surface The reduction of sensitivity is in this case simply due to a reduction of contrast, for it is clear that, if the brightness of the luminous surface be equal to that of the object looked at, small contrasts on the latter are reduced in effect to one-half of their true values. It is probable, too, that a tendency of the eye to focus on the "veiling" surface produces a further loss of efficiency.

Variations in the Sensitivity of the Extra-Foveal Retina.—What has been stated above has referred almost exclusively to foveal vision, naturally the most important for practical purposes. It is seldom realised how very imperfect is the outer part of the retina for accurate image perception Its main function is to give a more or less vague impression of the field surrounding the object looked at, and so to act as a "finder" for the macula, which, by means of very slight and rapid movements of the eyeball, is made to "feel over" the whole of the image just as the finger feels over a surface in order to obtain an accurate knowledge of its details. The limits of the visual field for a fixed right eye are shown in Fig 30, where the curves marked B, Y, R and G show the limits of the field over which perception of the colours blue, yellow, red, and green is possible with moderate brightness ([40]). As might be expected, the visual acuity varies greatly over this field, and the full curve of Fig 31 shows the order of the variation across the retina along a line passing through the fovea (F) ([41]) This curve applies to the light-adapted, or photopic, eye ([42]). In the dark-adapted, or scotopic ([42]), eye the acuity is much more constant over a range of 50° on either side of the fovea (broken line of Fig 31), and, in consequence, with the dark-adapted eye the advantage of central vision is much less marked. It need scarcely be remarked that the curves of Figs. 30 and 31 are liable to considerable variations from one individual eye to another.

Talbot's Law.—So far it has been assumed that the illumination of the object looked at, and therefore of the retinal image, has remained steady, but the effect of rapid variations of illumination is of very great importance in photometric work, as will be seen when considering (a) the sector disc, in which the effective brightness of a surface is reduced by a rapid periodic extinction of the illumination, and (b) the flicker photometer, in which the illumination of the field under observation is rapidly alternated from one intensity, or one colour, to another.

The sector disc is very widely used, and the principle on which it is founded was first proposed by Fox Talbot in 1834, and may be thus stated "If a point of the retina is excited by a light which undergoes regular and periodic variations, and which has the duration of its period sufficiently short, it produces a continuous impression equal to that produced if the light emitted during each period were distributed uniformly throughout the duration of the period" [43] It follows that the apparent intensity of an intermittent light bears to the actual intensity the ratio of the time of

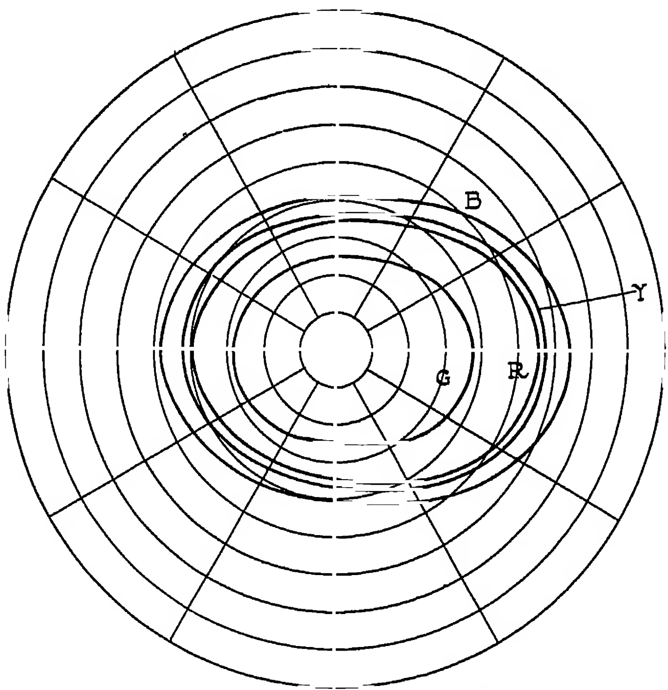

FIG. 30—The Field of Vision for Different Colours at Moderate Brightness: Right Eye. (After Abney *Reproduced by permission of the Royal Society*)

Approximate Brightnesses

Blue, 1 2 c/sq m Red, 0 9 c/sq m Yellow, 13 c/sq m Green, 6 3 c/sq m

exposure to the total time, provided the speed of alternation exceeds that at which flicker ceases to be perceptible. In this form it is known as Talbot's law [44] (or sometimes the Talbot-Plateau law), and it has been found to hold at all flicker speeds above the necessary lower limit [45], and down to a ratio of at least 3 per cent. for light of all colours, with an accuracy of at least 0·3 per cent [46]

This law would be a natural consequence if the eye responded instantaneously to changes of illumination, or if the rates of sensation change were identical for both increasing and decreasing illuminations. The first alternative cannot be true, or flicker would never disappear, and the well-known phenomenon of persistence of vision would be absent [47] Like all physiological actions, the response of the eye to light, though rapid, is far from instantaneous [48]. It

has been found, in fact, that when light is admitted to the retina the sensation rises rapidly to a maximum which is in excess of its final steady value This is possibly due to the fact that the final steady state is the result of an equilibrium between a "positive" luminous sensation effect and a "negative" fatigue effect, and this equilibrium is only attained at an appreciable time after the incidence of the illumination The length of this time depends on the intensity of the illumination, being measured in seconds, or even minutes, for

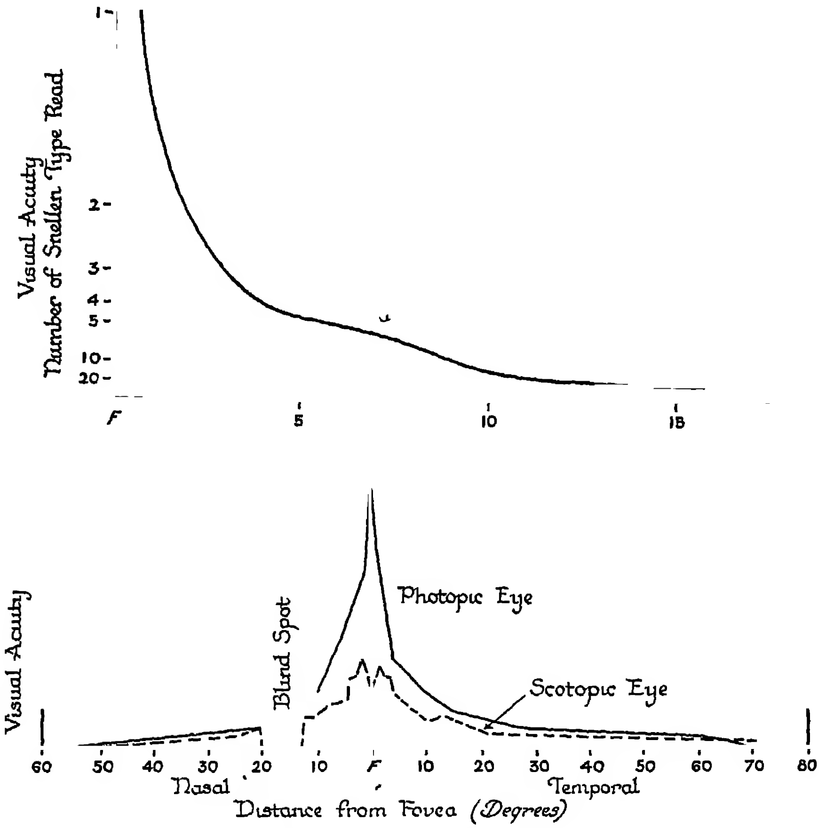

FIG 31 —The Variation of Visual Acuity across the Retina (Upper curve after H Dor Lower curve after H Piéron)

very low illuminations, while at ordinary intensities it is only a few hundredths of a second The results of experimental work on this point ([49]) are given in Fig 32, in which is shown the behaviour of lights of different colours at certain definite intensities ([50]). Blue light produces by far the greatest overshoot, the sensation at 0 07 sec after initial exposure being at least five times as great as the final equilibrium value (94 photons) The effect is less marked for white and red lights (equilibrium value 124 photons), while with green it is comparatively slight In the case of point sources of light there is no overshoot, except a very slight one for blue light ([51]). The time lag in the perception of a decrease of

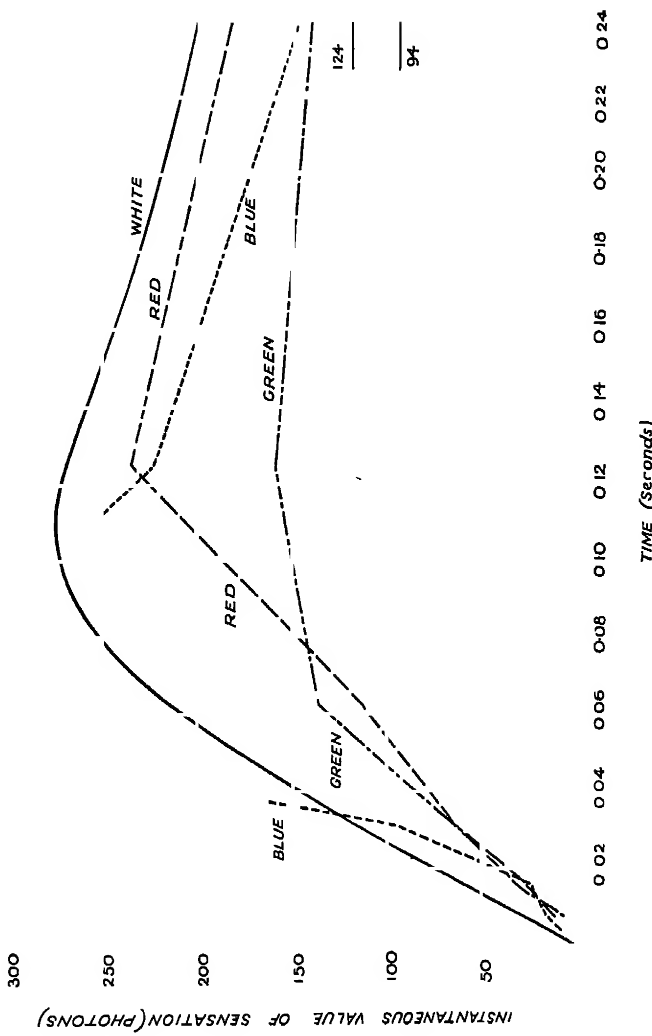

FIG 32 —The Rise of Visual Sensation.

brightness is of importance in the photometry of a rapidly decaying luminescence (52)

A cognate problem is the determination of the time necessary for the perception by the eye of a given amount of detail This also depends on the brightness and colour of the field of view (53)

Flicker Sensitivity : Persistence of Vision.—In the statement of Talbot's law given above the condition was laid down that the alternations of brightness should be sufficiently rapid to avoid any sensation of flicker The alternation period at which this sensation disappears depends on both the colour and intensity of the light. The period which may elapse between successive exposures without the appearance of flicker has been termed the duration period, or

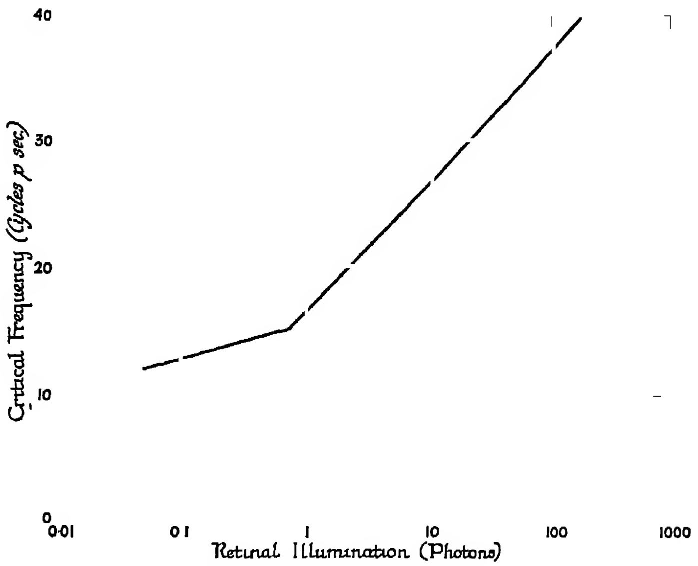

FIG. 33 —Critical Frequency

the persistence. It has been measured at various intensities (54), and the results found are shown in Fig. 33, in which the abscissæ are retinal illuminations in photons (to a logarithmic scale), while the ordinates are the number of complete cycles per second at the limit of the flicker sensation for white light, *i.e.*, half the number of alternations between light and darkness which the eye is just incapable of distinguishing from a steady exposure when the period of light is equal to the period of darkness This curve refers only to foveal vision. The persistence is somewhat less for the extra-foveal parts of the retina, so that flicker may be perceived by indirect vision when it is absent by direct vision (55) For coloured lights it has been found that the persistence is of about the same order throughout the visible spectrum so long as the luminosity remains the same (56). (See also p 249.)

In practical photometry a more important condition than that of alternation of light with complete darkness is the alternation of

two lights of slightly differing intensities. Since one method of photometry depends on the disappearance of flicker when the alternating brightnesses are equal, it is naturally of the utmost importance to determine for variously coloured lights, and at various speeds of alternation, the difference of brightness which just produces the flicker sensation. This has been studied extensively by Dow ([57]), who has found that the percentage difference of brightness detectable as flicker depends on both the intensity and frequency of alternation, the minimum for white light being about $1\frac{1}{2}$ per cent. at intensities of 10 metre-candles or over (natural pupil), between alternation

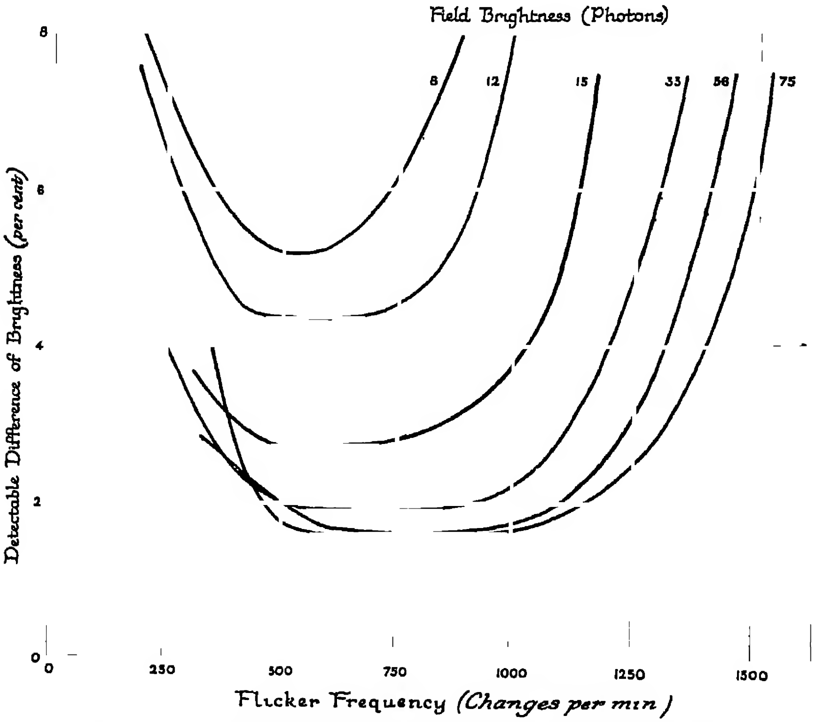

FIG 34 —Detectable Brightness Difference at various Flicker Speeds

frequencies of ten to fifteen per second. The results for white light are shown by the curves of Fig 34 ([58]).

Visual Diffusivity.—The time lag in the perception of light and the phenomena of flicker have led Ives to propose the theory that the transmission of impressions from the retina to the brain takes place in a manner analogous to the conduction of heat through matter having a coefficient of diffusivity, this coefficient in the case of the physiological phenomenon differing in value with the frequency and intensity of the incident radiation ([59]). This subject will be dealt with further in Chapter VIII, when the theory of the flicker photometer is considered.

Colour Sensitivity : The Luminosity (Visibility) Function.—It has already been stated (p 17) that the eye as a receptor of energy is not equally sensitive to light waves of all frequencies, in other

words, equal quantities of energy in the forms of blue and yellow lights do not produce the same amount of visual sensation In fact, since radiations of different frequencies produce effects on the eye which differ in kind (colour) as well as in degree (intensity), it follows that no real equality can ever be attained On the other hand, if the difference of colour be not too pronounced it is a matter of common experience in photometry (where, in fact, exact identity of colour is the exception rather than the rule) that the eye can give an equality judgment which can not only be repeated with reasonable accuracy by the same individual, but which will also agree with similar measurements made by other eyes so long as these do not possess any marked peculiarity as regards colour vision. "The mind appreciates the illuminations as equally bright when they make equal claims on the attention What determines this claim is obscure, and in any case does not concern us here, the important point is the experimental fact that for radiations of different wave-lengths there is an unique relation between the quantities received by unit area of the retina for which the radiations will be regarded as equally bright" ([60]) The inverse ratio of the energies, in radiation of two different frequencies, which will produce the same visual effect is defined as the relative luminosity (or visibility) at these two frequencies, and the reciprocal of the energy required to produce a certain degree of visual sensation at any frequency may, therefore, be termed the "luminosity" of radiation of that frequency The luminosity of a given heterochromatic radiation is, therefore, the sum total of the products found by multiplying the energy at each frequency by the luminosity of radiation at that frequency. This function is the factor which converts the physical quantity "energy" into the psycho-physiological quantity "light" It is, therefore, of the utmost importance in photometry, and great care has been devoted to its accurate determination at all frequencies within the visible spectrum (see later, Chapter X, p 294) The values obtained for normal values of field brightness (30 photons or over) are shown in tabular form in Appendix IV, p. 471, and graphically in Fig 186, p 295

The original determinations made by Konig ([61]) included also results at various degrees of brightness, and from these the table shown on page 65 has been calculated ([62]) to show the variation of the luminosity function with intensity The table shows very clearly that from the threshold of vision up to about 0 5 photon there is but little alteration in the luminosity function, while from 15 photons upwards a second steady state is reached. Between these limits, however, there is a region of marked change, as shown in Fig 35, where the frequency of maximum luminosity is plotted against intensity This region is, as will be seen in the next paragraph, of great importance in photometry

Before leaving the luminosity curves mention must be made of the mechanical equivalent of light. This has been variously defined, but it is now generally taken to mean the rate of energy flow (measured in watts, or in ergs per sec) which, if entirely concentrated in luminous radiation of the frequency of maximum luminosity, would be equal to one unit of luminous radiation as evaluated photometrically (see also p. 296)

Table of Values of the Luminosity (Visibility) Function throughout the Spectrum at various Values of Field Brightness. (Recalculated from A. König's values)

Brightness (Photons).		0 00016	0 0015	0 0242	0 385	1 54	6 16	24 6	98 6	398·5
λ (mμ)	ν −100									
430	233	0 081	0 093	0 127	0 128	0 114	0 114	—	—	—
450	222	0 33	0 30	0 29	0 31	0 23	0·175	0·16	—	—
470	213	0 63	0 59	0 54	0 58	0 51	0 29	0 26	0 23	—
490	204	0 96	(0 89)	(0 76)	(0 89)	(0 83)	0 50	0 45	0 38	0 35
505	198	1 00	1 00	1 00	1 00	0 99	(0 76)	0 66	0 61	0 54
520	192	0 88	0 86	0 86	0 94	0 99	(0 85)	0 85	0 85	0 82
535	187	0 61	0 62	0 63	0 72	0 91	(0 98)	0 98	0 99	0 98
555	180	0 26	0 30	0 34	0 41	0 62	0 84	0 93	0 97	0 98
575	174	0 074	0 102	0 122	0 168	(0 39)	(0 63)	(0 76)	(0 82)	(0 84)
590	169	0 025	0 034	0 054	0 091	0 27	0 49	0 61	0 68	0 69
605	165	0 008	0 012	0 024	0 056	0 173	0 35	(0 45)	0 54	0 55
625	160	0 004	0 004	0 011	0 027	0 098	0 20	0 27	0 35	0 35
650	154	0 000	0 000	0 003	0 007	0 025	0 060	0 085	0 122	0 133
670	149	0 000	0 000	0 001	0 002	0 007	0 017	0 025	0 030	0 030
λ_{max}		503	504	504	508	513	530	541	543	544
ν_{max}	−100	199	198	198	197	195	189	185	184	184

The Purkyně Effect.—It has long been known that if a red field and a blue field are illuminated so as to have the same brightness, a reduction of the illumination of both fields in the same proportion will cause the red field to appear darker than the blue after a certain limit of reduction has been reached ([63]) This is on account of the

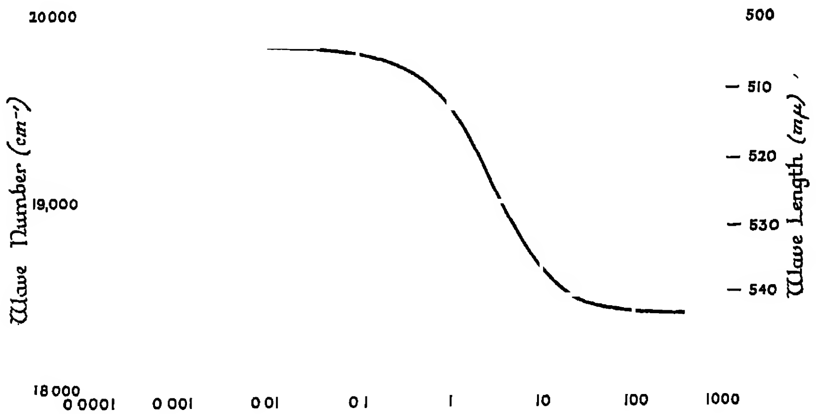

Fig 35 —Frequency of Maximum Luminosity

change in the luminosity function at low intensities (see previous paragraph) It is not noticeable for frequencies from $\nu = 20{,}000$ to $\nu = 25{,}000$, as the ratio of the ordinates of the luminosity curves at high and low intensities is approximately constant over this range ([64]). The ratio, however, changes rapidly between 15,000 and

20,000, so that a ratio of 1 : 3 at 21,000 becomes a ratio of 15 1 at 17,000. The intensity at which this effect becomes noticeable is found from Fig 36, which shows the relation between the ratio K_{ν_1}/K_{ν_2}, and intensity for the pair of wave-numbers $\nu_1 = 16{,}900$ and $\nu_2 = 19{,}200$ (65).

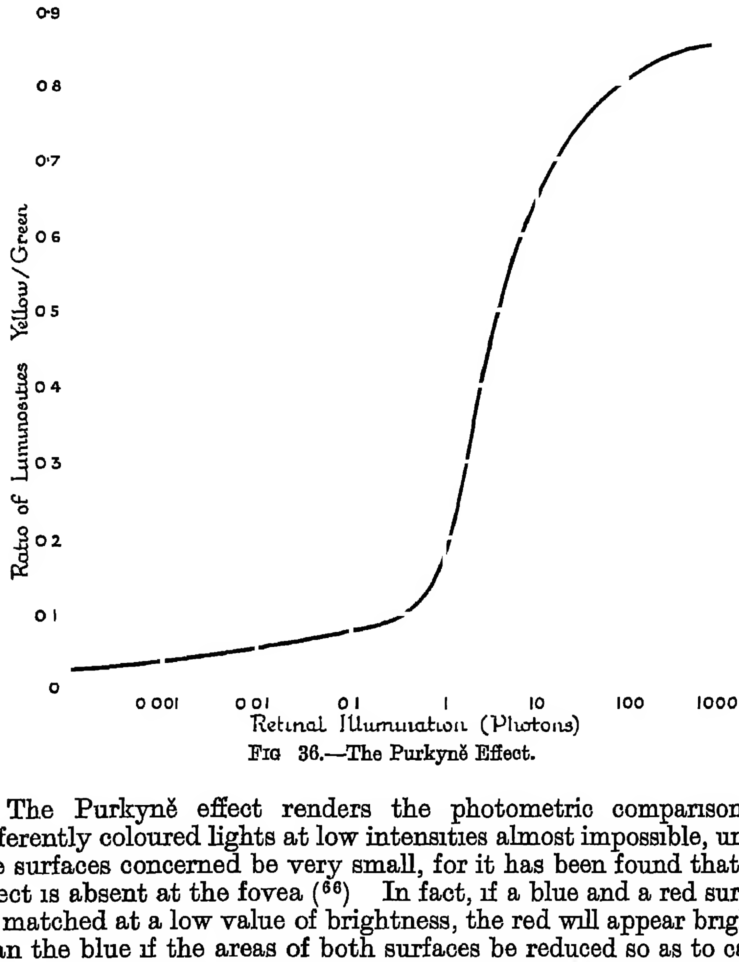

FIG 36.—The Purkyně Effect.

The Purkyně effect renders the photometric comparison of differently coloured lights at low intensities almost impossible, unless the surfaces concerned be very small, for it has been found that the effect is absent at the fovea (66) In fact, if a blue and a red surface be matched at a low value of brightness, the red will appear brighter than the blue if the areas of both surfaces be reduced so as to cause both images to fall on the central (rod-free) part of the macula lutea This phenomenon is known as the "yellow-spot effect"

Chromatic Sensitivity.—The number of parts into which the visible spectrum can be divided so that each part can just be distinguished in hue from the parts next to it varies greatly with individuals, *viz*, from about thirty to fifty, even for those with what would be termed "normal" colour vision (67). A considerable percentage of individuals have defective colour-vision to a greater or less extent, though, unless pronounced (as in the really "colour blind"), it often remains unsuspected for a long time The eyes of such individuals naturally have not the same form of luminosity curve as that shown in Fig. 186, and their readings in photometry where there is any

considerable colour difference are consequently unlike those that would be given by the normal eye (see p. 263).

It has been found that the just perceptible difference of frequency which can be detected by the normal eye is different at different parts of the spectrum The curves of Fig. 37 show the "chromatic sensitivity" (least perceptible wave-number and wave-length difference respectively) throughout the spectrum ([68])

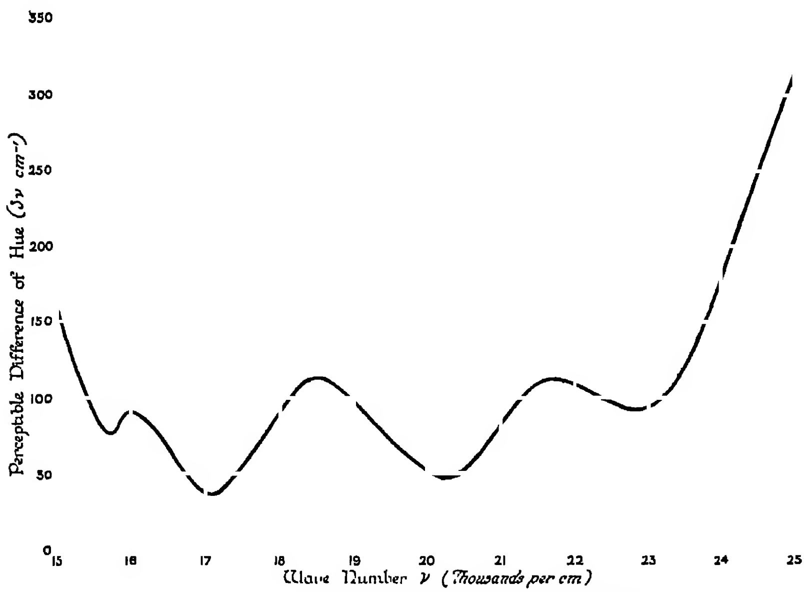

FIG 37A —Chromatic Sensitivity (Wave-Number Basis)

In connection with the measurement of colour (see Chapter X, p. 307), it is of interest to know what is the least proportion of white light which can be added to any given coloured light before the change of hue becomes perceptible to the eye This subject has been studied by several workers, but the results obtained are, as might be expected, somewhat diverse ([69]). The least perceptible increment of colour depends on the intensity and degree of coloration of the field, and on the hue of the added colour The least amount of a pure colour which is just noticeable when added to white is of the order of 1 per cent for the more saturated colours, red and blue, and about 2 per cent for yellow and green

Extinction of Colour : Threshold of Vision.—It is readily shown by experiment that the sensation of colour is lost before the sensation of light, especially in the case of the higher frequencies ([70]). Curve *A* of Fig 38 shows the brightness at which recognition of colour ceases (mean of two observers) at different parts of the spectrum. Curve *B* shows the values at which the sensation of light ceases for a scotopic eye ([71]) (see also p 56). The ordinates are the logarithms (to base 10) of the intensities in photons It will be seen that at the red end of the spectrum both sensations disappear almost simultaneously, while at the blue end the colour is lost long before the

light has disappeared ([72]) At intermediate intensities the sensation is that of a nondescript grey. This interval between the threshold of light sensation and the lower limit of colour perception is known as the "photochromatic interval," and has been found to increase greatly from fovea to periphery ([73]). At the other end of the scale, it is to be noticed that at a sufficiently high intensity the tendency of all colours is to become yellowish-white.

Limit of Visibility.—It is not often in photometry that recognition

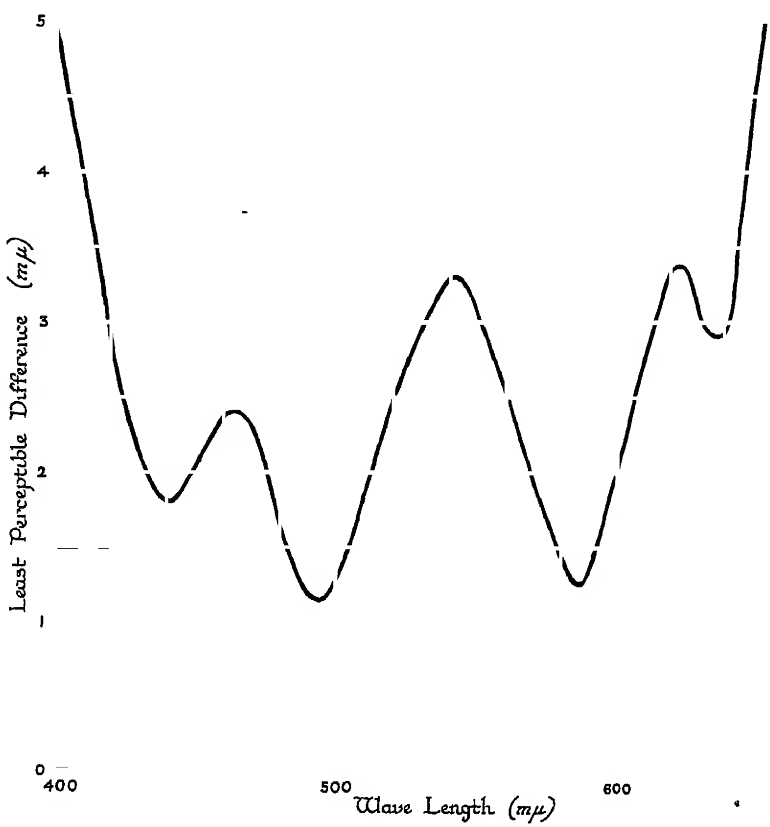

FIG. 37B—Chromatic Sensitivity (Wave-Length Basis)

of extremely small and faint sources of light is required, but this is of great importance in problems connected with the visibility of faint signal lights of various colours. At very low intensities it has been found that the visibility depends mainly on the total candle-power of the object, and very little on either its actual brightness or its size alone so long as it subtends an angle of less than 50′ at the eye ([74]) This means that for very low intensities the physiological effect of light is cumulative and independent of the area over which it is distributed up to the limit mentioned As the source increases in size to a diameter of about 4° the effect of accumulation becomes less, and above this limit the apparent brightness is proportional to the real brightness, provided the limiting amount of light (*i e.*, the

threshold amount required to produce visibility) is incident on the area occupied by a single cone of the retina. The amount of white light required just to produce visibility (the "threshold quantity") is that given by a source of a candle-power between 0 1 and 0 2 at a distance of about 1,000 metres This amount increases as the square of the distance between the source and the observer ([75]), so that results of this kind may conveniently be expressed in terms of the illumination of a surface placed in the position of the eye of the

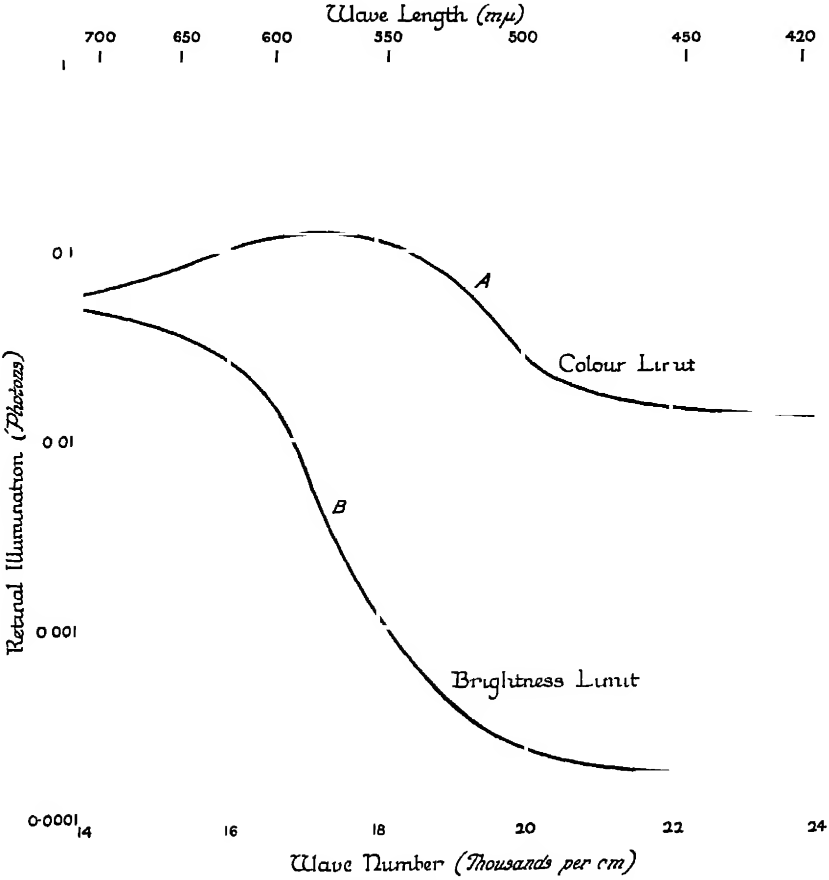

FIG 38 —Extinction of Light and Colour

observer The above figures of visibility for white light, *viz*, 1 to 2×10^{-7} metre-candles, refer to foveal vision. In the case of green or blue light, however, the threshold quantity is much less for regions of the retina at a short distance away from the fovea, so that these lights, even if invisible when directly looked at, may be "squinted" by directing the attention to a point at a small angular distance away from them

The visibility of coloured lights has been studied by Broca and Polack ([76]), who have found that for red lights the visibility is better by foveal vision, the illumination required at the eye in this case

being about 2×10^{-7} metre-candles, while for peripheral vision more than twice this illumination is required. The extinction point for colour is practically the same as that for light For blue-green light, peripheral vision is better than foveal, the illuminations required being about 2×10^{-8} metre-candles (77) and $1\ 2 \times 10^{-6}$ metre-candles respectively for recognition of light, and 8×10^{-5} metre-candles and 5×10^{-6} metre-candles for recognition of colour With white light recognition is easier by peripheral vision, the limiting illumination being 3×10^{-8} metre-candles For foveal vision the light appears white down to a limit of about 4×10^{-6} metre-candles, and below this it appears reddish, but is still recognisable as light down to the limit stated above. It follows that the Purkyně effect found in the case of extended surfaces is reversed for point sources when seen by *foveal* vision (78). The effect of a neighbouring bright light on the ability to distinguish the colour of a small source has been investigated by F Gotch (79)

The Visibility of Light of Brief Duration.—The figures just given refer to steady lights continuously visible for sufficiently long periods to avoid any time effect. Much work has been done on the visibility of flashing lights (80). It has been found experimentally that a point source giving an illumination E at the eye for a brief period of time t will be visible so long as $(E - E_0)t > aE_0$, where E_0 is the limiting illumination for visibility of a steady source of the same colour, and a is a constant which may have any value between 0 15 and 0 30 sec., according to the observer The expression is not valid if t be less than one-tenth or greater than about 2 to 3 secs (81).

Contrast and After-Image ; Spatial and Temporal Induction.—The effect produced in one portion of the retina by the illumination of a contiguous portion (spatial induction) is often referred to as " contrast " (82), since it is of the same kind as that which would result on the supposition that any stimulus applied to one part of the retina produces a partial fatigue of all parts in its immediate neighbourhood for that particular kind of stimulus (83). Thus the dark background of a small bright area appears to be darker near the edge of the bright patch than elsewhere, a bright red patch on a darker colourless background appears to be surrounded with a faint halo of a colour which is not far removed from its complementary green (84), and so on (85) The explanation of contrast effects may be partly psychological (86).

The peripheral part of the retina is particularly sensitive to brightness contrast, and this fact has been made the basis of a method of heterochromatic photometry (87).

Corresponding with the above phenomenon of " simultaneous contrast " there is a temporal effect known as " successive contrast." It is a matter of common observation that after the eye has gazed for some time at a bright object it will, on the gaze being transferred to a featureless white surface, show a more or less blurred image of the object previously looked at. If the original object be colourless, the after-image will be dark grey (88), while if the original object be strongly coloured, the after-image will be of the complementary colour (see p. 303, *infra*). This phenomenon is known as the " negative " or " complementary " after-image (89)

A rather more difficult after-image to observe is that which may

be noticed on closing the eye quickly after gazing at a bright light. The bright image seen is known as the "positive" after-image. It gives place to the negative after-image after a time interval depending on the relative brightnesses of the exciting object and of the surface to which the gaze is transferred ([90]). The colour of the positive or "homochromatic" after-image is the same as that of the exciting source.

A related phenomenon is that which may be noticed when a very small bright object, such as an illuminated pinhole surrounded by a dark field, is gazed at steadily for some time. The brightness appears gradually to diminish with lapse of time, and in the case of an object near the limit of visibility, it frequently appears to wax and wane if the gaze be directed to it continuously.

The Mechanism of Vision.—Vision is the result of the incidence of light on the retinal layer While self-luminous bodies are visible by reason of the light which they emit in the direction of the observer's eye, an illuminated body is only visible by reason of the light which it receives from some self-luminous body and returns to the eye by reflection from each element of its surface Frequently the light suffers many reflections in its passage from the luminary to the eye, but in all cases it is an image of the last reflecting surface (unless this be highly polished so as to act as a mirror) which is formed on the retina The retina receives radiant energy in the form of electromagnetic waves, the brain receives, by means of the optic nerve, the sensation of light It is the intermediate mechanism that must be considered in this paragraph, as on this depends the explanation of the relation between the energy stimulus and the visual perception, which is an all-important factor in the measurement of light

It is here that the physiological and psychological links in the chain connecting stimulus with impression must be considered In both of these regions, but especially the latter, the existing state of knowledge is still very incomplete. The connecting link between the retina and the cerebral region, which is the seat of vision, is the optic nerve, and this, although itself insensitive to light waves, can be stimulated apart from the retina, and then the sensation of light is produced according to Muller's law ([91])

The Trichromatic Theory.—Various theories have been put forward as to the manner in which the physical stimulus, the light energy, acts on the different elements in the physiological receptor, the retinal cerebro-neural system, so as to stimulate it in such a way as to evoke the sensations of light and colour Of these the earliest, and the one which still seems to afford a satisfactory groundwork, is the trichromatic theory of Thomas Young ([92]), elaborated by Hermann von Helmholtz ([93]) According to this theory, the receiving mechanism in the eye is three-fold, each part, when stimulated, giving rise to one of the primary sensations of *red*, *green* and *blue*. Each mechanism has a "luminosity curve" similar to that of the eye as a whole (see p. 295), the maximum in each case being in the spectral region which gives its name to the sensation These luminosity curves are shown in Fig 188 (p. 299) Their form has been arrived at by studying the phenomena of colour mixture and the vision of colour-blind subjects in whom, it is supposed on this theory, one or

more of these primary sensations is absent or deficient. The resultant effect produced on a normal eye by light of any given frequency is the mixed sensation due to the stimulation of the three primary sensations in the proportion indicated by the ordinates, at that frequency, of the three curves of Fig. 188 The sensation white results from the simultaneous stimulation of all three sensations in the correct proportions, and the curves of Fig 188 have been redrawn in Fig 190 (p. 301) in such a way that, when sensations represented by equal ordinates on each of these three curves are combined, the resultant sensation is white. The use of these curves for the specification of colour on a scientific basis and the study of the effects of mixing lights of different colours will be described in Chapter X., but, as an example of the adequacy of the Young-Helmholtz theory to account for the phenomena of colour mixture, the effect of superposing a red and a green light on the retina may be considered here

It will be seen from Fig. 190 that light for which $\nu < 14,900$ excites only one sensation, the red, while light of any other frequency excites two ($\nu < 17,200$), or all three sensations in varying proportions. It follows that when light from the red end of the spectrum is added in suitable proportion to spectrum green, the result will appear to the eye as a mixture of spectrum yellow together with white, for the blue sensation evoked by the green light will combine with equal amounts of red and green sensation to give white, and the remaining red and green sensations will, in combination, give the same sensation as a spectrum yellow. The effect thus predicted by the theory is in complete agreement with the experimental facts ([94]).

Many of the phenomena of colour vision described in the foregoing paragraphs may also be accounted for satisfactorily on the Young-Helmholtz theory. The Purkyně effect, for instance, may be explained on the assumption that the limits of the red, green and blue sensations are reached in the order named as the intensity of the light is reduced. The theory fails, however, to account for the colourless vision at low intensities To overcome this difficulty, J. von Kries proposed ([95]) the "duplicity" theory of vision, according to which the rods are the percipient agents for the sensation of luminosity, but have nothing to do with the appreciation of colour which resides in the cones alone. This theory, which must be regarded as an extension of the Young-Helmholtz theory, has the additional advantage of satisfactorily accounting for the regional variations of colour and light sensitivity mentioned in previous sections of this chapter (*e g*, pp. 58, 66), as a result of the manner in which the rods and cones are distributed over the retinal surface.

The growth of visual sensation and positive after-image must be regarded as due to a time-lag in the mechanism which translates stimulus into sensation "Over-shoot" and negative after-image may readily be explained by assuming that the action involved in the production of sensation also sets up an opposing action, which may be of the nature of fatigue of the physiological action or inhibition of the psychological perception ([96]). The equilibrium sensation resulting from continued constant stimulus is, then, the balance between action and inhibition when both have attained a steady value. The case is exactly analogous to a reversible chemical

system in which a supply of energy at a given rate results in a shift of the balance point between the two opposed reactions Contrast effects could result from the spread of inhibition to neighbouring areas, otherwise their basis must be placed in the psychological phase, *i.e*, they must be classed as "illusions," having no physical or physiological origin

The chief difficulty in reconciling the Young-Helmholtz-von Kries theory with the known facts of physiology is the total absence of any apparent triplicity in either the rods or the cones This has led F W. Edridge-Green ([97]) to make the visual purple (see p 51) the essential element in light perception Photo-chemical action in this substance stimulates the ends of the cones, and the nature of this stimulus, being dependent on the range of frequency of the light, is translated into colour sensation by a special perceptive centre in the brain On this theory the rods, under the influence of light, produce and circulate the visual purple, but have no perceptive function, the cones being the sole percipient rêtinal elements An interesting fact in connection with this theory is the similarity, already mentioned, between the scotopic luminosity curve of the eye and the rate of bleaching of visual purple by equal quantities of light energy of different frequencies (see Fig 39) ([98]) Dark adaptation is the result of a gradual increase in the concentration of

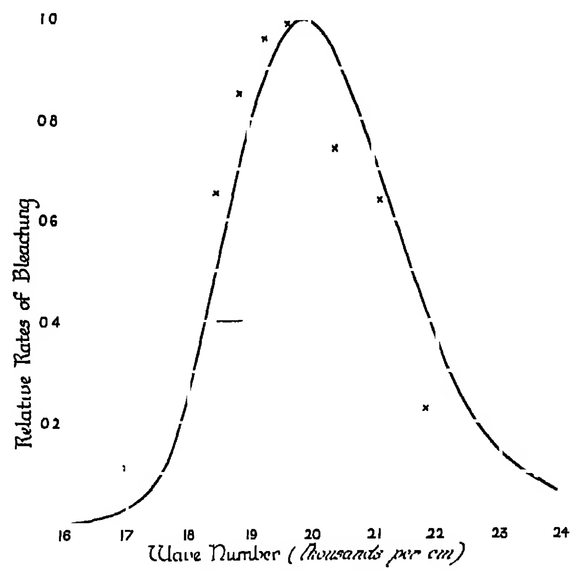

FIG 39 —The Bleaching of the Visual Purple The crosses show observed rates of bleaching The full line is the scotopic luminosity curve

visual purple in the fluid surrounding the cones The increase of acuity of central vision produced by illumination of the background (see p 57) is explained as being due to the increased production of the visual purple by the rods under the stimulating action of the general illumination, and the diffusion into the rodless fovea of the visual purple thus formed ([99]).

It will be noticed that the Edridge-Green theory does not define the nature of the stimulus communicated by the visual purple to the cones, nor the way in which the nature of this stimulus varies with the frequency of the light (100) Theories depending on electrolytic dissociation (101) and photo-electric action (102) have been propounded, but for an account of these and the other principal theories of vision the original papers should be consulted (103).

One other theory of which particular mention must be made is that of E. Hering (104), in which the possible sensations in vision are divided into three pairs, *viz*, black-white, red-green, and yellow-blue Corresponding with each pair is a hypothetical visual substance which undergoes chemical change in one direction when the retina is illuminated by light giving the first-named sensation in the pair (*e g.*, red), while the other sensation (green) is the result of chemical change in the opposite direction. This theory provides an adequate explanation of the most important phenomena of contrast, but it shares with the three-colour theory the disadvantage of requiring triplicity in the retinal sensitive structure

In conclusion it must be emphasised that all that it has been possible to give in the foregoing sections is a disjointed and totally inadequate description of some of the phenomena of vision, most attention having necessarily been devoted to the particular aspects of the subject which are of greatest importance in everyday photometry. A lack of appreciation of the peculiarities of the visual process has led repeatedly to the publication of results which are totally lacking in value, either because they refer to a condition of the eye which is different from that prevailing in ordinary circumstances or because insufficient details are given to enable the results to be correlated with other data obtained, probably, under quite different visual conditions The influence of field size and brightness on the results obtained by means of a flicker photometer may be cited as a particular example It is more than likely that in many of the branches of practical photometry there are unrecognised sources of uncertainty which may, later on, be traced to some at present unsuspected phenomenon of vision

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(9) See also P. G. Nutting, Bureau of Standards, Bull., 3, *1907,* p. 59, El. World,

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(60) J Guild Article on "The Eye as an Optical Instrument" in Dict Appl Phys, vol 4, p 60

(61) A König "Gesammelte Abh zur Physiol Optik" (Leipzig, *1903*), p 144, Deut Phys. Gesell, Verh, *1892* (Ann d Phys, 45, *1892*, p 604)

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(70) A. Charpentier Article in "Traité de Physique Biologique" (Masson, Paris, c *1900*), vol. 2, p 905 See also *Addenda*, p. 83

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Note (67). A Konig Z f Psychol u Physiol d Sinnesorgane, 8, *1895*, p 375 The colour discrimination is sensibly the same at all except very low intensities See T A Neelin, Roy Soc Canada, Trans., 7, *1913* (Sect III), p 221.

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CHAPTER IV

THE PRINCIPLES OF PHOTOMETRY

PHOTOMETRY may be defined as the measurement of light by visual comparison, and the two preceding chapters of this book have been devoted respectively to a consideration of the nature of light and to a description of the way in which the eye responds to luminous radiation In this chapter the various photometric quantities will be defined and described, and their relations to one another will be derived For this purpose the definitions adopted by the International Commission on Illumination in 1921 and 1924 will be employed, and definitions not yet formulated internationally will be taken from the reports presented to that Commission by the British, French or American National Illumination Committees (1).

Luminous Flux and Luminous Intensity.—In any system of photometric definitions it is possible to start either with the luminous intensity (candle-power) of a light source or with the rate of flow of the luminous energy produced by a source The first of these arrangements possesses the advantage that the only quantity which can conveniently be used for the purpose of standardisation is the luminous intensity of a specified light source (2). On the other hand the most fundamental notion in photometry is that of the radiant energy which, when it reaches the eye, produces the visual sensation, and very often this radiant energy is considered without reference to its source

It will be most convenient for the purposes of this chapter to consider both the energy and its source together, and, therefore, the internationally agreed definitions of luminous flux and of luminous intensity are here given as follows .—

"LUMINOUS FLUX *is the rate of passage of radiant energy evaluated by reference to the luminous sensation produced by it.*

"*Although luminous flux should be regarded, strictly, as the rate of passage of radiant energy as just defined, it can, nevertheless, be accepted as an entity for the purposes of practical photometry, since the velocity may be regarded as being constant under those conditions.*

"*The* LUMINOUS INTENSITY *(candle-power) of a point source in any direction is the luminous flux per unit solid angle emitted by that source in that direction. (The flux emanating from a source whose dimensions are negligible in comparison with the distance from which it is observed may be considered as coming from a point*)"

It will be seen from the foregoing definitions that the luminous flux at a given point in space is the amount of radiant energy passing that point in unit time, but with the proviso that the energy is not measured in the ordinary physical unit, the erg, but in luminosity units. Hence the number of ergs per second which is equivalent to a given amount of luminous flux is dependent on the frequency of

the waves by which that radiant energy is conveyed (see Chapter III., p. 63), and unless these waves be of a frequency which is able to stimulate vision the luminous flux is zero, whatever be the amount of radiant energy measured physically (3).

Many analogies have been proposed to illustrate the conception of luminous flux (4) Magnetic flux is not comparable since it involves no transfer of energy. Perhaps the most satisfactory analogies are to be found either in hydraulics or in electricity. If the energy be regarded as similar to an incompressible fluid such as water (5), then luminous flux corresponds with the flow past a point, and the unit of luminous flux, the lumen (to be defined later), corresponds with gallons of water per hour, the speed of the current being assumed constant. Similarly, if the energy be regarded as analogous to electricity, so that when evaluated according to its luminosity (ergs $\times K_v$) it is represented by quantity of electricity in coulombs, then the luminous flux corresponds with the current, and the lumen is represented by the ampère

Neither of these analogies is really satisfactory, since the essential fact of the rectilinear propagation of radiant energy is not implied, and therefore the vector nature of luminous flux is not brought out. In order to avoid misconception when using the term "luminous flux," it is often useful to recall the real nature of this quantity, for "it has frequently been loosely used in the past as if it represented the entity itself, and not, as it does in fact, a rate of passage of the real entity, *viz.*, energy. It is true that since the velocity of propagation of this energy may be taken as constant, the rate of passage is proportional to something which may be conveniently looked upon as an entity. In the same way electric current, which is the rate of passage of electricity, is almost invariably looked upon as an entity. Provided its real nature be kept in sight, there is much to be gained in conciseness of expression by using the word 'flux' in the way proposed Care must, however, be taken to preserve the distinction between luminous flux, as now defined, and energy" (6)

In the definition of luminous intensity it will be noticed that a point source of light is assumed, *i e*, a source whose dimensions are negligible in comparison with the distance at which the flux is measured The nearest practical approach to a point source is a fixed star as viewed from the earth The energy emitted from such a source may then be regarded as travelling outwards radially in the form of waves which are perfect spheres It should be noticed, in passing, that the luminous intensity is not here assumed to be the same in all directions A source of which this was true would be termed a "uniform point source" A variable star is an illustration of a non-uniform point source, for at any instant its luminous intensity is not the same in all directions in space. Since in this definition the luminous intensity in a single direction only is under consideration, it follows that it is only the flux density in that direction which is referred to

The luminous intensity in any direction is measured in terms of flux per steradian (7), for in the definition of luminous intensity it is the angular density of the flux along a line that is spoken of. This may be regarded as analogous to pressure at a point. The actual flux along a line is the rate of energy transfer across any point of

that line, and, like the pressure on a mathematical point, has no meaning, except as a limit (8) It may be regarded as $Lim._{\omega=0} F/\omega$, or $dF/d\omega$, where F represents the flux within a solid angle ω containing the line under consideration

It will be noticed that in the definitions given above for the quantities "luminous flux" and "luminous intensity" the second of these ideas is made dependent on the first When the units are defined, however, it is necessary to reverse this relationship, since the unit of luminous intensity (the candle) is the primary reproducible quantity The units are therefore defined as follows —

"*The unit of luminous intensity is the* INTERNATIONAL CANDLE, *such as resulted from agreements effected between the three national standardising laboratories of France, Great Britain and the United States of America, in* 1909 * *This unit has been maintained since then by means of electric incandescent lamps in these laboratories, which continue to be entrusted with its maintenance.*"

"* *These laboratories are · the Laboratoire Central d'Electricité in Paris, the National Physical Laboratory in Teddington, and the Bureau of Standards in Washington*" (9).

"*The unit of luminous flux is the* LUMEN. *It is equal to the flux emitted in unit solid angle by a uniform point source of one international candle.*"

The term which has been most commonly used in the past to express the light-giving power of a source is "candle-power" This word, including as it does the name of the unit, is not altogether satisfactory as a general expression for the quantity itself It is, therefore, preferable to use the term "luminous intensity" where no actual measure is involved, and to reserve the word "candle-power" for luminous intensity expressed in international candles Thus the statement that the candle-power of a lamp is 16 is equivalent to the statement that its luminous intensity is 16 international candles A "16 c p lamp" is an abbreviated form of expression which may sometimes be convenient

The method by which the unit of luminous intensity is actually maintained will be considered in detail in Chapter V It must be remarked here, however, that in the definition of the lumen a uniform point source is postulated This is clearly impossible of attainment under practical conditions The actual international candle, therefore, is a source which, under the conditions of use to be defined later, will give results equal to those that would be obtained with a uniform point source to an accuracy well within the limits of experimental error In this way it may, perhaps, be likened to the unit of length, which is the length between two lines on a bar of metal Theoretically those lines should be mathematical lines, without breadth, and therefore invisible Practically, their centres give a value for the unit which is correct within the limits of experimental error

From the definitions of the lumen and the candle just given the following defining equations result —

$$I = dF/d\omega \text{ and } F = \int I d\omega.$$

where F is the luminous flux measured in lumens, I is the candle-

power measured in international candles, and ω is a solid angle measured in steradians ([10])

Candle-Power Distribution: Mean Spherical Candle-Power.—It has already been said that a uniform point source, or even a point source, is unattainable under practical conditions. At the same time it is often sufficiently accurate to regard many small sources, though of appreciable dimensions, as point sources for photometric purposes, the limit to what is allowable being set by (*a*) the accuracy of measurement desired, and (*b*) the ratio of the linear dimensions of the source to the distance from it at which the measurements are made ([11]). While this is true, the question of *uniformity* is quite another matter This does not at all depend upon distance, and for many practical purposes a source which may quite well be regarded as a point source can by no means be regarded as uniform. For example, although an electric lamp of ordinary size may be regarded as a point source at distances exceeding a metre, the candle-power of such a lamp in the direction of the cap is very small compared with that in the direction of the pip, and this again often differs considerably from that in a direction perpendicular to the axis of the lamp

The expression "candle-power of a source" is therefore indefinite unless either the value in a single definite direction or the average value within a given region be specified ([12]) The most commonly used values of candle-power at the present time are (*a*) the candle-power in some well-defined direction, (*b*) the average value of the candle-power in all directions in space. Of these two systems, the first is at present capable of more accurate determination, and is therefore generally used for lamps which are to serve as standards or sub-standards of candle-power (see p. 136). The second system is generally adopted for the ordinary commercial measurement of light sources It will be noticed that the average candle-power in all directions is, by the defining equation given above, the same as the total luminous flux emitted by the source divided by 4π, since $\omega = 4\pi$ for the total solid angle at a point. This quantity is known as the "mean spherical intensity" (or, when expressed in international candles, as the "mean spherical candle-power—m.s.c p.), and is thus defined —

"THE MEAN SPHERICAL INTENSITY *of a source is the average value of the intensity of that source in all directions in space.*"

In certain cases (*e g*, street lighting or lighting by indirect units) the candle-power of a source in the lower or in the upper hemisphere is of more importance than the m.s c p. ([13]). The following quantities have, therefore, been defined —

"THE MEAN UPPER HEMISPHERICAL INTENSITY *of a source is the average value of the intensity of that source in all directions above the horizontal plane passing through its centre*

"THE MEAN LOWER HEMISPHERICAL INTENSITY *of a source is the average value of the intensity of that source in all directions below the horizontal plane passing through its centre*"

A system of candle-power specification which has been much used with vacuum electric lamps and upright gas mantles is that of the "mean horizontal candle-power" (m h.c p), which is thus defined :—

"THE MEAN HORIZONTAL INTENSITY *of a source is the average value of the intensity of that source in all directions in the horizontal plane passing through its centre*"

This system, however, is now being rapidly superseded by the mean spherical candle-power system. Methods of measuring mean spherical candle-power and mean horizontal candle-power will be described in Chapter VII. The ratio (m s c p.)/(m h c p) for a source is often termed its "spherical reduction factor" or, more briefly, its "reduction factor" (see p. 107) ([14]).

The Polar Diagram.—In addition to a knowledge of the mean spherical candle-power of a source it is often desirable, for purposes of illumination calculation, to know the value of the candle-power in any given direction This information can only be obtained, theoretically, by means of an infinite number of candle-power measurements, but for practical purposes it is generally found that the sources ordinarily in use are sufficiently symmetrical about their axes for measurements in a single plane to give the information to as great an accuracy as is desired. The results of such measurements are most conveniently represented by means of a polar diagram in which the source is considered to be at the origin, and the length of the radius vector in any direction represents the candle-power in that direction.

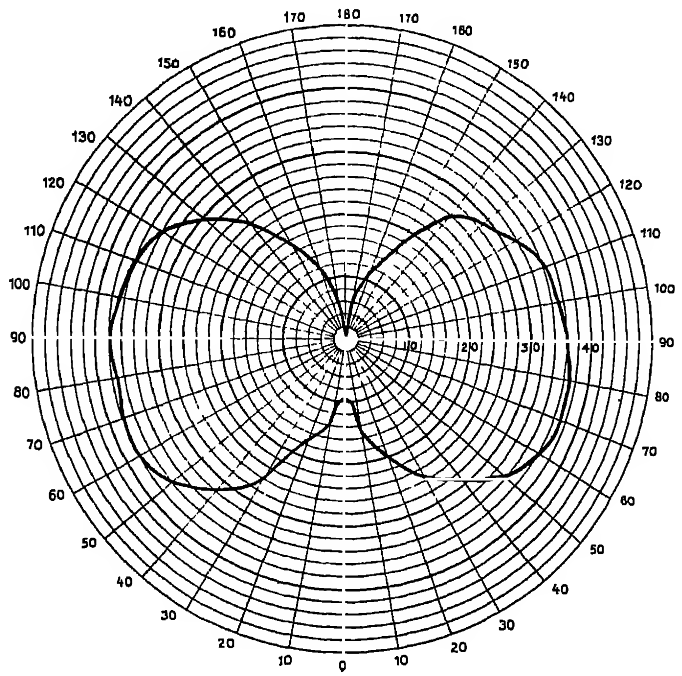

FIG 40 —A Polar Diagram of Candle-power

Such a curve is shown in Fig 40. The ordinary convention is that the vertically downward direction is the zero line, and angles are reckoned upwards, positive on one side and negative on the other, to 180° at the zenith ([15]). Sometimes, in order to give

a better representation of the average distribution about the source, the radius vector at any angle is drawn to represent the average value of the candle-power in all directions along the surface of a cone having its axis coincident with the axis of the source, and its semi-vertical angle equal to the angle of the radius vector on the diagram.

The Iso-Candle Diagram.—When the source is not sufficiently symmetrical to allow of the use of a polar diagram, recourse must be had to an "iso-candle" diagram (16). In constructing this diagram the source is considered to be surrounded with an imaginary

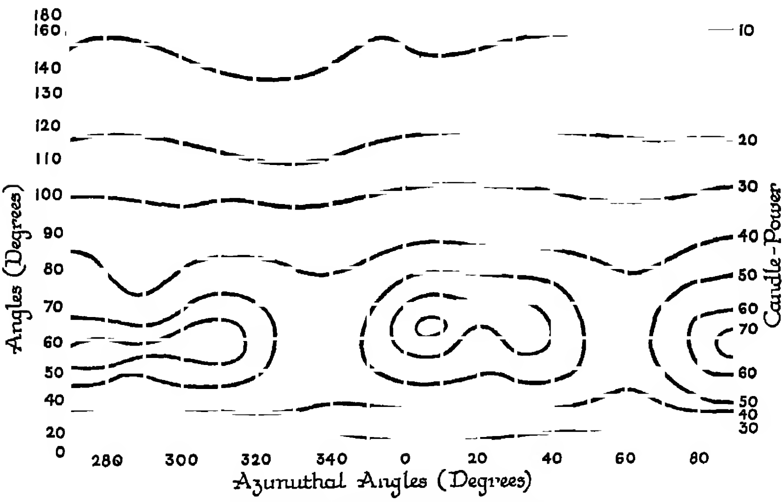

FIG 41A —An Iso-Candle Diagram on the Cylindrical Projection.

sphere whose radius is such that at every point on the surface of the sphere the source may be regarded as a point source. Contour curves are then drawn on this sphere through the points from which the source appears to have the same candle-power. These are curves of equal candle-power, or "iso-candle" curves. The spherical diagram thus obtained is reproduced on a flat surface by any of the methods of projection used in map making It is, however, desirable that some type of equal-area projection should be used, so that two diagrams may give by inspection an approximate idea of the relative values of the luminous flux emitted by the sources to which they refer If distortion of the curves be not regarded as any disadvantage a simple cylindrical equal-area projection may be used, but if it be desired to reduce the distortion as much as possible, the sinusoidal equal-area projection is best (17). Figs 41A and 41B show the iso-candle diagrams on these two projections for a source with four opaque bars reducing the candle-power at positions 90° apart in the lower hemisphere Only one-half of the complete diagram is shown in each figure.

The methods used for making the candle-power measurements necessary for the preparation of a polar distribution diagram or an iso-candle diagram will be described in Chapter VII. In the following section a description will be given of two methods which are used

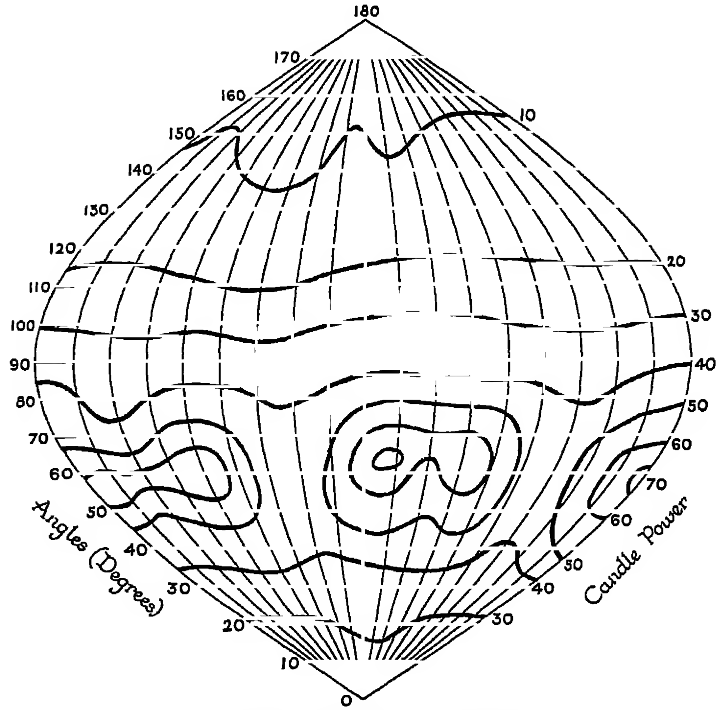

FIG. 41B —An Iso-Candle Diagram on the Sinusoidal Projection.

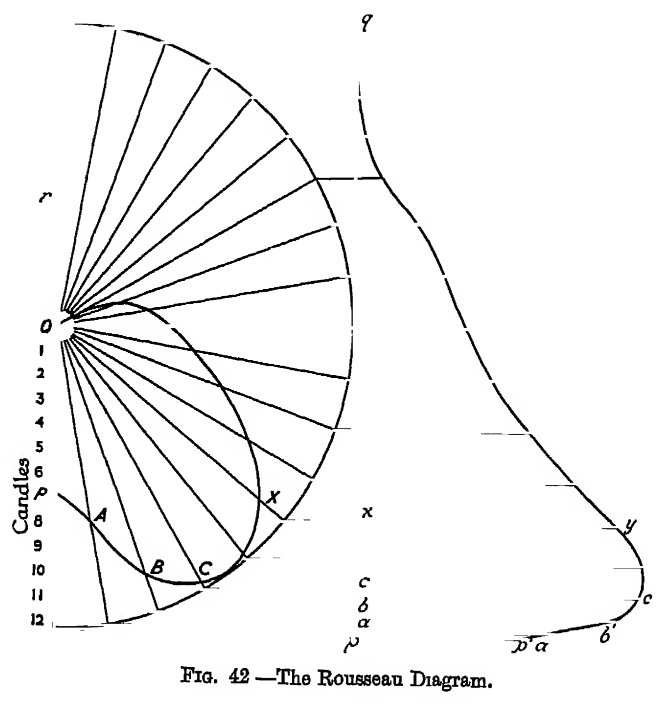

FIG. 42 —The Rousseau Diagram.

for the calculation of the mean spherical candle-power of a source from a knowledge of its polar distribution curve.

The Rousseau Diagram.—It might at first be thought that the mean spherical candle-power of a source could be obtained by finding the area of the polar diagram, or the volume of its solid of revolution about the 0°—180° axis, but a moment's consideration will show that this is not the case ([18]) For let curve ABC of Fig. 42 represent the polar curve of a source supposed symmetrical about the axis OP. Since the radius vector is everywhere proportional to the candle-power, the area of the curve is proportional to $\int_0^\pi I_\theta^2 d\theta$, while the volume of the solid of revolution is proportional to $\int_0^{2\pi}\int_0^\pi I_\theta^3 \sin\theta \, d\theta d\phi$ The mean spherical candle-power, however, is $(1/4\pi)\int_0^{2\pi}\int_0^\pi I_\theta \sin\theta \, d\theta d\phi$, i e, $\frac{1}{2}\int_0^\pi I_\theta \sin\theta \, d\theta$, since I_θ is here independent of ϕ.

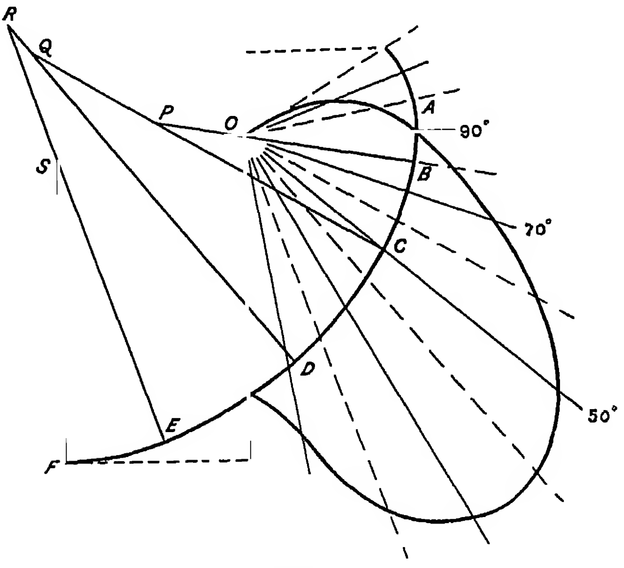

FIG 43 —The Kennelly Construction

The value of this expression may be obtained graphically from the polar curve by means of a simple construction due to Rousseau ([19]), and known as the Rousseau diagram. It is shown in Fig. 42 The left-hand part of the diagram is a polar curve. At the ends of the radii vectores OA, OB, OC, OX, . . horizontal lines are drawn to cut a vertical axis at a, b, c, x . Along these lines lengths aa', bb', cc', xy . . are cut off equal to OA, OB, OC, OX . . and the ends a', b', c', y . . . joined by a smooth curve Now if the angle XOP be θ, $\overline{px} = r(1-\cos\theta)$, so that $d\,\overline{px} = r\sin\theta\, d\theta$ Further, $\overline{xy} = I_\theta$, so that the area of the curve $pp'a'b'c'yqp$ is equal to $r\int_0^\pi I_\theta \sin\theta \, d\theta$, and, therefore, on the scale connecting OP with I, the area of the

Rousseau diagram represents twice the mean spherical candle-power of the source.

Co-ordinate paper can be obtained ([20]) with one set of divisions proportional to sin θ, so that the right-hand part of Fig 42 can be drawn directly from the observed values of candle-power The area of the Rousseau diagram may be found by means of a planimeter, or by the summation of equidistant ordinates, or by a graphical construction due to A E. Kennelly ([21]).

This construction will be clear from Fig. 43. The quadrant is divided into $(n + \frac{1}{2})$ equal zones, say four of 20° and one of 10°.

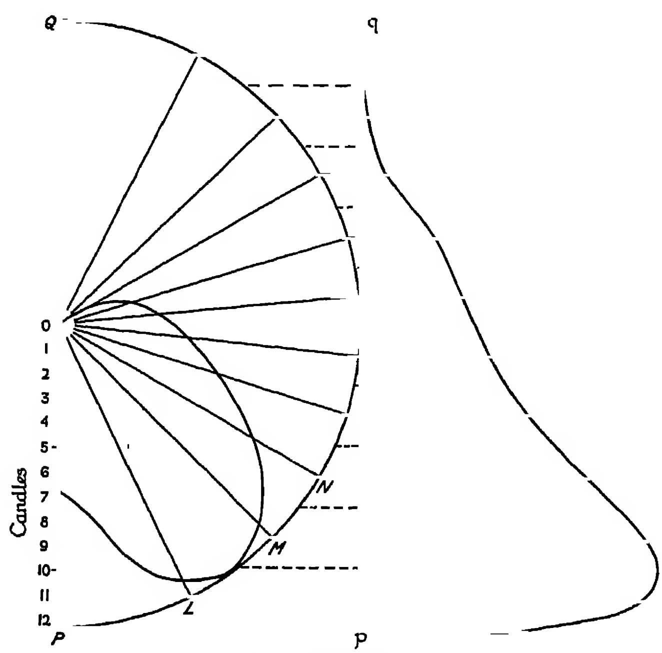

FIG 44 —The Russell Angles.

AB is a 20° arc about O as centre, and of radius I_{90}. BP is marked off on BO produced, so that $PB = I_{70}$. BC is a 20° arc with P as centre CQ is marked off on CP produced, so that $QC = I_{50}$, and CD is a 20° arc with Q as centre. A similar construction is followed above the horizontal, and the vertical distance between the outer ends of the extreme 20° arcs is equal to twice the mean spherical candle-power I_o on the scale chosen for I_{90}, I_{70}, *etc.* It is easy to see that the element of vertical distance added by each complete step in the construction is $I_\theta\{\cos(\theta - \frac{1}{2}\delta\theta) - \cos(\theta + \frac{1}{2}\delta\theta)\}$, where $\delta\theta$ is the angular interval between the steps (20° in the figure). Hence the total height is $\Sigma 2I_\theta \sin\theta \sin\frac{1}{2}\delta\theta$, which, when $\delta\theta$ is small, becomes $\Sigma I_\theta \sin\theta \,.\, \delta\theta$, or, in the limit, $\int I_\theta \sin\theta \, d\theta = 2I_o$. This method avoids the necessity for the calculation or measurement of an area ([22]).

Instead of drawing the Rousseau diagram and calculating the area by the addition of equidistant ordinates, it is clearly more simple to make the candle-power measurements at the angles which correspond to equidistant ordinates on the diagram, so that the mean spherical candle-power can be found at once from the average of the measured values of candle-power ([23]).

In Fig. 44 the same polar curve as that shown in Fig. 42 has been reproduced, but in this case the axis *pq* has been divided into a number of equal parts. Horizontal lines have been drawn through the mid-points of these parts to cut the circle of the polar diagram at *L*, *M*, *N* . . . and the points *L*, *M*, *N* . . . have been joined to *O*. It follows that if the candle-power measurements be made in the directions *OL*, *OM*, *ON* . . the mean spherical candle-power may be obtained simply by taking the arithmetic mean of the values thus obtained, for since these would form equidistant ordinates on the Rousseau diagram, their mean gives the area of the curve to a close approximation Other sets of angles may be used to obtain an equally good approximation with fewer measurements ([24])

The values of the Russell angles for 20, 10, 8 and 6 ordinates are given in the following table .—

TABLE OF RUSSELL ANGLES FOR CALCULATION OF MEAN SPHERICAL CANDLE-POWER *

20 angles	10 angles	8 angles	6 angles
18·2	25·8	29 0	33 6
31 8	45 6	51 3	60 0
41 4	60 0	68 0	80 4
49·5	72·5	82 8	99·6
56 6	84·3	97 2	120·0
63 3	95 7	112 0	146 4
69 5	107·5	128 7	—
75 5	120 0	151 0	—
81·4	134 4	—	—
87·1	154 2	—	—
92 9	—	—	—
98 6	—	—	—
104 5	—	—	—
110 5	—	—	—
116 7	—	—	—
123 4	—	—	—
130·5	—	—	—
138 6	—	—	—
148 2	—	—	—
161 8	—	—	—

* *N B.*—The angles here given are measured from a vertically downward zero, assuming measurements made in a vertical plane through a vertical axis of symmetry of the source. The angles given in the paper above quoted, and in most literature, are measured upwards and downwards from a horizontal zero.

The following example will serve to show the method of calculating mean spherical candle-power (*a*) by means of a Rousseau diagram, and (*b*) by the use of Russell angles The polar curve is that illustrated in Figs. 42 and 44, the scale of candle-power being indicated along *OP*

Rousseau Diagram		Russell Angles	
Angle	Candle-Power	Angle	Candle-Power.
0°	6 8	18 2°	9·6
10°	7·8	31·8°	12·0
20°	10·6	41·4°	12 1
30°	11·9	49·5°	11·2
40°	12·1	56·6°	9·9
50°	11 0	63·3°	8·7
60°	9 2	69·5°	7 4
70°	7 3	75·5°	6·3
80°	5·8	81·4°	5·6
90°	4 7	87·1°	5·0
100°	3 7	92 9°	4·4
110°	2 5	98 6°	3·8
120°	1 1	104 5°	3 4
130°	0 4	110 5°	2·6
140°	0 1	116 7°	1·5
150°	0 0	123·4°	0 7
160°	0·0	130 5°	0 4
170°	0·0	138 6°	0·2
—	—	148 2°	0·0
—	—	161 8°	0 0
		Total of 20 readings	104·8

(Area of Rousseau diagram) — $\overline{pq} = 5{\cdot}25$.
. . Mean spherical candle-power = 5 25
Mean reading at Russell angles, 5·24 candles.

It will be clear from the description given above that the accuracy of the mean spherical candle-power determination by both these methods depends on (*a*) the accuracy of the candle-power measurements, (*b*) the degree to which the lamp is symmetrical about the axis, and (*c*) the absence of sudden variations of candle-power in the regions between the angles at which measurements are made, *i e*, it depends on the "smoothness" of the polar curve. For most sources in common use the set of twenty Russell angles used in the above example gives a result which is correct within the limits of experimental error ([25]). When the source cannot, to the accuracy required, be regarded as symmetrical about its axis, measurements must be made in more than one plane. From theoretical reasoning it can be shown ([26]) that for a source with two vertical planes of

symmetry (*e.g.*, a gas-filled electric lamp or a two- or four-mantle gas lamp) the measurement should be made in one half (0° to 180°) of each of the four vertical planes, which respectively make the following angles with either plane of symmetry 9°, 144°, 236°, and 279° For a source with three planes of symmetry (*e g* , a three-source lighting unit) the corresponding angles are 9°, 150°, and 291°.

Illumination.—In the foregoing paragraphs a description has been given of luminous flux and candle-power and their relationship with each other. When luminous flux reaches a surface, that surface is said to be illuminated, and the illumination at any point of it is thus defined —

"*The* ILLUMINATION *at a point of a surface is the density of the luminous flux at that point, or the quotient of the flux by the area of the surface when the latter is uniformly illuminated*"

Thus the illumination at any point of a surface is the luminous flux density at the point ([27]), or the quotient of the flux by the area if the illumination of the surface be uniform. Thus illumination is analogous to pressure at a point in that it is $Lim_{\ s=0}\ (F/s)$, where s is the area (containing the point in question) which receives the flux F. There are several units of illumination, according to the unit of length adopted for the measurement of s If s be measured in square centimetres, the unit is called the *phot*, but this unit is of an inconvenient magnitude, and the *milliphot* has been proposed for practical use It is, however, seldom employed The ordinary metric unit is the *lux*, or *metre-candle*, which is 1 lumen per square metre The unit on the British system is the *foot-candle* ([28]), which is 1 lumen per square foot, and therefore equal to 10·76 metre-candles. The defining equation is clearly $E = dF/ds$, where, if F be measured in lumens, and s in square metres or square feet, E is measured in metre-candles (lux) or foot-candles respectively.

It was pointed out in Chapter II. (p 19) that, as a consequence of the rectilinear propagation of radiation, the quantity of radiant energy received by any area which is normal to the direction of propagation varies inversely as the square of the distance of this area from the source, it being understood that the area is so small in comparison with its distance from the source that it may be considered as a part of the spherical wave surface emanating from the source, which is regarded, again, as a point source Since it has been agreed to regard the rate of propagation of light energy as constant in photometric work, it follows that the rate of reception of radiant energy by a surface under the conditions above described, *i e.*, the luminous flux it receives (see p 84, *supra*), also varies inversely as the square of its distance from the source ([29]) Now if the small area s be at a distance d from a source of which the candle-power in the direction of s is I, then the luminous flux incident on s is $F_s = I \times (s/d^2)$, and the illumination $E = F_s/s = I/d^2$, where d is measured in the same units as s ([30]). This relationship gives an alternative definition of the unit of illumination as "that illumination which is produced at the surface of a sphere of unit radius, due to a uniform point source of one international candle placed at its centre"

In the above definition the illumination of the surface of a sphere is considered, as this avoids the stipulations, necessary in the case

of a plane surface, that the area s shall be negligibly small compared with d, and normal to the incident light; for another consequence of the rectilinear propagation of radiant energy is that the rate of energy reception by a surface is proportional to the cosine of the angle between the normal to that surface and the direction of propagation of the incident waves (see p 19). Hence the above relationship may be widened to include any area s of which the normal makes an angle θ with the direction of the incident light by writing it $E = I \cos \theta/d^2$

This equation is the symbolic expression of the two fundamental laws of photometry, *viz*, the inverse square law and the cosine law of illumination [31], which may be formally stated thus.—

The illumination of an elementary surface due to a point source of light is proportional to the candle-power of the source in the direction of that surface, and to the cosine of the angle between this direction and the normal to the surface, and it is inversely proportional to the square of the distance between the surface and the source

When it is impossible to regard the surface as small in comparison with its distance from the source, the illumination will be different at different parts of the surface, and the average illumination may be found either by calculating the total flux incident at the surface and dividing by the area, or, alternatively, by finding the illumination at each point and averaging over the surface In the case of a symmetrical arrangement the latter course is often more convenient, and as an example it will be useful to consider the simple case of a circular disc of radius a illuminated by a uniform point source of candle-power I placed at a distance d from the disc along the axis of the latter. Let L (Fig 45) be the source of light, and P any point in the plane of the disc distant r from the projection of L on the plane, then the illumination at P is $I \cos \theta/(r^2 + d^2)$, and since $\cos \theta = d/\sqrt{r^2 + d^2}$, this may be written $I \cos^3 \theta/d^2$. It should be remarked in passing that this result shows that the illumination at any point of a plane due to a source at a distance d from the plane is proportional to the cube of the cosine of the angle of incidence θ of the light when the source is uniform, and varies as $I_\theta \cos^3\theta$ when I varies with θ. This is sometimes referred to as the "cosine-cubed" law [32]. For the disc, when I_θ is independent of θ, it follows that the

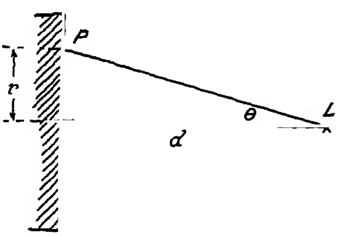

FIG. 45 —The "Cosine-cubed" Law

average illumination is $(1/\pi a^2)\int_0^a (I \cos^3 \theta/d^2) 2\pi r\,dr$, where $r = d \tan \theta$.

This equals $(2I/a^2)\int_0^{\tan^{-1}(a/d)} \sin\theta\, d\theta$ or $(2I/a^2)\{1 - d/\sqrt{a^2 + d^2}\}$. When a is small compared with d, this expression reduces, as it should, to I/d^2. The same expression may be obtained by the flux method, for assuming the disc to have its edge on a sphere of radius $\sqrt{a^2 + d^2}$ with the source as centre, the flux reaching the disc is $AI/(a^2 + d^2)$, where A is the area of the spherical sector limited by the disc. Since

$A = 2\pi\sqrt{a^2 + d^2}\{\sqrt{a^2 + d^2} - d\}$, the expression for the flux becomes $2\pi I\{1 - d/\sqrt{a^2 + d^2}\}$, so that the average illumination of the disc becomes $(2I/a^2)\{1 - d/\sqrt{a^2 + d^2}\}$ as before

The Calculation of Illumination.—A very important problem in practical engineering is that of finding the illumination produced at different points on a given plane (*e g* , the surface of a street, the level of a table, *etc.*) by one or more light sources of known candle-power situated at known positions with respect to the plane and to the points considered

From what has been proved above it follows that the illumination of a horizontal plane at P, due to a source L (see Fig 45) is $I_\theta \cos^3 \theta/h^2$, where h is the vertical height of the source above the plane and I_θ its candle-power in the direction θ If a number of sources contribute to the illumination at P the total illumination is $\Sigma I_\theta \cos^3 \theta/h^2$.

The Illumination Curve.—The formula just quoted makes it possible, from a knowledge of the polar curve of a source and its height h above a plane, to calculate the curve of variation of illumination along any line situated in that plane and passing through a point vertically below the source ([33]) Such a curve is shown in Fig 46, where the heavy line is the curve of illumination along a

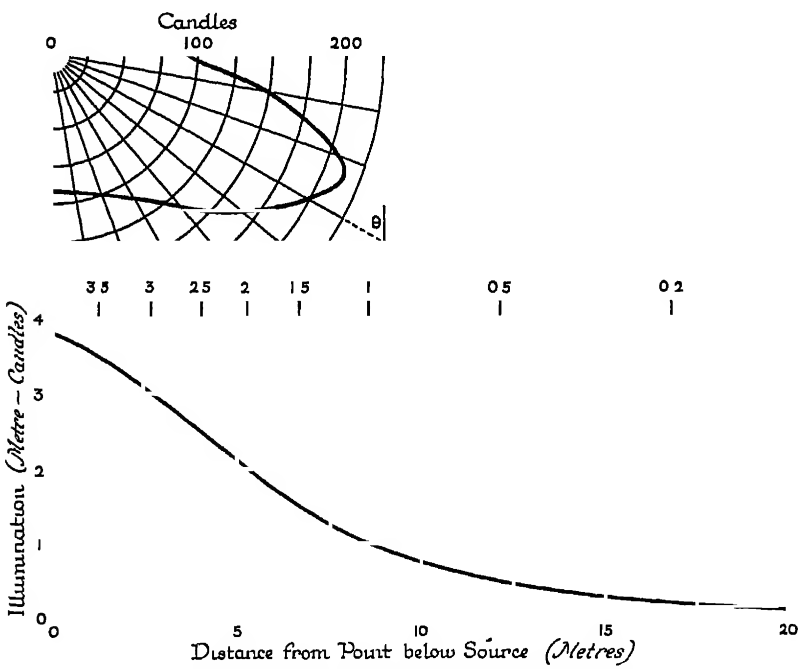

FIG 46 —The Illumination Curve for a Single Source

line 5 metres below a source having the polar diagram shown in the top left-hand part of the figure If, now, there be, instead of one source, a number of similar sources spaced at 10-metre intervals over the line for which the illumination curve has been drawn, the curve giving the total illumination due to all the sources is obtained

by superposing a number of curves like that of Fig 46, the maxima being placed at distances apart corresponding to 10 metres This has been done in Fig 47, where the full line curve, obtained by adding the ordinates of the simple (broken line) curves, shows the distribution of illumination due to the whole line of sources Clearly, if the sources be not all similar, a separate illumination curve must be drawn for each single source, and these individual curves, when superposed at the proper intervals, will then give the final illumination curve It will be obvious that a similar method may be applied generally, *i e* , when the sources are arranged in any manner whatever with respect to the line considered, but the calculations involved will be very much more complicated than in the example given above [34]

The Iso-Lux Diagram.—The illumination curve of Fig 47 shows

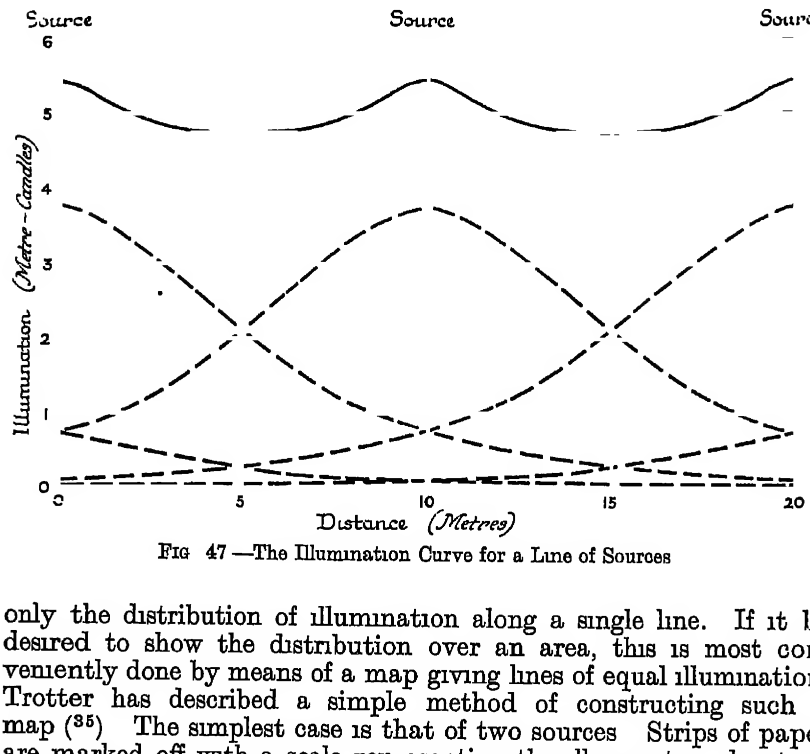

FIG 47 —The Illumination Curve for a Line of Sources

only the distribution of illumination along a single line. If it be desired to show the distribution over an area, this is most conveniently done by means of a map giving lines of equal illumination Trotter has described a simple method of constructing such a map [35] The simplest case is that of two sources Strips of paper are marked off with a scale representing the illumination due to a single source at various horizontal distances from that source Such a scale corresponding to the curve of Fig 46 is shown above the curve in that figure Two such scales are pinned to a sheet at points representing the positions of the two sources (see Fig 48), and marks are made at the points of intersection of the scales, where the sum of the graduations has a given value For example, the 4 metre-candle contour is the line joining the points where the scale marks 0 5, 3·5 , 1, 3 , 1·5, 2 5 on the two scales respectively are coincident. The complete contour map is shown in Fig. 48 The larger the number of sources the more complicated become the necessary

calculations Examples of such maps for different arrangements of sources may be found in the literature of illumination [36].

Brightness.—In the above section, dealing with illumination, it will have been noticed that the nature of the surface was not mentioned This is because the illumination of a surface does not depend at all on the nature of that surface. If a piece of white paper and a piece of black cloth receive the same amount of luminous flux per unit area, they are equally illuminated, however different may be the appearance they present to the eye. This difference of

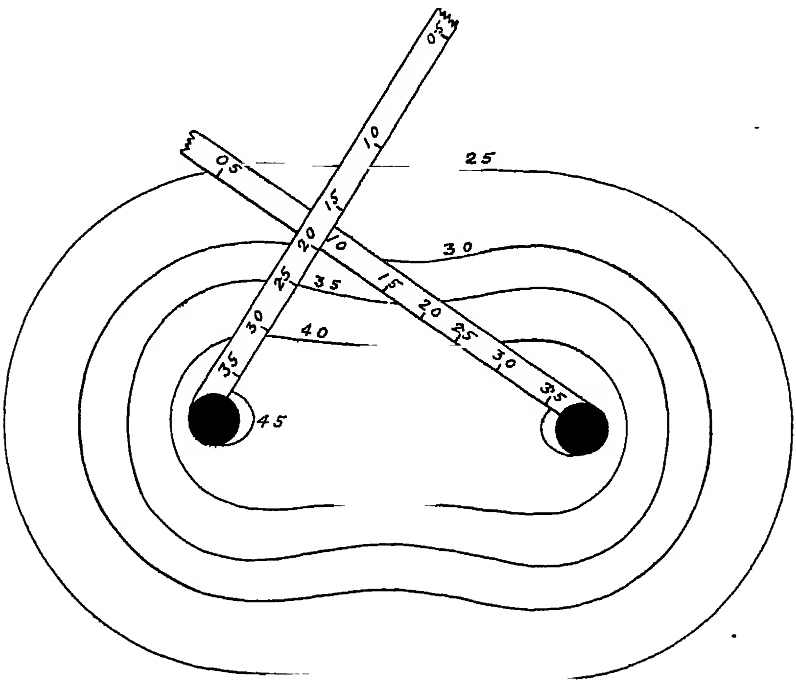

FIG 48 —The Construction of an Iso-Lux Diagram

appearance is entirely due to the fact that the surfaces behave differently as regards the proportion of the incident light which they are able to reflect to the eye The apparent brightness of a surface depends solely on the illumination of the retinal image, and this, for any given condition of the eye, depends solely on the angular density of the luminous flux which unit area of the surface emits in the direction of the eye [37] The distance from which the object is viewed is immaterial [38], since, although the flux entering the eye from unit area of the surface varies inversely as the square of this distance, so does the superficial area of the retinal image (see p. 24), and hence the illumination of this image is constant The brightness of a surface may, therefore, be defined thus [39] —

"*The brightness in a given direction of a surface emitting light is the quotient of the luminous intensity measured in that direction by the area of this surface projected on a plane perpendicular to the direction considered. The unit of brightness is the candle per unit area of surface*"

Thus if O in Fig 49 represent a surface whose brightness in the direction OP is B, then the candle-power of the surface per unit projected area is B, *i e*, the candle-power per unit of actual area in this direction is $B \cos\theta$. In this it is assumed that the size of the surface is small compared with the distance at which the candle-power is measured The defining equation for brightness is therefore $B_\theta = (dI_\theta/ds) \sec\theta$.

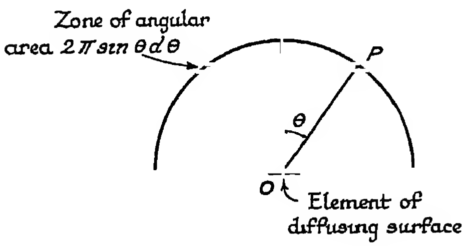

FIG 49 —The Relation between Brightness and Flux Emission

Thus it will be seen that, while illumination is the measure of the light *received* by a surface, brightness is the measure of the light *emitted* in any given direction ([40]) This light may be due either to self-luminosity, as in the case of a flame or other luminous source, or it may be due to reflection by a non-self-luminous surface of some of the light received by that surface from a self-luminous body The two units generally employed are the candle per square millimetre or the candle per square metre The first, which is a million times as large as the second, is generally used for self-luminous surfaces, the second for non-self-luminous surfaces, but this arrangement is purely a matter of convenience. As an example of the order of magnitude to be dealt with, it may be mentioned that the brightness of an acetylene flame is about 2 candles per sq mm, and that of the crater of an arc with pure carbon electrodes is about 160 to 170 candles per sq. mm The brightness of a piece of white paper illuminated sufficiently for writing purposes (30 to 50 metre-candles) is of the order of 10 to 15 candles per square metre On the British system of units the candle per square inch and the candle per square foot are used.

So far only the brightness of a surface in a given direction has been dealt with, and in practice it is found that all surfaces, to a greater or less extent, vary in brightness according to the direction in which the measurements are made From the defining equation it will be seen, however, that if $I_\theta = I \cos\theta$, where I is the candle-power per unit area in the direction of the normal to the surface, then $B = dI/ds$, and is independent of θ. Such an ideal surface is known as a "perfectly diffusing surface" or, more shortly, a "perfect diffuser" Although, as has been said above, no such surface is known in practice, a large number of surfaces exist for which the relationship holds approximately over a wide range of values of θ Such surfaces are said to be "matt," or "good diffusers," and have many applications in photometry A perfectly diffusing surface is said to obey the *cosine law of emission*, which was first enunciated by Lambert ([41]), and may be thus stated —

A perfectly diffusing surface is one for which the candle-power per unit area in any direction varies as the cosine of the angle between that direction and the normal to the surface, so that it appears equally bright whatever be the direction from which it is viewed

The brightness of a perfect diffuser may very conveniently be

expressed in terms of the flux emitted by it per unit area, for (Fig 49)

$$F = \int_0^{\pi/2} I \cos\theta \, d\theta \quad 2\pi \sin\theta = \pi I \int_0^{\pi/2} \sin 2\theta \, d\theta = \pi I,$$

so that the flux emitted per unit area by a perfect diffuser whose brightness is I candles per unit area is πI lumens. The brightness on this system may be defined as "the total luminous flux emitted by the surface per unit area" and measured in lumens per square metre or per square foot The defining equation is thus $B_F = dF/ds$. When F is measured in lumens and s in square centimetres, the unit of B_F is called in America the lambert (42) This unit is, however, of an inconvenient size, and its thousandth part, the millilambert, is generally employed on this system From the defining equations of the two systems it follows that $B_F = \pi B$, so that a perfectly diffusing surface which has a brightness of 1 candle per square metre has a brightness of π lumens per square metre, *i e*, $\pi \times 10^{-4}$ lamberts or $\pi/10$ millilamberts Brightness expressed in millilamberts may thus be expressed in candles per square metre by multiplying by $10/\pi$ Although a brightness of 1 lambert implies an emission of 1 lumen per unit area only in the case of a perfect diffuser, it is clear that the brightness of *any surface* in *any direction* can be expressed in the same units by simply expressing it in terms of the brightness of a perfectly diffusing surface having a brightness in all directions equal to 1 lambert "To say that the brightness of a surface as viewed from a given point is n lamberts signifies that its brightness is the same as that of a perfectly diffusing surface emitting or reflecting n lumens per square centimetre" (43)

A similar unit on the British system is the foot-lambert, which may be defined (44) as "the average brightness of any surface emitting or reflecting 1 lumen per square foot, or the uniform brightness of a perfectly diffusing surface emitting or reflecting 1 lumen per square foot"

It is regrettable that the two methods of expressing brightness should have come into use, for, while the flux system has the advantage that for a surface which may, for practical purposes, be regarded as a perfect diffuser, the brightness in foot-lamberts is numerically equal to the illumination in foot-candles multiplied by the reflection factor of the surface (see p 110), there can be no doubt that much confusion is caused owing to the somewhat artificial conception needed on the flux system, and to the fact that it tends to obscure the essentially similar nature of brightness and luminous intensity These two quantities differ only in that one is the area density of the other, and it seems only logical that if one be measured in candles, the other should be expressed in candles per unit area No name has been adopted for the unit of brightness on this system, and a name seems unnecessary except for the sake of brevity (45) In this book, therefore, brightness will always be expressed either in the units described in the first definition, *viz*, candles per square metre or per square millimetre, or, where the British system is being employed, in candles per square foot or per square inch

It has been said already that no surface obeys the cosine law of emission perfectly Nevertheless, many of the surfaces dealt with

in photometry obey this law to a sufficiently close approximation for the purposes of calculation in certain classes of problems The closest approximation is that given by a "black-body" cavity, the opening of which behaves very accurately as a surface radiating according to the cosine law, so that, at a distance which is large compared with the size of the opening, the luminous intensity may be taken as proportional to the cosine of the angle between the line of measurement and the axis of the surface. If P (Fig 50) be an element of such a surface, whose area is a and whose normal brightness is B candles per unit area, then the flux emitted per unit solid angle in any direction PQ is $Ba \cos\theta$, where θ is the angle which PQ makes with the normal to P The flux incident on an element of surface of area b situated at Q, and having its normal at an angle ϕ with PQ, is therefore $(b \cos\phi/d^2)$ $Ba \cos\theta$, where $PQ = d$

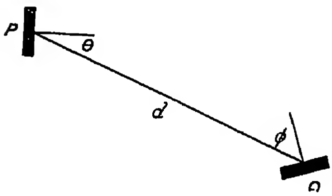

FIG 50 —The Radiation of Flux from one Surface to Another

The symmetrical form of this expression, $Bab \cos\theta \cos\phi/d^2$, shows that the flux reaching Q from P is equal to that which P would receive from Q if the latter had a normal brightness of B candles per unit area.

When one of the surfaces is not negligibly small compared with the distance d, each element may be taken as radiating according to the cosine law, and the aggregate result may be obtained by integrating the above expression over the surface, thus in the case of a radiating circular disc (46) of radius r the flux received by a parallel and coaxial element of area b is $\pi bBr^2/(d^2 + r^2)$, where B is the normal candle-power per unit area of the disc and d the distance from it at which the elementary area is situated. For (Fig 51) if P be an element of the disc at a distance x from the axis and of area a, the flux reaching Q from P is

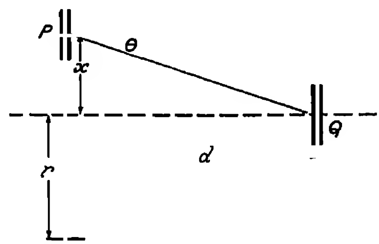

FIG 51 —The Radiation from a Disc

$$Bab \cos^2\theta/(d^2 + x^2) = Babd^2/(d^2 + x^2)^2$$

The flux reaching Q from an annulus of radius x and breadth dx is therefore $Bbd^2\, 2\pi x dx/(d^2 + x^2)^2$ Thus the total flux reaching b from the disc is $2\pi Bbd^2\int_0^r x(d^2 + x^2)^{-2}dx = \pi Bbr^2/(d^2 + r^2)$ It will be seen that this differs from the result which would be obtained according to the simple inverse square, *viz*, $\pi r^2 Bb/d^2$, by the factor $d^2/(d^2 + r^2)$ It follows that the true candle-power, as measured when $d = \infty$, may be deduced from the candle-power measured at a finite distance by multiplying by the factor $(d^2 + r^2)/d^2$. The values of this factor for different values of r/d are given in

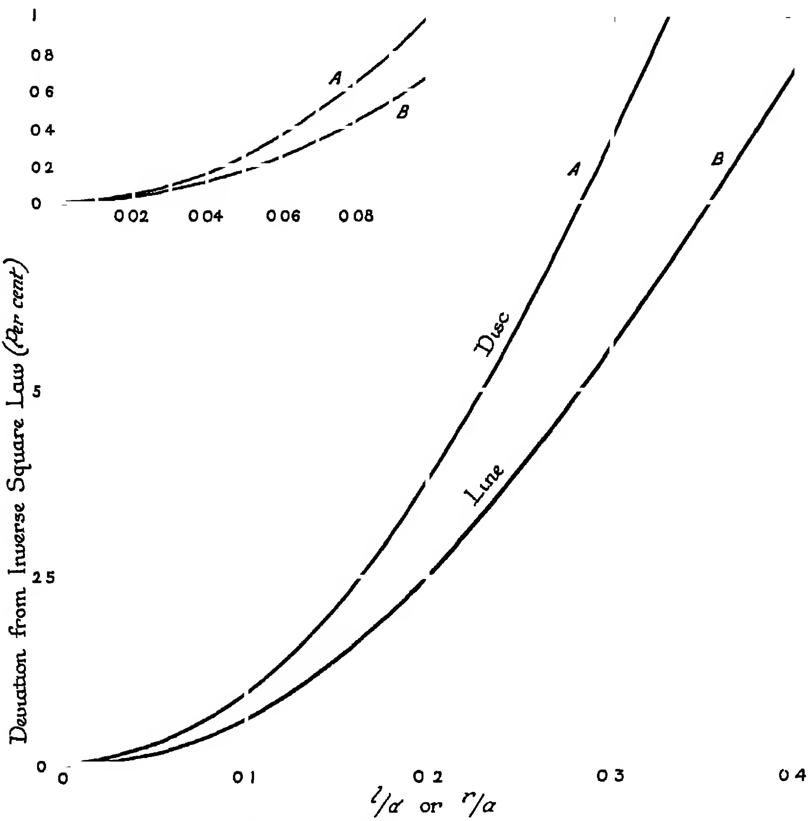

FIG 52 —The effect of Size of Source on the Inverse Square Rule

curve A of Fig 52 When the receiving element is not on the axis of the radiating disc, but is at a distance ρ from this axis, the expression for the flux received becomes $\frac{1}{2}\pi bB[1 - c/\sqrt{c^2 + r^2}]$, where $c \equiv (d^2 - r^2 + \rho^2)/2d$ This may be written

$$\tfrac{1}{2}\pi bB\{1 - \operatorname{cosec}\theta + (r/\rho)\cot\theta\},$$

where $\sec\theta = (d^2 + \rho^2 + r^2)/2\rho r$.

When the receiving element is *perpendicular* to the plane of the disc instead of parallel to it, the flux received can be shown to be $\frac{1}{2}\pi bB(d/\rho)\{1 + \operatorname{cosec}\theta\}$ Of this amount the contribution from the half of the disc shown unshaded in Fig 53 is

$$(Bbd/\rho)\{\operatorname{cosec}\theta\tan^{-1}\sqrt{(\sec\theta - 1)/(\sec\theta + 1)}$$
$$+ (\rho/\sqrt{d^2 + \rho^2})\tan^{-1}(r/\sqrt{d^2 + \rho^2}) - \pi/4\}$$

When $\rho = 0$ this gives the radiation from a semicircular disc to an element situated on the axis, but perpendicular to the plane of the disc, *viz* ,

$$Bb\{\tan^{-1}(r/d) - rd/(r^2 + d^2)\}.$$

This result may readily be obtained by direct integration ([47]).

From the above results it is quite easy to deduce that the total flux which reaches any disc from another parallel disc radiating according to the cosine law [48] is

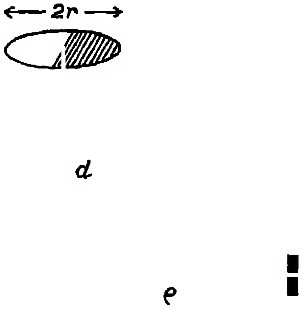

FIG 53 —The Radiation from a Disc to a Perpendicular Surface

$$\tfrac{1}{2}\pi^2 B\left\{(r_1^2 + r_2^2 + d^2) - \sqrt{(r_1^2 + r_2^2 + d^2)^2 - 4r_1^2 r_2^2}\right\}$$

Since the total flux from the radiating disc is $\pi^2 r_1^2 B$, it follows that the fraction of the whole flux which reaches the second disc is

$$(1/2r_1^2)\left\{(r_1^2 + r_2^2 + d^2) - \sqrt{(r_1^2 + r_2^2 + d^2)^2 - 4r_1^2 r_2^2}\right\}$$

It is easy to show that when two discs are so placed that their edges form small circles of the same sphere, the flux received by either disc from the other is independent of the relative positions of the two discs on the sphere, so that it can be at once found from the formula just given It may also be shown that the amount reflected back to the radiating disc is

$$\frac{1}{2}\rho\pi^2 B\left[(r_1^2 + r_2^2 + d^2) - \sqrt{(r_1^2 + r_2^2 + d^2)^2 - 4r_1^2 r_2^2} - dr_1 \tan^{-1}\frac{2dr_1 r_2^2}{(d^2 - r_1^2)r_2^2 + (d^2 + r_1^2)^2}\right],$$

where r_1 is the radius of the radiating disc and r_2 that of the reflecting disc [49]

Another important case is that of a radiating cylinder of radius a and length $2l$ The radiation from such a cylinder to an elementary

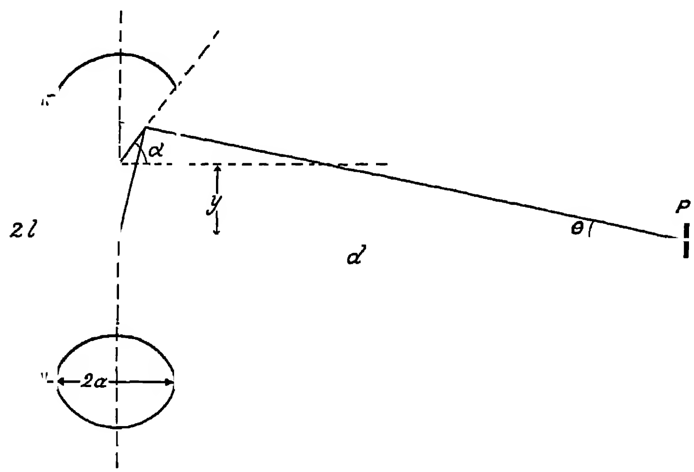

FIG 54 —The Radiation from a Cylinder

surface P at a distance d from the axis of the cylinder, and situated on and perpendicular to a line passing through the centre of the

cylinder and perpendicular to its axis, has been given by Hyde ([50]) as (Fig 54)

$$2Ba\iint (d\cos\alpha - a)(d - a\cos\alpha)\{a^2 + d^2 - 2ad\cos\alpha + y^2\}^{-2}d\alpha dy$$

where the limits of y are $\pm l$, and those of α are $\pm \cos^{-1}(a/d)$ This reduces to

$$\frac{2B}{d}\left[a\cos^{-1}\frac{d^2 - a^2 - l^2}{d^2 - a^2 + l^2} + l\left\{\frac{q+1}{\sqrt{q}}\cot^{-1}\sqrt{\frac{p}{q}} - 2\cot^{-1}\sqrt{p}\right\}\right].$$

where $p \equiv (d+a)/(d-a)$ and $q \equiv \{(d+a)^2 + l^2\}/\{(d-a)^2 + l^2\}$.

When, as is commonly the case in practice, a is small compared with d, the cylinder may be regarded as equal to an elementary strip of breadth $2a$ and length $2l$ The radiation from such a strip to P is clearly (Fig 55), putting $\tan^{-1} y/d \equiv \theta$

$$4aB\int_o^l \{\cos^2\theta/(d^2+y^2)\}dy = (4aB/d)\int_o^{\tan^{-1}l/d}\cos^2\theta d\theta$$
$$= (2aB/d)\ \{\tan^{-1}(l/d) + ld/(l^2 + d^2)\},$$

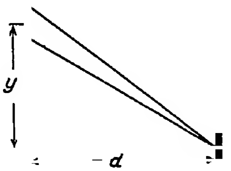

FIG 55 —Radiation from a Straight Flat Strip, or from a Rod of Flame.

which is also the limiting value of Hyde's expression when a becomes a small quantity so that squares and higher powers of a may be neglected

If l also be small compared with d, this becomes, as it should, $4alB/d^2$ It follows that if l be not small compared with d the result given by the inverse square rule is too large, being divided by the factor

$$(d/2l)\{\tan^{-1}(l/d) + ld/(l^2 + d^2)\}\ ([51])$$

The magnitude of this error for different values of l/d is shown in curve B of Fig 52, p 103

The above examples will serve to show the general principles upon which may be based calculations of the illumination at a given point due to the flux from an extended source, and the methods by which those calculations may be carried out ([52]) Other results which are of some importance in photometry are (i) the illumination due to a plane rectangular source at a point situated on the line perpendicular to the source and passing through one corner of it, (ii) the illumina-

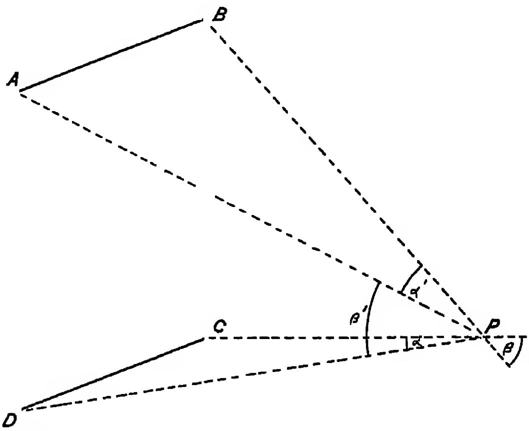

FIG 56.—Radiation from a Rectangular Area to a Point opposite One Corner

tion due to a flat or a cylindrical flame each part of which is perfectly transparent to the radiation emitted from the other parts ([53]).

The former of these problems arises in finding the illumination at a point in a room due to a rectangular window through which is visible a sky of uniform brightness. The solution is ([54]) (Fig 56)

$$E_H = \frac{1}{2}\frac{B}{d^2}\{\alpha - \alpha' \cos\beta\}, \text{ and } E_V = \frac{1}{2}\frac{B}{d^2}\{\alpha' \sin\beta + \beta' \sin\alpha\},$$

where B is the normal brightness of the rectangle, $d = PC$, and E_H or E_V represent respectively the flux received per unit area on an elementary horizontal or vertical surface placed at P It is clear that the general case, when the position of P is not restricted, may be treated as shown in Fig 57 When finding the illumination at P, the rectangle $ABCD$ is divided into the four rectangles PA, PB, PC and PD In the case of Q, rect $ABCD$ = rect QB + rect QD − rect QA − rect. QC

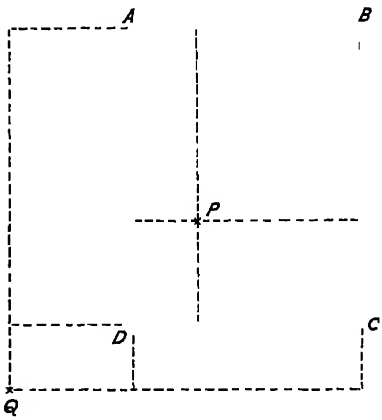

FIG 57.—Radiation from a Rectangular Area to any Point

It is to be noticed that in finding E_H at P the rectangles PD and PC have to be neglected, since they contribute nothing to the illumination of the upper side of a horizontal surface at P

The problem of the radiation from a flame is of importance in the photometry of flame sources and of electro-luminescent sources, such as the mercury vapour lamp or the Moore tube, where the condition as to transparency of the radiating body is approximately met ([55]) The latter case is of considerable practical importance in photometry ([56]), and has been investigated by several workers, some of whom, however, have regarded this form of source as radiating according to the cosine law ([57]).

It is clear that the illumination of P (Fig. 55) due to the flux from a rod of flame, each part of which is perfectly transparent, is equal to $I_v\int Ad(d^2 + y^2)^{-3/2}dy$, where A is the cross-section of the rod, and I_v the candle-power per unit volume of the flame (necessarily the same in all directions) The integral equals $I_vAl/d\sqrt{d^2 + l^2}$

Similarly it can be shown ([58]) that the illumination due to a disc of radius r and elementary thickness t is $2\pi I_v t\{1 - d/\sqrt{d^2 + r^2}\}$, while that due to a rectangular flame of the dimensions shown in Fig 56 is $I_v t \tan^{-1}(\sin\alpha' \tan\beta)$.

The solution for the cylindrical flame of radius r and semi-height h is less simple, leading to a form containing elliptic integrals. The final result for the illumination due to the whole flame (height $2h$) is (d being measured to the axis of the cylinder)

$$\{4I_v rht/d\sqrt{h^2 + (r + d)^2}\}\{F_1(k) + \Pi_1(n, k) . (d - r)/(d + r)\},$$

where F_1 is the complete elliptic integral of the first kind and

$$k^2 = 4rd/\{h^2 + (r + d)^2\}.$$

$\Pi_1(n, k)$ is equal to the expression

$$F_1(k) + \frac{\sqrt{1 - k'^2 \sin^2 \theta}}{k'^2 \sin \theta \cos \theta}\left\{\frac{\pi}{2} + F_1(k) \quad F(k', \theta) - E_1(k) \,.\, F(k', \theta) - F_1(k)E(k', \theta)\right\},$$

where $\quad k'^2 = 1 - k^2 = \{h^2 + (d - r)^2\}/\{h^2 + (d + r)^2\}$

and $\quad k' \sin \theta = (d - r)/(d + r)$

For the special case $r = h = d/10$ the illumination is about 3 per cent greater than that found by assuming the light concentrated at the centre of the base of the flame ([59])

In all ordinary cases the flat flame is not sufficiently large to necessitate any correction on account of departure from the inverse square law In the case of a cylindrical flame the problem is only of practical importance when flame standards such as the Harcourt pentane lamp (see p. 127) are used In this case the flame is 20 mm. in diameter and 47 mm high, and the candle-power certified for the lamp is that calculated from the illumination measured at a surface exactly 1 metre from the central axis of the flame ([60]). If any other distance is used, a correction should, theoretically, be applied In practice, however, this correction may be neglected at all distances exceeding 500 mm.

The analogous case of the "squirrel cage" filament lamp, in which the filament consists of a number of vertical limbs disposed on the surface of an imaginary cylinder, has been discussed by U Tanaka ([61]), who finds that the first order correction on the distance is reduced to zero if the semi-height of the filaments and the radius of the cylinder are related by the equation $h = 0\ 86_5 r$

Reduction Factor.—It will be seen from the above results that the total flux emitted by a disc radiating *on one side* with a normal brightness of B candles per unit area is πBs. Its mean spherical candle-power is therefore $\frac{1}{4}\ Bs$ Its normal candle-power is Bs, so that the factor connecting the two is $\frac{1}{4}$ This may be termed the "normal" reduction factor for a disc.

Similarly, the total flux emitted by a straight filament of length l and radius r having a normal brightness of B candles per unit area is $\pi B.2\pi rl$, so that the mean spherical candle-power is $\frac{1}{2}\ \pi Brl$ The candle-power in any direction perpendicular to the filament is $2Brl$, so that the factor in this case is $\pi/4$. This may be termed the mean horizontal reduction factor, since it is the factor connecting the mean horizontal candle-power of a vertical filament with the mean spherical candle-power ([62]) An electric lamp with squirrel-cage filament and an upright incandescent gas mantle may be regarded approximately as cylinders, and therefore these sources have a spherical reduction factor (see p 88) equal to $\pi/4$ or 0 785.

It is also of interest to draw the polar curve (p 88) of an

elementary radiating disc and cylinder. In the case of the disc the axis of symmetry is the axis of the disc, and the candle-power at any angle θ is proportional to $\cos\theta$, so that the polar curve is simply a circle touching the disc and having its diameter to represent the candle-power in the normal direction (63) (Fig 58) In the case of an elementary cylinder the axis of symmetry is the axis of the cylinder, so that the polar curve becomes two touching circles, of which the filament forms the common tangent, each circle having its diameter to represent the candle-power of the filament in the normal direction The two polar curves of Fig. 58 represent respectively a disc and a cylinder giving the same total flux, for in the first case the normal candle-power $I_N = F/\pi$, and in the second case $\frac{1}{4}\pi I_N' = F/4\pi$, so that $I_N' = F/\pi^2$ This diagram illustrates very forcibly the difficulty of judging total flux from a polar curve (see p 91)

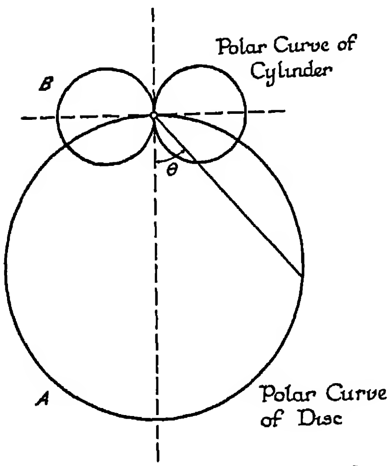

FIG 58 —Polar Curves for a Disc and a Cylinder

Photometry of a Screened Source.—When a diaphragm is placed in front of a uniformly bright diffusing source, the illumination at any point in front of the diaphragm may be found by regarding the opening in the diaphragm as a plane diffusing source of brightness equal to that of the surface behind it, whatever the shape of this latter surface may be, for it is clear that to an eye placed anywhere in front of the diaphragm the opening appears to have a uniform brightness equal to that of the source It follows that if the finite size of the opening be neglected the illumination varies inversely as the square of the distance from the *diaphragm* (64) This is a principle often used in photometry. It can naturally be applied only as long as the diaphragm aperture is completely filled by the source when the combination is viewed from the point where the illumination is being considered In the case of a vertical diffusing strip masked by a diaphragm with a long horizontal opening, the position of the "effective light-centre" is not definite It may generally be taken as approximately half way between the strip and the diaphragm (65), for if l be the distance from the photometer to the diaphragm, and d the separation from diaphragm to source, the distance of the effective light source from the photometer is clearly $\sqrt{l(l+d)} = \left(l + \frac{1}{2}d\right)$ approximately so long as d is small compared with l.

A similar principle may be used in finding the light distribution from an opaque *diffuse* reflector placed over a light source If the total flux intercepted from the source by the reflector be Φ and the reflection factor ρ, it follows that the reflector, if its edge be plane and internal reflections negligible, has a luminous intensity in the

direction θ equal to $(\rho\Phi/\pi)\cos\theta$, where θ is measured from the normal to the plane containing the edge of the reflector The method may clearly be extended ([66]).

Brightness of an Image formed by a Lens or Mirror.—When a lens or mirror is used to form an image of a source or other bright object on a diffusing surface, the illumination of this image is proportional to the brightness of the source, B, and to the aperture of the lens or mirror It is independent of the area s of the source, for the flux reaching a lens of area a from the source is Bsa/u^2, if s and a be both small compared with u, the distance of the source from the lens The flux transmitted to the image is therefore $\tau Bsa/u^2$, where τ is the transmission factor (see p 116) of the lens Since the area of the image is v^2s/u^2 (see p 24 , s is assumed to be the projected area of the source as viewed from the lens), its illumination is $\tau Ba/v^2$. This principle is used in some forms of photometer in which a variable diaphragm is placed over a lens so that the value of a can be changed at will (see p 183).

Brightness of a Projector System: The Maxwellian View.—Another principle which is sometimes useful in photometry is that, if a lens or mirror be caused to form a real image of a surface at the pupil of the eye, the whole surface of the lens or mirror appears to have a uniform brightness equal to that of the original object surface B, multiplied by the transmission or reflection factor of the system and by a factor depending on the relative distances of the object and the eye from the lens or mirror The case of a biconvex lens may be considered by reference to Fig 59 Let S be an element of the surface and L the lens forming an image of S at P Let P and R be respectively the pupil and retina of the eye. The eye is focussed on L so that an image of the surface of L is formed on R. If both S and L be small compared with the distance between them, the flux reaching an element (area a) of the lens at Q is $sB\,a/u^2$ Hence the flux reaching P from S through Q is $\tau saB/u^2$ Now if L were replaced by a diffusing surface of brightness B', the flux reaching P from Q would be paB'/v^2, where p is the area of P (the pupillary aperture). Hence the apparent brightness of Q is $\tau sBv^2/pu^2$ It is here assumed that v^2s/u^2, the size of the image of S, is smaller than P,

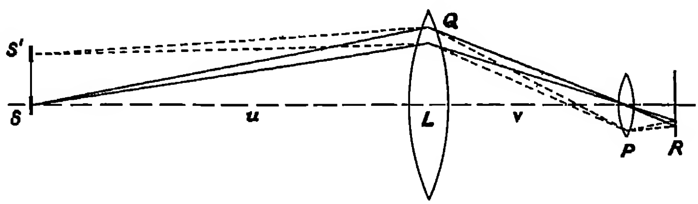

FIG 59 —The Principle of the Maxwellian View

otherwise the iris acts as a stop, limiting the apparent brightness of L to the value τB.

This principle is made use of in some photometric instruments designed to measure sources of light of low candle-power, or those which are situated at a considerable distance, a lens being arranged to form an image of the source at the pupil of the eye. The arrangement is sometimes termed the "Maxwellian view," it having been

adopted by Clerk Maxwell in his colour-mixing apparatus ([67]) By its use an extended field of view may be obtained, with a brightness very much greater than that of the ordinary photometric comparison surface used in most instruments ([68]) In spectrophotometry or colorimetry the Maxwellian view is used to obtain an extended field of uniform colour

Referring again to Fig 59, if S' be a neighbouring element of the same surface as that of which S forms a part, the light reaching R from S' through Q is shown by the broken lines It will be seen that the rays by which Q is viewed at R pass through different parts of P unless S be very small In this latter case the light from each part of L only passes through a very small area of P In consequence, when the Maxwellian view is used with a source which subtends a very small angle at the lens, the field of view is marred by the imperfections in P, such as variations in transmission which cause patchiness, and specks or other opaque bodies which give shadows moving apparently across the field of view ([69]).

There is a precaution to be observed in photometric instruments in which the Maxwellian view is employed, particularly when the photometric balance is achieved by rotation of part of the optical train If, as is generally the case, some shift of the image is liable to occur during the rotation, it is necessary to ensure that the eye-piece does not act as a stop of variable aperture This is achieved either by making the image so small that it is always completely within the aperture, or, alternatively, by having the image more then large enough to cover the aperture completely with a patch of uniform illumination. Generally this can be arranged by using as the object a small diffusing surface of uniform brightness or a small uniform patch of an extended diffusing surface.

Reflection, Absorption and Transmission.—When radiation (including light as a special case) is incident upon the surface of a body, then some of it is, in general, regularly reflected, some is diffusely reflected, while the remainder passes on into the substance of the body Of this latter, some is absorbed, some is reflected back into the body at the surface of emergence, while the remainder emerges, some of it regularly, according to the law of refraction, and the rest diffusely.

It is convenient to divide surfaces into two classes · those which reflect regularly, and may therefore be termed "polished," and those which reflect diffusely, and are called "matt" or "unpolished" No surfaces behave in practice as either perfectly polished or perfectly matt, and most surfaces depart very markedly from the ideal in either direction It is, however, convenient to consider the behaviour of these two classes of theoretical surfaces, and thence to deduce the general behaviour of surfaces met with in practice

"THE REFLECTION FACTOR *of a body is the ratio of the flux reflected by the body to the flux incident upon it.*

"*The flux reflected according to the laws of specular reflection is called specularly reflected flux, and the corresponding reflection factor is called the factor of specular reflection. The flux diffused,* i e , *that sent out in directions other than that of regular reflection, gives the diffuse reflection factor. The total reflection factor is obtained by considering the whole of the flux reflected by the body.*"

The reflection factor of a body may differ very greatly, according to the frequency of the radiation; in fact, the colour of a non-self-luminous surface depends entirely upon the power of that surface to reflect light of certain frequencies more strongly than that of other frequencies. A blue object, for example, has a higher reflection factor for frequencies in the neighbourhood of $\nu = 20{,}000$ than for any other part of the visible spectrum. From this it will be seen at once that the colour of such a surface is dependent upon the colour of the light it receives, for it is obvious that it cannot reflect light of frequencies which are absent from the light which it receives ([70]), so that a normally "blue-green" object, if illuminated by a light containing no frequencies above about 17,000 appears quite black,

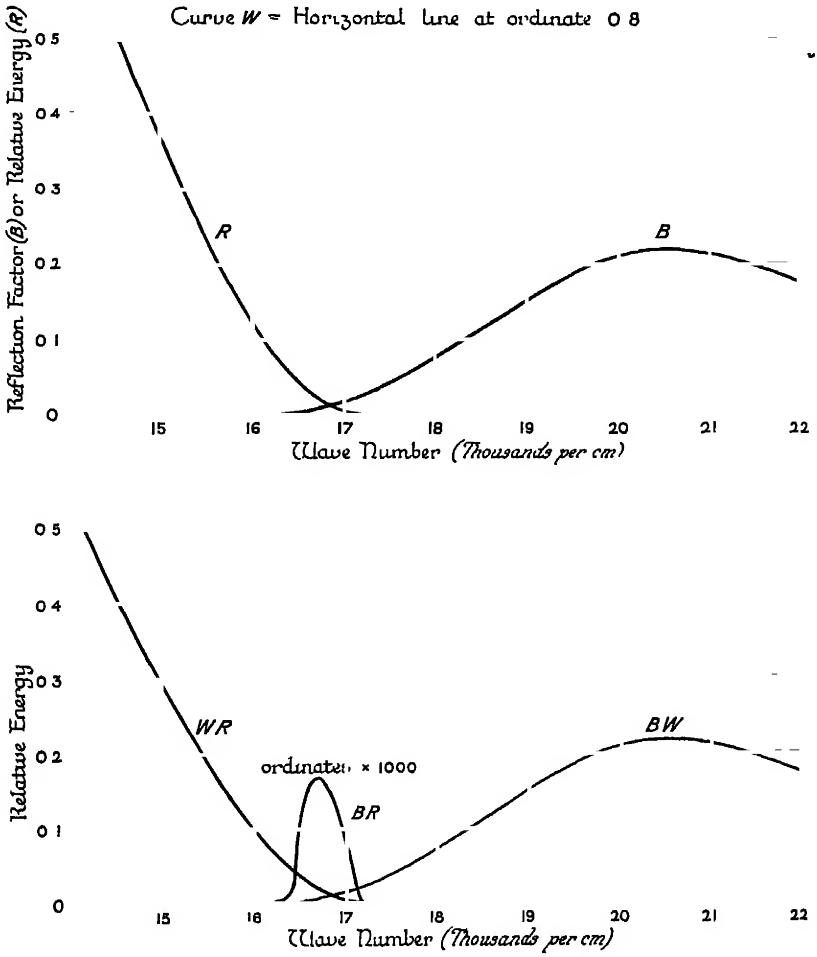

FIG. 60.—The Reflection of Light from a Coloured Surface.

since its reflection factor for the lower frequencies is very small ([71]). A white surface is one which has an equal reflection factor for all frequencies in the visible spectrum. It therefore appears to be of exactly the same colour as the light illuminating it ([72]).

The foregoing statements are illustrated by Fig 60, in which curve R shows the energy distribution in a certain kind of red light, and curve B is a curve of reflection factors for a blue surface The similar curve W, for a white surface (not shown), would be a horizontal straight line through, say, the ordinate 0 8 In the lower diagram the energy distributions in the light reflected from the two surfaces are exhibited by curves BR and WR. The ordinates of these two curves are obtained by multiplying the ordinates of curve R by the corresponding ordinates of curves B and W respectively. Similarly, curve BW represents the spectral distribution of *white* light reflected from the blue surface.

It is to be noticed that a second reflection results in a further multiplication of the ordinates of the spectral distribution curve This is illustrated by Fig 61, where the curve B^2W is obtained by

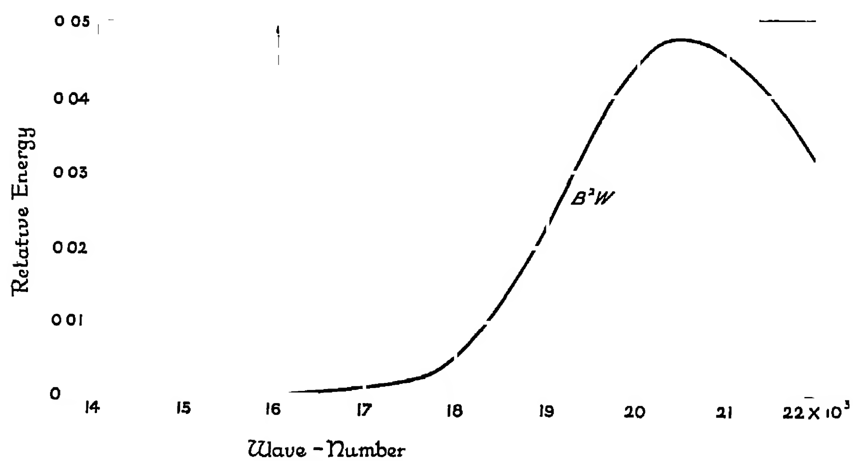

FIG 61 —Double Reflection from a Coloured Surface

squaring the ordinates of curve B or BW in Fig 60. It will be seen that the colour of the reflected light is accentuated by this second reflection, and it is clear that the colour would be further pronounced after a third or subsequent reflection This fact is of importance in connection with the choice of an inner coating for a photometric integrator (see Chapter VII, p 216)

Regular Reflection.—The reflection factor of a polished surface depends not only on the frequency, but also, in the case of polarised light, on the angle between the plane of incidence and the plane of polarisation Fresnel's law ([73]) states that if ρ_1 and ρ_2 be respectively the reflection factors for light polarised in and perpendicular to the plane of incidence, then

$$\rho_1 = \sin^2 (i - r)/\sin^2 (i + r)$$

and

$$\rho_2 = \tan^2 (i - r)/\tan^2 (i + r),$$

where i and r are the angles of incidence and *refraction*. For perpendicular incidence it follows that $\rho_1 = \rho_2 = (n - 1)^2/(n + 1)^2$, and for glancing incidence ($i = 90°$) $\rho_1 = \rho_2 = 1$.

With glass of refractive index 1·5 the values of ρ_1 and ρ_2 are given in the following table ([74]) —

$i =$	0°	15°	30°	45°	56 3°	60°	75°	90°
$\rho_1 =$	0 040	0 044	0 058	0·092	0 148	0·177	0 399	1 000
$\rho_2 =$	0 040	0 036	0 025	0·008	0 000	0 002	0 107	1·000
$\tau_1 =$	0·960	0 956	0 942	0·908	0 852	0 823	0 601	0 000
$\tau_2 =$	0 960	0 964	0 975	0·992	1 000	0 998	0 893	0 000

Light polarised in a plane making an angle ϕ with the plane of incidence may be regarded as consisting of two components, whose intensities are proportional to $\cos^2 \phi$ and $\sin^2 \phi$, polarised respectively in and perpendicular to the plane of incidence, so that the reflection factor of the surface is in this case $\rho = \rho_1 \cos^2 \phi + \rho_2 \sin^2 \phi$ Unpolarised light may be regarded as made up of equal components polarised in two perpendicular directions, so that in this case $\rho = \frac{1}{2}(\rho_1 + \rho_2)$

From this table it will be seen that at a certain angle, for which $\tan i = n$ (known as the polarising angle), the reflection factor is zero for light polarised perpendicular to the plane of incidence, so that ordinary light reflected at this angle should be completely polarised in the plane of incidence ([75]). This phenomenon was made use of in some early forms of polarisation photometer (see p 4). In practice the polarisation is never complete, owing, probably, to the fact that the transition from one medium to another does not take place with absolute suddenness, but that there is an extremely thin transition layer ([76]).

While the above treatment is applicable to transparent bodies for which n can be measured, for opaque reflectors, such as metals, there is no such theoretical basis The reflection factor varies with the state of polish of the surface, and it also depends on the frequency, direction of plane of polarisation, and angle of incidence of the light. In the case of silver it may be as high as 0·9 or more Some values of reflection factors are given in Appendix VII , p 476.

It is easy to show that if an element of surface of brightness B be viewed by means of a polished plane surface of reflection factor ρ, the light appears to come from an element of surface of brightness $B\rho$ situated symmetrically with the real surface on the other side of the reflecting surface (Fig 62) This surface is known as the image of the original surface in the reflector, and if P be a part of the original surface, and P' its image, then $\angle PMN = \angle NML = \angle N'MP'$, and so for any other direction of the reflected light. Hence PP' is bisected perpendicularly by MQ at Q, and so for any other point of the surface. The flux emitted from P in the direction PM appears to come from P' along

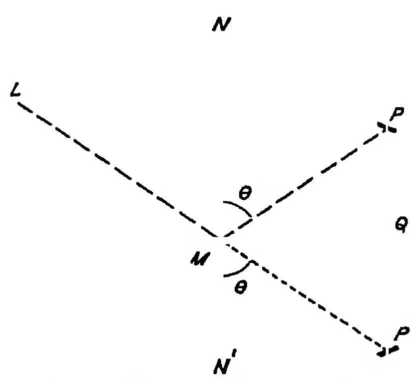

FIG 62 —Reflection in a Polished Surface

the path $P'L$. If the brightness of P in the direction PM be B, the flux reaching L would be the same as that from an equal element of surface at P', assuming perfect reflection. If the reflection factor is ρ, however, the flux is reduced in this ratio, that is, it is now the same as that which would be received from a surface at P' of brightness ρB. It is to be noted that this is also the apparent brightness of the reflecting surface at M

Diffuse Reflection.—A perfectly matt surface has been defined (p. 100) as one which emits light according to the cosine law. Similarly, a perfectly diffuse reflecting surface may be defined as one which redistributes the light it receives in such a manner that, whatever be the directional distribution of the incident light, the light reflected from each element of the surface in any given direction is proportional to the cosine of the angle which that direction makes with the normal to the surface In other words, the perfectly diffuse reflector acts as if it absorbed all the incident radiation and then re-emitted a definite fraction of it in accordance with the cosine law of emission Such a surface, therefore, appears equally bright in all directions (77) If illuminated to the extent of 1 metre-candle, it receives 1 lumen per square metre It therefore emits ρ lumens per square metre, and so has a brightness of ρ/π candles per square metre, or $\rho \times 10^{-4}$ lamberts. It may be treated exactly as the perfectly diffuse radiator was dealt with previously (pp 100, 102, *et seq*)

Reflection by Matt Surfaces.—It has already been said that the simple cosine law of reflection or emission enunciated by Lambert is not accurately followed by any known surface In every case the emission is a maximum in the direction of specular reflection (78), and in fact it is not easy to see any physical reason for supposing the law to be of the simple cosine form, unless it be assumed that the reflected or emitted light emanates equally in all directions from particles within the substance of the reflecting or emitting body. In this case, if α' be the absorption factor of the material, the light emitted in the direction θ from the particles at a distance x below the surface is proportional to $e^{-\alpha' x \sec \theta}$ if refraction be ignored (see p 116, *infra*) Hence the total light emitted in this direction

is (79) $\int_0^\infty e^{-\alpha' x \sec \theta}\, dx = (\cos \theta)/\alpha'$

The experimental evidence is, as has been said, against the cosine law, and various empirical expressions have been proposed to fit the results obtained by various workers,

e.g., $$B = C \,.\, \cos i \cos r/(C' \cos i + \cos r)$$

where C and C' are constants (80).

Bouguer supposed (81) that a matt surface was made up of innumerable elementary surfaces, each acting as a mirror (82), and this theory has been elaborated by E M Berry (83), who assumes that the elementary surfaces are distributed in slope according to the Gaussian probability law, so that if δA be the total area of the surfaces whose slopes lie between θ and $\theta + \delta\theta$, then $\delta A = \kappa e^{-a^2 b^2 \theta^2} \delta\theta$. This leads to the following expression for the brightness at an angle

θ' when the light is incident at an angle θ, θ' being taken in the plane of incidence, and B_0 being the brightness when $\theta = \theta' = 0$:—

$$B = B_0 \sec^2 \tfrac{1}{2} (\theta - \theta') \sec\theta \ \sec\theta' . e^{-a^2 \tan^2 \frac{1}{2} (\theta - \theta')}.$$

When this is compared with Lambert's cosine law, for which $B = B_0$, very good agreement is found up to $\theta' = 80°$ for the case of normal incidence ($\theta = 0$), putting $a^2 = 3$ For $\theta = \theta'$, $B = B_0 \sec^2\theta$, and for $\theta = -\theta'$, $B = B_0 \sec^4\theta \ e^{-3 \tan^2 \theta}$

These expressions, while showing marked departures from Lambert's law, are of the kind generally found in practice with so-called matt surfaces

In many problems of photometry it is desirable to have a surface which behaves as nearly as possible like a perfect diffuser Surfaces of magnesium carbonate, plaster of Paris, white blotting paper, sand-blasted celluloid, and many others have been used, as in most cases a surface having equal reflection factors for lights of all frequencies (*i e.*, a white surface) is required In general, departure from the cosine law is greatest (*a*) when the angle of incidence of the light is over 45°, (*b*) when the angle of emission considered is large and, particularly, (*c*) in the neighbourhood of the direction of specular reflection

These points are referred to further in Chapters XII. and XIII, where the use of diffusing surfaces for measuring illumination and the method of determining reflection and transmission factors of diffusing surfaces are described

Absorption.—Of the light entering a body from an external medium, part is absorbed within the substance of the body, and its energy is converted into other forms, *e g*, heat, chemical action, or some other process requiring the supply of energy If the body absorb all the light entering it, it is said to be opaque, otherwise it is termed transparent or translucent Clearly these terms can only be used in a very rough and ill-defined way The absorption factor of a body is defined as the ratio of the flux absorbed by the body to the flux incident upon it As in the case of reflection, this ratio depends on the frequency of the radiation entering the body, some frequencies being more readily absorbed than others

It will be noticed that the above definition of absorption factor concerns a particular *body*, and not a substance in general It is, therefore, dependent on the form of the body, and, in fact, depends entirely on the length of path travelled by the radiation within the body. This, again, depends on the thickness, and therefore for any given substance it is the absorption factor per unit thickness which must be employed

In the case of homogeneous transparent substances, the light passes through them undeviated, while in translucent bodies it suffers internal reflections, so that the length of its path may be many times the thickness of the body In the former case, if the absorption factor for a thickness of 1 cm be α, and the luminous flux entering any given area of the body be F, then the intensity absorbed in passing through 1 cm thickness is αF, and the amount entering the second layer of 1 cm thickness is $(1 - \alpha)F$, the amount absorbed in this second layer is therefore $\alpha(1 - \alpha)F$, and that trans-

mitted is $(1 - \alpha)^2F$, and so on The amount transmitted through a layer of thickness t cm is therefore $(1 - \alpha)^tF$, and that absorbed is $F\{1 - (1 - \alpha)^t\}$ If, instead of using the absorption factor per unit thickness, the absorption in a very thin layer be expressed as a multiple of the thickness of that layer, $\alpha'\delta t$, the transmitted flux is [84] $F(1 - \alpha'\delta t)^{t/\delta t} = Fe^{-\alpha' t}$ [85], and the absorbed flux is therefore $F(1 - e^{-\alpha' t})$

The quantity α' in the above relationship is generally termed the absorption factor (or coefficient) of the substance [86] This name, however, has sometimes been applied misleadingly to the quantity $e^{-\alpha'}$, which is written k, so that the above expression for the absorbed light becomes $F(1 - k^t)$, and for the transmitted light Fk^t k is more properly termed the transmission factor (or coefficient) *of the substance* [87], or sometimes its "transmissivity" [88] For most glass of good quality k is in the neighbourhood of 0 9 when t is in centimetres [89]

For substances in homogeneous solution in a solvent it might be expected *a priori* that increase in concentration would be exactly equivalent to increase in thickness, *i e*, that the transmission factor would be proportional to k^{tc}, where t was the thickness, and c the concentration This relation, known as Beer's law, is found, however, to be only approximately true in many cases [90] The constant k in the equation $\tau' = k^{tc}$ may be called the specific transmission factor or specific transmissivity of the dissolved substance [88], τ' being now the ratio of the transmission factor of a thickness t of the solution to the transmission factor of an equal thickness of the solvent alone

In the case of a translucent material, owing to the backward scattering of a certain portion of the light traversing the medium, the simple exponential expression given above for transparent media does not hold It has been found empirically that, to a close approximation, $\log \tau = at^b$, where a and b are constants of the material and t is the thickness of the plate [91].

A perfectly absorbing body is one that absorbs all the radiation incident upon it, it must, therefore, be not only perfectly opaque, but it must not reflect any of the incident radiation It follows that it must have the same index of refraction as the medium surrounding it, and, in fact, the only perfect absorber is a "black-body" cavity (see p. 34)

When light reaches the second bounding surface of a body, it suffers a certain degree of internal reflection on its passage out of the body, in fact, this is merely another case of reflection on passing from one medium to another It will be noticed, from the form of the expressions given above for the reflection factor, that ρ is the same on passage from medium I to medium II as for passage from medium II. to medium I. when the angle is the same in the same medium

Transmission.—The definition of the transmission factor of a body or a substance is exactly analogous to the definition of absorption factor, and the dependence of one of these quantities on the other has already appeared In general, the transmission factor is of more practical importance than the absorption Like the latter, it depends on the frequency of the incident radiation and it is this which

determines the colour of a substance as seen by transmitted light. A blue glass has its highest transmission factor for frequencies in the blue part of the spectrum, while most of the red light is absorbed The function connecting thickness and transmission of a transparent medium has already been found, but in any particular body the light lost by reflection at the two surfaces must be added to the light lost by absorption within the substance of the body, so that $\tau = 1 - \alpha - 2\rho$, *where ρ denotes the reflection factor for a single surface.* Generally, however, part of the light reflected at the second surface is added to the light absorbed, and the remainder to the light reflected at the first surface, so that the relationship is written in its usual form, $\tau = 1 - \alpha - \rho$, in which α denotes the proportion of the light absorbed, and ρ the proportion returned from both surfaces in the direction opposite to that of the incident light

The curves of Fig 60 may be used to illustrate the dependence of the colour of a substance seen by transmitted light on the variation of its transmission factor with frequency. If R refer again to the incident light, while B and W are respectively the transmission curves of blue and colourless transparent bodies, then the curves BR and WR of Fig 60 give the spectral distribution of the transmitted light

It is to be noticed that this curve only holds for a certain thickness of the transparent material. Doubling the thickness clearly results in a second multiplication of the ordinates of the spectral distribution curve by the corresponding ordinates of the transmission curve (just as in the case of a double reflection) Curve B^2W in Fig 61 shows the distribution resulting from the passage of white light through two thicknesses of a medium having the transmission curve B in Fig 60, and from this it will be seen that the greater the thickness of the transmitting substance, the greater the accentuation of the light in the region of maximum transmission This observation is of importance in the determination of the transmission factors of the coloured media used in photometry (see Chapter VIII , p 242).

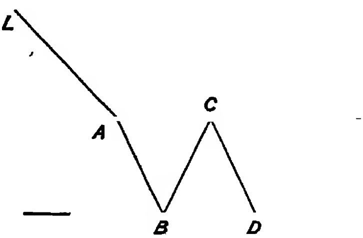

FIG 63—Transmission through a Plate

The light transmitted by a single transparent plate of glass may be calculated fairly simply thus If the incident light follow the direction LA, the incident flux is F Of this $F\rho$ is reflected, and $F(1 - \rho)$ enters the glass along the path AB Of the part $F(1 - \rho)k^t$, which reaches B, $F(1 - \rho)\rho k^t$ is reflected along BC, and $F(1 - \rho)^2k^t$ emerges Of the part reaching C, *viz*, $F(1 - \rho)\rho k^{2t}$, one part, $F(1 - \rho)\rho(1 - \rho)k^{2t}$, emerges, and $F(1 - \rho)\rho^2k^{2t}$ is reflected along CD to D, where $F(1 - \rho)^2\rho^2k^{3t}$ emerges

Hence the total light emerging on the side BD is [92]

$$F(1 - \rho)^2\{k^t + \rho^2k^{3t} + \rho^4k^{5t} + \quad . \} = F(1 - \rho)^2k^t/(1 - \rho^2k^{2t})$$

which, if $\rho = 0\ 04$ and $k^t = 0\ 9$, is equal to 0·83 [93]

When light traverses two or more plates in succession, the

expressions for the transmission factor become more complicated G G Stokes has given the formula [94]

$$\rho_N/(b^N - b^{-N}) = \tau_N/(a - a^{-1}) = 1/(ab^N - a^{-1}b^{-N})$$

where ρ_N and τ_N are respectively the reflection and transmission factors for N parallel plates, and a, b are constants found from the equations

$$\rho_1/(b - b^{-1}) = \tau_1/(a - a^{-1}) = 1/(ab - a^{-1}b^{-1})$$

This formula applies no matter what the angle of incidence of the light or the absorption factor of the substance of the glass When there is no absorption, the above expressions become indeterminate In this case, writing ρ' for $(1 - n)^2/(1 + n)^2$,

$$\rho_N/2N\rho' = \tau_N/(1 - \rho') = 1/\{1 + (2N - 1)\rho'\} \text{ [95]}$$

The light transmitted by a triangular prism of base-length t is clearly equal to $(1/t)\int_0^t k^t dt$, i e , to $(k^t - 1)/t \log_e k$ if reflection at the surfaces be neglected [96] In the case of a constant deviation prism (Fig 14) of which the length of the reflecting side is $t\sqrt{2}$, the transmission is, similarly, found to be $k^t(k^{nt} - 1)/nt \log_e k$ where $n \equiv \sqrt{3}/2$

Diffuse Transmission.—In the case of a translucent body, or of a transparent body one or both surfaces of which have been rendered matt, diffusion of the incident light takes place to a greater or less extent [97]

The transmitted light, as might be expected, is not distributed according to the cosine law of emission, the intensity being always a maximum in the direction of the incident light if the substance be in the form of a plate [98] The approximation to perfect diffusion increases with the density of the substance and the thickness of the plate, and the nearest approach to a perfectly diffuse transmitter is a sheet of dense opal glass sand-blasted or etched on both sides Diffuse and colourless transmission is required for the window of any form of photometric integrator (see Chapter VII), but it is difficult to obtain a material which combines a sufficiently high transmission factor with a satisfactory degree of diffusion

Definition of "Diffusing Power."—The "diffusing power" of a substance, opaque or translucent, may be qualitatively defined as the degree to which the light it reflects or transmits is distributed according to the cosine law A satisfactory quantitative definition of diffusing power is, however, very difficult to arrive at, and it has generally been expressed by means of a polar curve showing the candle-power of a given element of surface as viewed from all directions in a plane passing through the normal to that element This curve, sometimes termed the indicatrix [99], becomes a circle touching the element in the case of a perfect diffuser For all other surfaces it is distorted in a manner depending on the nature of the surface and on the direction of the incident light Various attempts have been made to define a "figure of merit" for expressing the diffusing power of a substance [100] Probably the most convenient system is that of Halbertsma [101], in which, with light incident normally on the surface, the candle-powers at different angles of view, θ, are plotted as abscissæ, the corresponding ordinates being

proportional to $(1 - \cos\theta)$, as in Fig 64. For a perfectly diffusing surface the resulting curve clearly becomes a straight line The area enclosed by the axes of co-ordinates and the representative

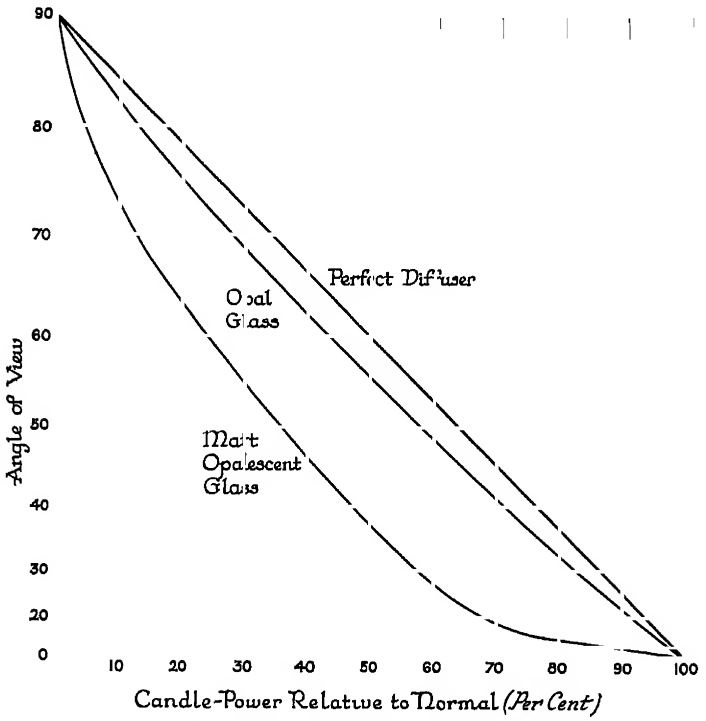

FIG. 64 —Diagram for the Definition of Diffusing Power.

curve for any surface may be termed the " diffusing power " of the surface, the area for a perfectly diffusing surface being taken as unity. Imperfect diffusion is sometimes referred to as " spread " reflection or transmission (102)

Atmospheric Diffusion.—The visibility of distant objects in the open depends mainly on the diffusing power of the intervening atmosphere, and instruments have been designed for measuring this quantity on an arbitrary scale by finding the amount of additional diffusion necessary in order to cause a given object just to become invisible This additional diffusion is obtained by inserting a number of sheets of material of graded diffusing power between the observer's eye and the selected object (103)

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(2) Illumination is a possible alternative, but a standard of illumination can only be set up through the medium of a standard source. See U Bordoni, Elettrot, 6, *1919*, p 430, C I E, Proc, 5, *1921*, p 53, Elettrot, 9, *1922*, p 37, N Campbell and B P Dudding, Phil Mag, 44, *1922*, p 577, J W. T Walsh, *ibid*, p 1165

(3) See Report of Standards Com on Photometry and Illumination, Opt Soc Am, J, 4, *1920*, p 230.

(4) L Bloch "Grundzüge d Beleuchtungstechnik" (Berlin, *1907*), p 1, Illum Eng, 2, *1909*, pp. 516 and 682, F W Wilcox, Illum Eng, 10, *1917*, p 10, A. R Meyer, "Lichttechnik" (Munich, *1921*), p 14

(5) The compressibility of water may be neglected for the purposes of illustration

(6) Report of the National Illumination Committee of Great Britain, Illum Eng, 15, *1922*, p 225

(7) The unit of solid angle, often called a "steradian," is the angle subtended at the centre of a sphere of unit radius, by unit area of the surface of the sphere. It follows that the total solid angle at a point is 4π steradians

(8) For a curious example of the confusion which may arise from a misconception of flux, see the Electrician, 29, *1892*, p 170

(9) See note (23), p 142.

(10) Another unit of candle-power used in Germany and some other countries is the Hefner candle, of which the value is 0 9 international candles (see p 130) Its symbol is HK

(11) J Ondracek Licht u Lampe, *1925*, p 231

(12) W J Dibdin Soc Chem Industry, J, 3, *1884*, p 277

(13) See, *e g*, E T Z, 27, *1906*, p 479, and 28, *1907*, p 304, Heyck, E T Z, 36, *1915*, p 620 It is to be noted that in the case of a body of uniform brightness, provided there be no reflections from cavities in its surface, the mean hemispherical candle-power is the same in each of the two hemispheres and is equal to the mean spherical candle-power. See, *e g*, H Heimann, E T Z, 27, *1906*, p 380, Ecl El, 47, *1906*, p 396

(14) G B Dyke. Phys Soc, Proc, 19, *1904*, p 399 See also Illum Eng Soc N Y, Trans, 5, *1910*, p 605

(15) British National Illum Committee, C I.E, Proc, 5, *1921*, p 98

(16) F A Benford Gen El Rev, 28, *1925*, p 271, Illum Eng, 18, *1925*, p 126, El World, 85, *1925*, p 1357, E u M (Lichttechnik), 43, *1925*, p 131

(17) See, *e g*, C F Close and A R Clarke, article "Map," in Encyc Brit (11th ed), vol 17, p 658

(18) F Uppenborn J G W, 48, *1905*, p 272, E T Z, 28, *1907*, pp 139 and 168
J Herzog and C Feldmann E T Z, 28, *1907*, p 93

(19) Rousseau C R des essais photométriques à l'exposition d'Anvers, *1885*
R H Ledeboer Lum El, 26, *1887*, p 58
E Liebenthal E T Z., 10, *1889*, p 337
See also H. Krüss, Central-Ztg f Opt u Mech, 5, *1884*, p 110, Z f Elektiot, 2, *1884*, p 404; Der Gastechniker, 2, *1884*, p 53, Centralbl f Elektrot, 6, *1884*, p 372

(20) E T Z, 37, *1916*, p 53 This paper is supplied by Messrs Drawing Office Supplies, Ltd, of Cheapside, or by Messrs Carl Schleicher and Schüll of Duren in Rhineland

(21) Illum Eng Soc N Y, Trans, 3, *1908*, p 243, El World, 51, *1908*, p 645, El Engineer (N. Y.), 41, *1908*, p 521; Z f Bel, 14, *1908*, pp. 175 and 185, E u M, 26,

1908, p 390, E T Z, 29, *1908*, p 844, Illum Eng, 7, *1914*, p 242, Electrician, 61, *1908*, p 371, Lum El, 2, *1908*, p 275 See also J Kuhn, E u M, 33, *1915*, p 500

(22) Other shortened methods for finding mean spherical candle-power (total flux), or flux within a given zone, have been described by L Wild, Electrician, 55, *1905*, p 936, E T Z, 27, *1906*, p 122, Ecl El, 46, *1906*, p 503, and by the following —

E W Weinbeer El World, 53, *1908*, p 1396, Illum Eng, 1, *1908*, p 559, Elektrot Anzeiger, 26, *1909*, pp 135 and 215

A Wohlauer Illum Eng (N Y), 3, *1909*, p 655, and 4, *1909*, p 491, Licht u. Lampe, *1923*, pp 60 and 84

J S Codman and T W Rolph Illum Eng (N Y), 4, *1909*, p 148

W E Barrows Illum Eng Soc N Y, Trans, 4, *1909*, p 433

J K Sumec E u M, 27, *1909*, p 318, Lum El, 6, *1909*, p 152

J S Codman Illum Eng (N Y), 5, *1910*, p 252, Illum Eng, 3, *1910*, p 138

H Strache Z d Vereines v Gas- u Wasserfachm in Ost-Ungarn, 51, *1911*, p 489.

W E Barrows El Rev (Chicago), 58, *1911*, p 440

J Teichmüller J G W, 55, *1912*, p 190

R ff Pierce El World, 61, *1913*, pp 297, 578 and 1046

W E Hodge and R W Chadbourne Illum Eng Soc N Y, Trans, 11, *1916*, p 22, Am Gas Light J, 104, *1916*, p 162

N A Halbertsma E u M, 35, *1917*, p 65

O Gerhardt E u M, 36, *1918*, p 359

A R Myhill Gas J, 161, *1923*, p 332

L Bloch E T Z, 44, *1923*, p 1071

See also L Bloch, Illum Eng, 2, *1909*, pp 516 and 682

(23) A Russell Inst El Eng, J, 32, *1903*, p 631, Ecl El, 37, *1903*, p 193

N Macbeth Illum Eng (N Y), 3, *1908*, p 27

See also N. Macbeth, Illum Eng (N Y), 5, *1910*, p 126, and E T Z, 35, *1914*, p. 274

(24) H Buckley. Illum Eng, 18, *1925*, pp 69 and 93

(25) See also J Ondracek, E u M, 36, *1918*, p 77

(26) H Buckley Illum Eng, 18, *1925*, pp 69 and 93

(27) See, *e g*, J Teichmüller, J G W, 57, *1914*, p 193, E T Z, 36, *1915*, p 417

(28) Sometimes "candle-foot" On the objections to a name of this form for the unit of illumination, see, *e g*, A P Trotter, "Illumination, etc," p 16, B Monasch, J G W, 50, *1907*, p 1143, C H Sharp, Illum Eng (N Y), 2, *1907*, p 470, etc (See references to Appendix II)

(29) It may, perhaps, be thought that this assumption of constant rate of propagation is invalid, in view of the fact that light travels in water, for example, with a velocity which is only three-quarters of its velocity in air, but it will be clear that although it travels more slowly in water, as much energy must reach a surface enclosed in water as would reach it were the water absent (neglecting losses due to surface reflection, and absorption), otherwise there would be accumulation of energy in the body of the water In this respect the energy propagation may be likened to a number of balls dropped from a height at regular intervals. The rate of their arrival at the bottom will be the same as the rate at which they are dropped, but owing to the increased velocity at the end of their journey they will be farther apart (less concentrated) at the bottom than at the top of their path Similarly decreased velocity of the light will be compensated by the decrease in the wave-length, the frequency, and therefore the number of waves arriving at the surface in any given time, being unchanged

(30) For an account of an experimental investigation of the inverse square law, see F. Carstaedt, Pogg Ann, 150, *1873*, p 551, J de Phys, 4, *1875*, p 61

(31) To be distinguished from the cosine law of emission (see p 100)

(32) Tables of $\cos^3\theta$ have been given by Van R Lansingh (Western Soc Eng, J, 8, *1903*, p 137), by A P Trotter ("Illumination, etc," p 274), and by W E Barrows ("Light, Photometry and Illumination," p 314) They are included in W Bertelsmann's "Rechentafeln f Beleuchtungstechniker" (Enke, *1910*)

(33) See, *e g*, H Krüss, Der Gastechniker, 5, *1885*, p 169, Centralbl f Elektrot, 7, *1885*, p 670, Elektrotechniker, 4, *1885*, p 481

L Weber E T Z, 6, *1885*, p 55

J Wybauw Soc Belge des Elect, Bull, 2, *1885*, p 79

C E Guillaume Lum El, 26, *1887*, p 101

A Blondel Le Génie Civil, 26, *1895*, pp 212, 228, 244, 259 and 278, Electrician, 35, *1895*, p 846, and 36, *1895*, pp 13, 39 and 89

A P Trotter Inst Civil Eng, Proc, 110, *1892*, p 69

G L'Hoest Assn des Ingénieurs El, Liège, Bull, 9, *1898*, p. 464

K Bunte J G W, 55, *1912*, p 1197

J S Codman El Rev (N Y), 51, *1907*, p. 551

Illum Eng, 8, *1915*, p 399

E Piérard Soc Belge des Elect, Bull, 37, *1923*, p 1

(34) A graphical method of finding illumination curves has been described by F A Benford, Illum Eng Soc N Y, Trans, 7, *1912*, p 695

See also F Loppé, L'Electricien, 14, *1890*, p 936

C H Lees Phil Mag, 40, *1895*, p 463

F W Carter El Rev, 45, *1899*, pp 994 and 1034

W E Hodge and R W Chadbourne Illum Eng Soc N Y, Trans, 11, *1916*, p 22

E C White Illum Eng Soc N Y, Trans, 1, *1906*, p 233, El Rev (Chicago), 49, *1906*, p 648, Illum Eng (N Y), 1, *1906*, pp 333 and 640

(35) A P Trotter "Illumination, etc," p 46

(36) A P Trotter Inst Civil Eng, Proc, 110, *1892*, p 69, "Illumination, etc," pp 47–51

H Maréchal "L'Eclairage à Paris" (Baudry, *1894*), pp 474 *et seq*

A Blondel *Loc cit*, note (33), *supra*, and Electrician, 37, *1896*, pp 14 and 42.

H Lux Z f Bel, 2, *1896*, p 282

C P Steinmetz "Radiation, Light and Illumination," p 252

W. E Barrows "Light, Photometry and Illumination," p 186

F Uppenborn—B Monasch "Lehrbuch d Phot," p 135

W E Wickenden "Illumination and Photometry," p 153

H Bohle "Electrical Photometry and Illumination," p 119

(37) See, *e g*, A Schmidt, Phys Z, 4, *1903*, p 453

(38) A Blondel Ecl El, 8, *1896*, p 8

(39) C I E, Proc, 6, *1924*, p 69, Report of Nat Illum Com of Gt Britain, Illum. Eng, 18, *1925*, p. 40

(40) See, *e g*, El. World, 84, *1924*, p 1295, Illum Eng, 17, *1924*, p 165

(41) "Photometria," p 324

(42) El World, 65, *1915*, pp 332, 460 and 715.

(43) Report of Committee on Nomenclature and Standards, Illum Eng Soc N. Y, Trans, 13, *1918*, p 512

(44) C I E, Proc, 6, *1924*, p 175, Illum. Eng Soc N Y, Trans, 19, *1924*, p 512

(45) Blondel has proposed the name "stilb" for the candle per sq cm, so that 1 millistilb = π millilamberts (C I E, Proc, 5, *1921*, p 89, and 6, *1924*, pp. 156 and 167, Rev Gén de l'El, 16, *1924*, pp 771 and 896, Licht u Lampe, *1925*, p 361)

(46) A F Salanson Soc techn de l'indust du gaz, C r, 16, *1889*, p 200

E P Hyde Bureau of Standards, Bull, 3, *1907*, p 88, Phys Rev, 24, *1907*, p 249 Science, 25, *1907*, p 511

P D Foote Bureau of Standards, Bull, 12, *1916*, p 583

See also H Krüss, E T Z, 8, *1886*, p 17, J G W, 29, *1886*, p 886, Central-Ztg f Opt u Mech, 8, *1887*, p 28, J. of Gas Lighting, 49, *1887*, p 156, J de l'Ecl au Gaz, 35, *1887*, p 148

D Coglievina J G W, 30, *1887*, p 88, Central-Ztg f Opt u Mech, 8, *1887*, p 97, Z f d chem Industrie, 1, *1887*, p 326

K Strecker. J G W, 30, *1887*, p 229, Centralbl f Elektrot, 9, *1887*, p 384, E T Z, 8, *1887*, p 17

Dingler's Polytechn J, 265, *1887*, p 236

A Palaz Lum El, 24, *1887*, p 174

Gas World, 25, *1896*, p 284, Progressive Age, 14, *1896*, p 559

S Frohner Z f d phys u chem Unterricht, 37, *1924*, p 145

On the effect of size of the photometer disc, see H Kruss, Central-Ztg f Opt u Mech 7, *1886*, p 218, also Elektrotechniker, 5, *1886*, pp 106, 145 and 157.

(47) See also U Bordoni, Rivista d'ottica e mechanica di precisione, 2, *1922* (II), p 1

(48) "Photometria," p 99

J W T Walsh Phys Soc, Proc, 32, *1920*, p 59

An extended investigation of similar problems has been carried out by C Hillebrand, Wien Ber (IIa), 118, *1909*, p 1399

See also V Fock, Z f Phys, 28, *1924*, p 102

Z. Yamanouti Electrot Lab, Tokyo, Researches, No 148, *1924*, Z f I, 45, *1925*, p 402.

J Ondracek E u M (Lichttechnik), 42, *1924*, p 336

(49) J W T Walsh Phys Soc, Proc, 32, *1920*, p 315

(50) E P Hyde. Bureau of Standards, Bull, 2, *1906*, p 1, and 3, *1907*, p 81

For the mathematical treatment see A S Chessin, Am Math Soc, Bull, 14, *1908*, p 212

(51) G B Dyke Phys Soc, Proc, 19, *1905*, p 616; Phil Mag, 10, *1905*, p 216

E W Weinbeer Z f Bel, 15, *1909*, p 305

(52) The radiation from a spherical diffusing surface has been investigated by W Saltzmann (E T Z, 8, *1887*, p 430) The illumination due to such a source at a point distant d from the geometrical centre of the sphere is r^2B/d^2 This formula naturally

applies also to a hemispherical source so long as the edge of the hemisphere is not visible from the point under consideration

(53) The case of the transparent radiator may, clearly, be treated as a problem in gravitation See A Beer, Pogg Ann, 88, *1853*, p 114, W von Bezold, Phil Mag, 41, *1871*, p 241, Pogg Ann, 141, *1870*, p 91, H Ebert, Gesell deut Naturforscher u Aerzte, Tagebl, 62, *1889*, p 200

(54) B Jones Illum Eng Soc N Y, Trans, 4, *1909*, p 216 [formula (13) in this paper is incorrect], 5, *1910*, p 281 [formula (20) is incorrect], and 6, *1911*, p 365 [gives the polar curve for a rectangle]

U Bordoni Assoc El Ital, Atti, 12, *1908*, p 265, and 17, *1913*, p 559

R Böker Z f Bel, 25, *1919*, p 95

(55) See, *e g*, R W Wood, Phil Mag, 11, *1906*, p 782, Phys Z, 7, *1906*, p 475

On the transmission of flames, see E Allard, C R, 82, *1876*, p 1300, Les Mondes, 40, *1876*, p 303, Ann des Ponts et Chaussées, 12, *1876*, p 5, Acad des Sci, Paris, Mém. Divers Savants, 25, *1877*, No 2

(56) For an example of the magnitude of the errors liable to be introduced by neglecting the dimensions of the light source, see O Bastian, Electrician, 57, *1906*, p 131, Ecl El, 47, *1906*, p 505, F Laporte, Electrician, 57, *1906*, p 549, Ecl El, 48, *1906*, p 276, Z f Bel, 13, *1907*, p 1, E Hospitalier, L'Industrie El, 16, *1907*, p 33

(57) See, *e g*, J Pole, Illum Eng Soc N Y, Trans, 6, *1911*, p 306, E T Z, 32, *1911*, p. 440, Z. f Bel, 18, *1912*, pp 26, 37, 49, 61 and 74, Lum. El, 15, *1911*, p 205

E B Rosa Bureau of Standards, Bull, 6, *1910*, p 543, Illum Eng Soc N Y, Trans, 5, *1910*, p 473

F Uppenborn Z. f Bel, 11, *1905*, p 71

K Norden E T Z, 28, *1907*, p 757, 29, *1908*, p 883, and 34, *1913*, p 292, El World, 61, *1913*, p 841

M D Cooper El World, 54, *1908*, p 718

A S McAllister Illum Eng Soc N Y, Trans, 6, *1911*, p 703

(58) Gas World, 25, *1896*, p 284, Am Gas Light J, 65, *1896*, p 407

(59) This case is approximately the same as that of the pentane standard at a distance of 10 cm from the photometer disc

(60) Notification of the Metropolitan Gas Referees, *1916*, p 24

(61) Dept of Communications, Tokyo Researches "On the Use and Selection of Standard Electric Lamps" [In Japanese], Feb, *1921*

(62) J A Fleming Phys Soc, Proc, 19, *1905*, p 681, Phil Mag, 10, *1905*, p 208, Ecl El, 45, *1905*, p 191, Chem News, 91, *1905*, p 301, Nature, 72, *1905*, p 215

(63) C Michalke E T Z, 43, *1922*, p 275

(64) See, *e g*, R Ulbricht, "Das Kugelphotometer" (Oldenbourg, Berlin, *1920*), p 18

N A Halbertsma Z f Bel, 22, *1916*, p 43

(65) E Liebenthal E T Z, 16, *1895*, p 655

A G Vernon-Harcourt J of Gas Lighting, 51, *1888*, p 371

H B Dixon J of Gas Lighting, 48, *1886*, p 615

C H Sharp El World, 27, *1896*, p 54, Am Gas Light J, 64, *1896*, p 125

D S Jacobus Am Soc Mech Eng, Trans, 17, *1896*, p 212, El World, 26, *1895*, p 643, Am Gas Light J, 63, *1895*, p 972, J of Gas Lighting, 67, *1896*, p 21, Progressive Age, 13, *1895*, p 592

(66) N A Halbertsma E T Z, 38, *1917*, p 482, E u M, 37, *1919*, p 197.

J Ondracek E u M, 36, *1918*, p 476

(67) J Clerk Maxwell Phil Trans, 150, *1860*, p 57

(68) See note (70), p 408 See also A. Blondel, C R., 181, *1925*, p. 310

(69) A Broca "Leçons d'Optique Physiologique" (Paris, *1923*), p 40

A A Michelson Opt Soc Am, J, 9, *1924*, p 197

(70) Photo-luminescence (see p 41) is not considered here since it is not a pure reflection phenomenon

(71) See, *e g*, H E Ives, Illum Eng Soc N Y, Trans, 7, *1912*, p 62

T E Ritchie Illum Eng, 5, *1912*, p 64

M Luckiesh Illum Eng Soc N Y, Trans, 17, *1922*, p 315

(72) L Weber Central-Ztg f Opt u Mech, 5, *1884*, p 53

(73) A J Fresnel Ann Chim Phys, 17, *1821*, p 167 (paged in error as 267), and 46, *1831*, p 225

T Preston "Theory of Light," §§ 208 and 209, A Schuster and J W Nicholson, "Theory of Optics," §§ 27 and 143

This law has been verified experimentally by E C Pickering (Am Acad Arts and Sci, Proc, 9, *1873*, p 1), and by D W Murphy (Wied Ann 57, *1896*, p 593)

(74) E Liebenthal "Prakt Phot," p 397 Experimental values have been btained by Rayleigh and others See, *e g*, Roy Soc, Proc, 41, *1886*, p 275, Nature, 5, *1886*, p 64. Engineering, 42, *1886*, p 282

(See over.

By special treatment of the surface the reflection factor can be reduced See F. Kollmorgen, Illum Eng Soc N Y, Trans, 11, *1916*, p 220

(75) It does not follow that this is the direction of maximum polarisation for the transmitted light See T Erhard, Wied Ann, 12, *1881*, p 655

(76) L Lorenz Pogg Ann, 111, *1860*, p 460, and 114, *1861*, p 238.
P Drude "Lehrbuch d Optik," p 266 (287)
R C Maclaurin Roy Soc, Proc, 76, *1905*, p 49

(77) J H Lambert "Photometria," part 3, chap II
D Dubrovsky Russian Phys-Chem Soc, J (Phys part), 42, *1910*, p 350

(78) P Bouguer "Traité d'Optique," p 175

(79) J C F Zöllner Photometrische Untersuchungen (Engelmann, Leipzig, *1865*), p 17
E Lommel Wied Ann, 10, *1880*, pp 449 and 631
Smoluchowski de Smolan J de Phys, 5, *1896*, p 488
L V King Phil Mag, 23, *1912*, p 237
See also P G Nutting, Illum Eng Soc N Y, Trans, 7, *1912*, p 616, and C Michalke, E T Z, 43, *1922*, p 275

(80) H Seeliger Vierteljahrsschrift d Astronomischen Gesell, 20, *1885*, pp 111 and 267, and 21, *1886*, p 217, Munchen Ber, 18, *1888*, p 201
See also E Lommel, Munchen Ber, 17, *1887*, p 95, Wied Ann, 36, *1889*, p 473, J of Gas Lighting, 64, *1894*, p 777, Am Gas Light J, 61, *1894*, p 661
G Müller "Photometrie der Gestirne" (Leipzig, *1897*), pp. 25 *et seq*
F Jentzsch Ann d Phys, 39, *1912*, p 997
G Meslin Ann Chim Phys, 1, *1914*, p 297
B Fessenkoff C R, 158, *1914*, p 1271

(81) Traité d'Optique, p 161 See, however, L Grabowski, Astrophys J, 39, *1914*, p 299, Astron Nachr, 199, *1914*, p 49

(82) On the fine structure of diffusing substances, see F L Godinez and A J Marshall, Illum Eng (N Y), 5, *1910*, p 526

(83) Opt Soc Am, J, 7, *1923*, p 627 See also G I Pokrowski, Z f Phys, 30, *1924*, p 66

(84) See, *e g*, Lambert, "Photometria," pp 222 *et seq* Also G Govi, C R, 85, *1877*, p 1100, J Bottomley, Manchester Lit and Phil Soc, Mem, 28, *1884*, p 198, C Chistoni, Soc dei Naturalisti di Modena, Atti, 1, *1900*, p 66

(85) This may also be written $10^{-\epsilon t}$ where $e^{a'} = 10^{\epsilon}$ ϵ is then the reciprocal of the thickness having a transmission factor 0 1 It was called by Bunsen and Roscoe (Pogg Ann, 101, *1857*, p 235) the "extinction coefficient" See also E Hagen and H Rubens, Ann. d Phys, 8, *1902*, p 432

(86) T Preston "Theory of Light," § 282 See also "Traité d'Optique," p 237

(87) See H Kayser, "Handbuch d Spectroscopie" (Leipzig, *1905*), vol 3, p 15
G and H Kruss "Kolorimetrie u quantitative Spektralanalyse" (Hamburg, *1891*), p 237
S E Sheppard "Photo Chemistry" (Longmans, Green, *1914*), p 144

(88) Report on Spectrophotometry, Opt Soc Am, J, 2, *1915*, p 177

(89) See, *e g*, J Conroy, Brit Assn Report *1886*, p 527, Roy Soc, Proc, 45, *1889*, p 101

(90) A Beer Pogg Ann, 86, *1852*, p 78
E C C Baly "Spectroscopy" (Longmans, *1912*), 1st ed, p 492
W C McC Lewis "System of Physical Chem" (Longmans, *1920*), vol 2, p 409
H S Taylor "Treatise on Physical Chemistry" (Macmillan, *1924*), p 1207.

(91) H J Channon, F F Renwick and B V Storr. Roy Soc, Proc, 94, *1917*, p 222, Phot J, 58, *1918*, p 121

(92) F Bernard Ann Chim Phys., 35, *1852*, p 385

(93) It is to be noted that no allowance need generally be made for the light reflected by a glass bulb or chimney surrounding a light source, for clearly the amount lost by reflection at the surfaces of the front glass is exactly compensated by the light gained owing to reflection in the surfaces of the back glass, neglecting absorption in the flame and in the substance of the glass itself See R A Fessenden, El World, 33, *1899*, p 231

(94) Roy Soc., Proc, 11, *1862*, p 545, Math and Phys Papers, vol 4, p 145 See also F R Benford, Opt Soc Am, J, 7, *1923*, p 1017

(95) H Lunelund Phys Zeit, 10, *1909*, p 222.
T. Y Baker Opt Soc, Trans, 22, *1921*, p 88
See also F de la Provostaye and P Desains, Ann Chim Phys, 30, *1850*, p 159
H Wild Pogg Ann, 99, *1856*, p 235, Bibliothèque Univ, 3, *1858*, p 341.
J A Fresnel Œuvres (Paris, *1868*), vol 2, p 789

(96) F Paschen Berlin Ber, *1899*, p 405

(97) For a theoretical treatment of the subject, see, *e g*, O Chwolson, Mélanges physiques et chimiques, 13, *1889*, p 83, Acad Imp des Sciences, St Pétersbourg, Bull, 33, *1890*, p 221, Repertorium d Phys, 26, *1890*, pp 364 and 385.
H J Channon, F F Renwick and B. V. Storr. Roy Soc, Proc, 94, *1918*, p 222; Phot J, 58, *1918*, p 121.

(98) T. W Rolph Illum Eng Soc N Y, Trans, 11, *1916*, p. 1144
(99) Traité d'Optique, p 170
A Blondel Ecl El, 3, *1895*, pp 406 and 583
(100) A v Obermayer Photographische Korrespondenz, 47, *1910*, p 66
M Luckiesh El World, 60, *1912*, p 1040, Electrician, *1913*, p 562
(101) N A Halbertsma J G W., 60, *1917*, p 651, E T Z, 39, *1918*, p 207; E u M, 36, *1918*, p 225
(102) Report on Spectrophotometry, Opt Soc Am, J, 10, *1925*, p 169
(103) A Wigand Phys Z, 20, *1919*, p 151, Meteorol Z, 36, *1919*, p 342, Phys Z, 22, *1921*, p 484, and 23, *1922*, p 277, Meteorol Z, 38, *1921*, p 349, and 41, *1924*, p 216, Phys Z, 23, *1922*, p 277, Ann d Hydrogr u. Marit. Meteorol, 47, *1919*, p 134, Das Wetter, 39, *1922*, p 54
A Wigand and K Genthe Phys Z, 25, *1924*, pp 212 and 263, Z f I, 44, *1924*, p 470, Meteorol Z, 40, *1923*, p 365
A Wigand Z f I, 45, *1925*, p 411

CHAPTER V

STANDARDS OF CANDLE-POWER

IT has already been said (p. 84) that the only practical photometric standard is one of luminous intensity. The standards to be dealt with in this chapter include two groups (*a*) primary standards by reference to which the candle-powers of all sources are expressed, and (*b*) secondary standards for practical use in everyday photometry as temporary custodians of the unit fixed by the primary standards.

The conditions which a primary standard of light should fulfil are those required of any physical standard, *viz*, ease of reproducibility from specification, maintenance of value over long periods, low correction factors for change of conditions, such as barometric pressure, temperature, *etc* In addition to these, a standard of luminous intensity should fulfil, as far as possible, the conditions that its candle-power should be of a convenient magnitude, and that the spectral distribution of its light should approximate to that of the light sources to be measured by comparison with it (1) This is important owing to the physiological aspect of photometry and the great difficulties introduced by a difference of colour between the lights being compared (see Chapter VIII).

Many suggestions have been made at different times for the production of a satisfactory primary standard of luminous intensity (see Chapter I, pp. 4 *et seq.*) None of these fulfils all the conditions outlined above. Most are difficult to reproduce with sufficient accuracy, and are greatly affected by change of exterior conditions. None of those which most nearly meet the other requirements fulfils, even approximately, the condition as to spectral distribution, the colour of the light being in every case much redder than that of the sources of light in common use to-day

The standards which have actually been employed at various times are (*a*) the flame of a candle of specified dimensions burning at a given rate (the standard candle, from which the unit derives its name); (*b*) the flame of a lamp of specified construction burning a specified fuel at a given rate; (*c*) a certain area of a specified radiating surface held at a specified temperature; and (*d*) electric lamps mutually compared so as to preserve an arbitrarily agreed value of candle-power as the unit

Flame Standards.—Of the standard candles adopted at different times nothing need now be said, they have been mentioned in Chapter I and are described in most books on photometry (2) Two of the flame standards, the pentane and the Hefner, possess considerable interest, particularly as regards the definition of the present unit of luminous intensity They are, moreover, still in use for certain photometric purposes, and a brief description must, therefore, be given of their construction and method of use (3)

The pentane lamp, first devised by Vernon-Harcourt in 1877,

and subsequently modified in many particulars (see p. 5), is shown diagrammatically in Fig. 65 The saturator *A* holds the fuel, liquid pentane (C_5H_{12}), a highly inflammable and very volatile hydrocarbon distilled from petroleum. This saturator is filled to about two-thirds of its capacity before the lamp is lighted. The level of the liquid (observed through the window in the side of the saturator) is never allowed to fall below one-eighth of an inch when the lamp is in use. The saturator is connected, by means of a wide india-

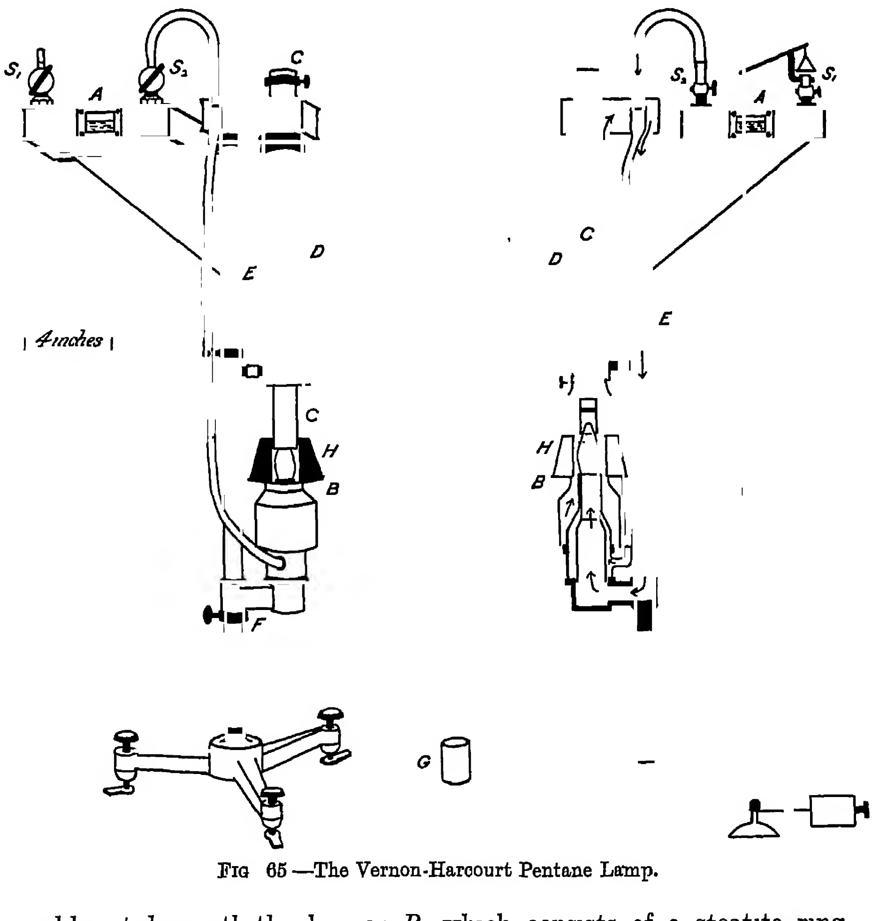

FIG 65 —The Vernon-Harcourt Pentane Lamp.

rubber tube, with the burner *B*, which consists of a steatite ring pierced with thirty holes, and at which the issuing mixture of air and pentane vapour is ignited. The rate of flow of the mixed vapour, and therefore the height of the flame, can be adjusted by means of the stop-cocks S_1 and S_2 on the saturator The chimney tube *CC* is furnished near its base with a mica window, across which is placed a horizontal bar 38 mm. above the bottom of the chimney The chimney is set, by means of a cylindrical wooden gauge *G*, so that its lower end is exactly 47 mm. above the steatite ring burner. Surrounding the chimney *CC* is a concentric tube *D*, up which a current of air is drawn by the heating of the chimney, and this heated

air passes into the hollow supporting pillar E, and so down through F to the centre of the steatite ring, where it is used in the combustion of the pentane. It is important that the chimney CC be brought centrally over the burner B, and three screws are provided at the base for the purpose of making this adjustment. H is a conical shade for protecting the flame from draught When in use the lamp is set up with the pillar E vertical, and the stop-cocks S_1 and S_2 are so adjusted that the tip of the flame just rises to a level halfway between the bottom of the mica window in CC and the crossbar (A slight variation in the height of the flame, however, does not affect its candle-power appreciably (4)) The mica window must be turned away from the photometer head, while H is turned so that the whole of the flame is visible from the photometer, except the portion at the top which is cut off by the lower part of the chimney The saturator A is at first placed upon its bracket as far from the central column as possible, and the lamp is left alight for at least a quarter of an hour before any photometric measurements are made (5) If it is found, at the end of this period, that the flame has a tendency to fall in height, the saturator is moved slightly towards the central column In making photometric measurements all distances are reckoned from the centre of the flame, *i.e* , the geometric centre of the steatite ring, but since this is not the position of the "equivalent light centre" of the flame (see p. 107), it is specified that the lamp shall be used at a fixed distance (1 metre) from the photometer surface If used at any other distance a correction should, theoretically, be applied to the value of illumination calculated on the inverse square law The effect, however, is negligible in practice, except at very short distances The same consideration applies also to the Hefner lamp (*infra*).

When burnt under standard conditions of temperature, pressure and humidity, the pentane lamp is recognised in Great Britain as having a luminous intensity of 10 international candles (6) For a more accurate and detailed specification of the lamp and of the preparation of the pentane (7), the Notification of the Metropolitan Gas Referees for the year 1916 (published by H M Stationery Office) should be consulted

The candle-power of the pentane lamp depends on the humidity and barometric pressure of the atmosphere in which it is burning. Several determinations have been made of the effect of these variables on the candle-power of the lamp (8)

The British determinations, by Butterfield, Haldane and Trotter, and by Paterson and Dudding, give the following formula for the candle-power ·

$$I = 10\{1 + 0\ 0063(8 - e) - 0{\cdot}00085(760 - b)\},$$

where I represents the candle-power of the lamp when burning in an atmosphere at a pressure of b mm of mercury with a humidity of e litres of water vapour per cubic metre of the moist air The American determination by Rosa and Crittenden, however, gives values for the constants which are notably lower than those given above, *viz.*, $0\ 0056_7$ and 0 0006 The value for the humidity correction found in 1917 by K Takatsu and M Tanaka (9) was $0{\cdot}0063_8$, and they suggest that the different value found in America may be

due to the use of a hood and ventilating duct, if the effect of this is not the same at all humidities (10) On the other hand, it has been suggested that this difference in the constants may be accounted for on the assumption that the lamp really possesses a temperature coefficient, but that, as humidity and temperature are so closely related in any one locality, their separate effects cannot be determined by the usual method of observation This has been confirmed by direct experiment on the effect of varying humidity and temperature separately by artificial means (11), and as a result the new formula for the candle-power is found to be

$$I = 10\{1 + 0\,0052(8 - e) + 0\,001(15 - t) - 0{\cdot}00085(760 - b)\}$$

where t is the temperature in degrees Centigrade. The values of the constants are obtained from a very large number of comparisons with an electric glow lamp sub-standard of the same colour, observations being made under all available conditions of pressure and humidity The most probable values of the constants are then found by the method of least squares.

The standard of candle-power adopted as legal in Germany and some other European countries is the lamp devised in 1884 by F von Hefner-Alteneck (12), and shown in Fig. 66. It consists of a container C made of brass, 70 mm. in diameter and 38 mm high It holds about 115 c c of amyl acetate ($C_5H_{11}C_2H_3O_2$), a specially pure grade of this compound being required for photometric purposes (13) The liquid should always be emptied out of the container when the lamp is not in use, as otherwise corrosion is liable to take place, even though the inside of the container be tinned or nickel plated A thin German silver tube T, constructed very accurately to the dimensions of 25 mm in height, 8 mm internal diameter, and 0·15 mm in thickness of metal, holds a wick of fifteen to twenty strands of untwisted cotton, which can be adjusted in height by means of the screw S G is a gauge consisting of a lens and ground glass screen with a horizontal cross-line (14) The lens forms an inverted image of the flame on the screen, and by this means the tip of the flame can be adjusted very accurately to the correct height of 40 mm above the level of the tube The candle-power of the flame depends appreciably on its height, and one of the chief disadvantages of this standard is the fact that the flame is very lambent and sensitive to draught, so that in the absence of any chimney its use is attended with great difficulty for practical

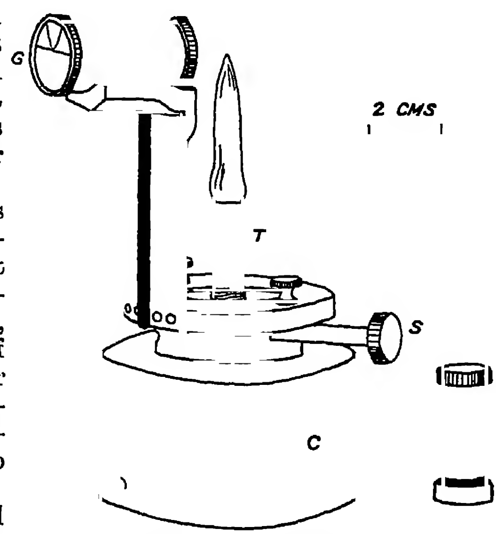

FIG 66 —The Hefner Lamp

measurement. Liebenthal finds ([15]) that a variation of 1 mm in the height of the flame causes 2·7 per cent change in candle-power The gauge on the lamp may be raised 4·2 mm to enable adjustment of the flame to a height of 44 2 mm , at which height the candle-power has been found to approximate very closely to the international candle, although the flame becomes even more objectionable in use at this height ([16])

With the Hefner, as with the pentane lamp allowance has to be made for atmospheric pressure and humidity ([17]) The formula in the case of the Hefner lamp is

$$I = 1 + 0\ 0055(8\ 8 - e) - 0\ 00011(760 - b),$$

where e and b have the same meaning as before (see p 128) ([18])

An allowance should also be made for the amount of carbon dioxide in the air if this substance be present in appreciable quantity, as it has a considerable effect on all flame standards ([19]) (including the pentane lamp). This correction may be made by adding a term $+ 0\ 0072(0\ 75 - k)$ to the above expression for I, k being the number of litres of CO_2 present per cubic metre of air The amount of the correction is so uncertain, however, that it is best to avoid it altogether by arranging efficient ventilation of the room in which the lamp is burning ([20]) (see p 446)

The light given by the Hefner lamp is much redder than that of the pentane, and, being without a chimney, the flame is more troublesome to maintain in a steady condition On the other hand, the lamp has the great advantages of simplicity and portability, which, however, are principally of value in a working standard, and do not enter into consideration as far as a primary standard is concerned ([21])

The flame standard adopted in France is the Carcel lamp, which has been briefly described on p. 4 It is, however, quite unsuitable as a standard lamp for use in modern photometry.

A large number of inter-comparisons between the various flame standards have been carried out at different times ([22]) with varying results. As a result of the intercomparison in 1907, it was found that the British standard, based on the 10-candle pentane lamp, was 1 6 per cent lower than the American candle, while the French *Bougie décimale*, derived from the Violle unit (see below), was greater than the British unit by about 1 per cent In 1909, by agreement between the standardising laboratories of Great Britain, France, and the U.S.A , the American candle was lowered 1·6 per cent to bring it into agreement with the British unit, while the French also agreed to adopt the resulting unit, which thenceforward became known as the international candle Germany and the other countries which had adopted the Hefner standard continued to use that unit, but its value was agreed to be exactly nine-tenths of the international candle ([23]). This unit has been still further stabilised by the action of the International Commission on Illumination in 1921 (see p 86).

Incandescence Standards.—Of the proposals in this class the best known is the melting platinum standard ([24]), first given a practical form by J Violle, and therefore generally known as the "Violle standard" ([25]). It is the light given by 1 sq cm. of a

surface of molten platinum at the temperature of solidification. The Violle standard, on account of its obvious theoretical advantages, has been regarded as a promising advance on the existing flame standards, and unsuccessful attempts have been made at various times to place it on a satisfactory basis ([26]) The most recent careful work on this standard is that of Petavel ([27]), who used a semicircular bar of platinum heated by an electric current to such a temperature that the inner core of the bar melted, while the outer shell remained solid. The second form of the standard on which he made measurements was an ingot of platinum fused in a crucible of pure lime by means of an oxy-hydrogen blow pipe. The metal was first completely melted ; heating was then stopped and photometric measurements of the brightness were made at intervals of ten seconds during cooling. The readings, when plotted, showed a constant value over the region corresponding to the time of solidification, and the mean of the observations at this period was taken as the value required It was found that the values obtained by this method did not depend on the shape or mass of the ingot, but that the effect of contaminating the platinum with either silica or carbon was very marked Petavel's final conclusion was that the probable variation in the light emitted by molten platinum under standard conditions was not greater than 1 per cent , and that with more experimental refinements an even greater accuracy than this might be attainable. It cannot be said, however, that 1 per cent. is sufficient to bring the Violle standard, with its additional disadvantage of redness of light, into serious competition with the existing standards It was, however, adopted by the International Electrical Congress in 1889, and its one-twentieth part was given the name "*bougie décimale*" ([28])

In order to avoid the possible variations in the temperature of fusion or of solidification of the platinum, it has been proposed to use the metal at a temperature below the melting point, and to define the temperature as that at which a layer of water 2 cm in thickness transmits 10 per cent of the total resultant radiation ([29]). This ratio is determined by means of a bolometer. Although this standard is used at the Physikalisch-Technische Reichsanstalt for the checking of Hefner lamps, Petavel has found that the bolometer method of temperature adjustment is not sufficiently fundamental to enable this apparatus to fulfil the conditions of a primary standard. He proposed a modification of the Lummer-Kurlbaum standard ([30]).

Another incandescence standard, on which a considerable amount of work has been done by many investigators ([31]), is that provided by a square millimetre of the positive crater of a carbon arc operating under conditions designed to ensure steadiness. In the Forrest arc (Fig. 67) two negatives are employed, each at an angle of about 100° with the positive Under these conditions, and using carbons of 8 mm diameter with a total current of 7 to 10 amperes, it has been found that the brightness of the crater is uniform over the whole of its surface, and measurements of the candle-power per sq. mm. can be made by inserting in front of the crater at *D*, Fig. 67, a small diaphragm of accurately known dimensions. Forrest found that the arc would work quite silently over a considerable range of currents, and that the crater brightness was independent of the

current under these conditions The value he obtained was 172 candles per sq. mm., using carbons of 6 to 8 mm. diameter and currents up to 5 ampères (32)

An advantage which this standard possesses over any other is that the colour of the light is bluer than that of most present-day

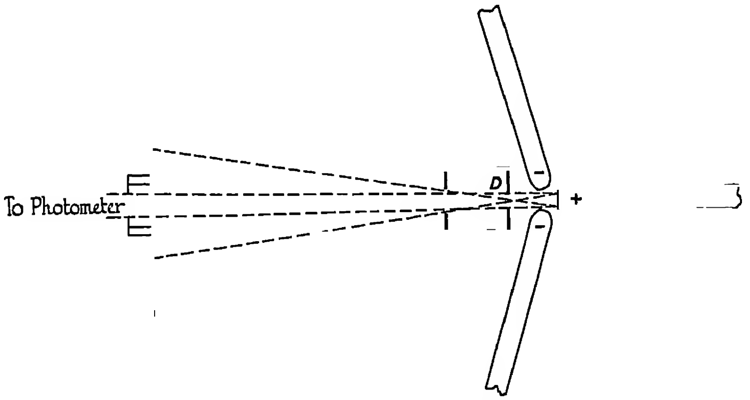

FIG. 67 —The Forrest Arc Standard

sources, a difference which will diminish as the efficiency of practical illuminants increases.

The "Black Body" Standard.—The proposal to use a definite area of a total radiator (black body) at some definite temperature has been made at various times by several workers (33) The great advantage of a total radiator as a primary standard is its reproducibility and its independence of small variations in construction. Moreover, its radiation follows definite and well-established laws (see p 37), not only as regards the total energy emitted, but also as regards the distribution of that energy throughout the spectrum Therefore, if measurements of the total energy radiated by a given area of a black body could be made to the necessary accuracy, the temperature, and thence the candle-power, could be at once deduced At temperatures in the region of 2,000° K. (34), however, a change of 1 per cent. in temperature produces a change of about 10 to 12 per cent in candle-power, so that for accuracy in the standard of candle-power the temperature measurements need to be exceedingly good.

Practical Forms of "Black Body."—The most commonly employed form of black body is cylindrical in shape, and for work up to a temperature of about 1,700° K it consists of a tube of porcelain electrically heated by means of platinum strip (35).

The construction is shown in Fig. 68 *A*, *B* and *C* are three concentric porcelain tubes, *B* being wound with platinum strip 0 02 mm thick and 1 cm wide The winding is closer at the ends than at the middle, in order to obtain a more uniform temperature *A* is uniformly and closely wound with platinum strip 0 1 mm thick. The shields *a*, *b*, *c* . . ensure that no radiation can be emitted from the opening at *P*, except that proceeding directly from the diaphragm *d*. Thermocouples attached to the back and front of this

diaphragm give the temperature of operation Water-cooled shutters are used in front of the opening P in order to cut off radiation from the surrounding porcelain walls. The heating of the two platinum coils is generally separately controlled by regulating the current through them The energy distribution curve obtained with

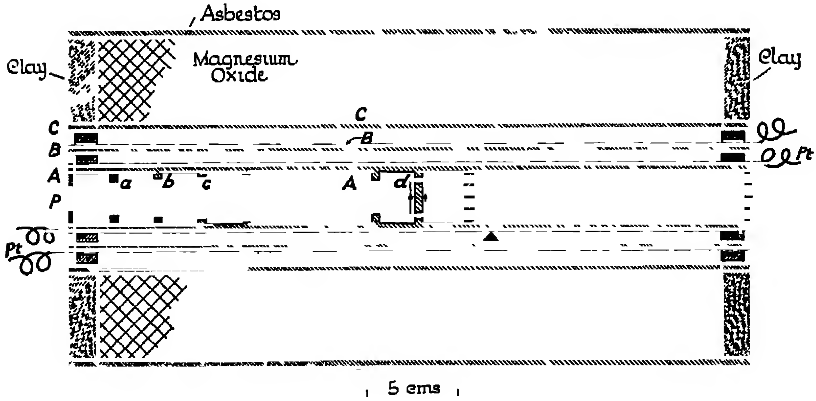

FIG. 68 —A Porcelain Tube Black Body Radiator

a black body of this kind at a temperature of 1,596° K is shown in Fig. 21 (36)

For higher temperature work a carbon tube is used (Fig 69) (37) This is heated by passing an electric current through the tube itself, which is 1 2 mm thick. The ends of this tube are copper plated and fitted into thick carbon cylinders These cylinders are securely clamped into the metal blocks B, B which convey the current The back wall of the radiating cavity is formed by the plug P_1, so shaped that its area of contact with the tube is as small as possible in order to prevent local inequalities of heating Access of oxygen is prevented at the back of the tube by the plugs P_2 and P_3, and in front by passing a stream of nitrogen into the cap at the mouth of the cavity A similar form of total radiator has been used up to about 2,800° K (38)

The Measurement of Temperature.—There are several methods of thermometry which may be used for measuring the temperature of a black body The gas thermometer, which may be used up to about 1,870° K (39), gives the basis for the standard temperatures now generally adopted, *viz* , the melting points of gold and palladium taken as 1,336° K and 1,829° K respectively (40) A wire of gold or palladium may then be made part of an electrical circuit and heated up within the black body When the wire melts, the circuit is broken and the temperature of the black body at that instant is assumed to be the same as the melting point of the metal. Some form of pyrometer may then be used to measure the radiation at the same instant, and by its means the temperature of the black body at higher temperatures may be deduced from the laws governing pure temperature radiation (41) These pyrometers may be of either the total radiation or the optical type. In the former, of which the Féry and the Foster are the best known forms (42), an image of the black-

body aperture is formed by means of a mirror on a small thermo-couple (see p 319) The deflection of a galvanometer in circuit with the thermocouple is then proportional to the rise in temperature of the latter, *i e.*, to the total radiation it receives, less that it emits. This is proportional to the normal radiation per unit area from the

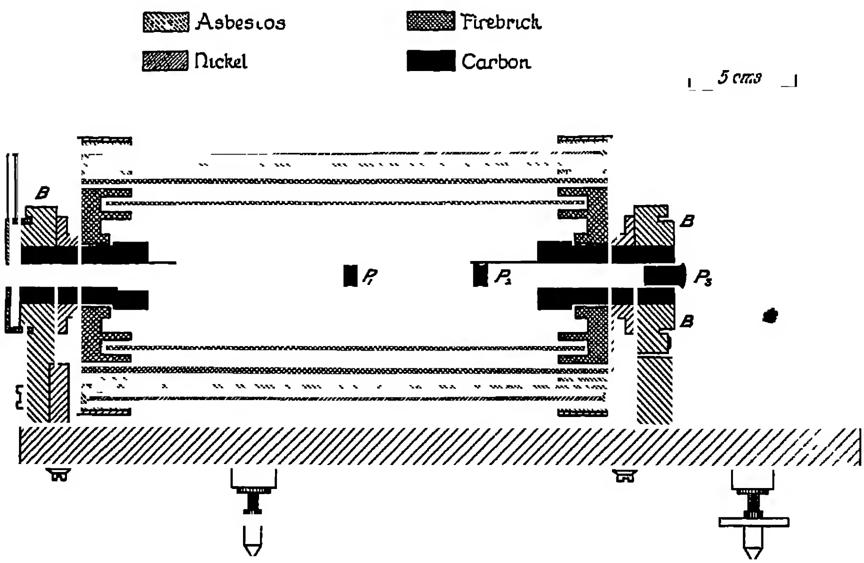

FIG. 69 —A Carbon Tube Black Body Radiator

black-body aperture [43], so that the galvanometer deflection varies as $(T^4 - T_0^4)$, where T is the absolute temperature of the black body, and T_0 the temperature of the surroundings. If, then, the deflection at the melting point of palladium be D_{Pd}, while that at temperature T is D_T, it follows that

$$(T^4 - T_0^4)/(1{,}829^4 - T_0^4) = D_T/D_{Pd}.$$

The optical pyrometer depends on the law connecting the radiation at any given frequency with the absolute temperature of the black body (see p. 37) The Wanner pyrometer, which is of this type, is a modified form of the Konig-Martens spectrophotometer (see p 281), and depends on a polarisation method for measuring the ratio of the brightness of the black body to that of a standard lamp maintained at a constant value [44]. With this instrument measurements are made with a red glass, which only transmits a limited portion of the spectrum, and may therefore be regarded as giving the values of brightness at a certain "effective" wave-number ν Alternatively, the light from both sources may be analysed by means of a prism and the ratio obtained at any desired wave-number.

If B and B_{Pd} be the relative brightnesses of the black body at the unknown temperature T and at the palladium point respectively,

then $B/B_{Pd} = (e^{c_2\nu/1829} - 1)/(e^{c_2\nu/T} - 1)$, and from this, since C_2 and ν are known, T may be found.

In practice it is simpler to use, instead of the full Planck expression employed above, a modified form (actually proposed by Wien before Planck's theory had been brought forward [45]), *viz.*, $E = C_1\nu^3 e^{-c_2\nu/T}$ or $E_\lambda = C_1\lambda^{-5}e^{-c_2/\lambda T}$ [46] The above equations therefore become, on taking logarithms to base e,

$$\log_e B - \log_e B_{Pd} = - C_2\nu(T^{-1} - 0\ 0005467)$$

Instead of assuming the value of C_2, it is clearly possible to measure the value of B at the gold point (B_{Au}) and so to obtain the following equation independent of both C_2 and ν —

$$(\log_e B - \log_e B_{Pd})/(\log_e B_{Pd} - \log_e B_{Au}) = \frac{0\ 0005467 - T^{-1}}{0\ 0002015}.$$

In the Holborn-Kurlbaum type of optical pyrometer [47] an image of the black-body aperture is formed on the filament of a small carbon lamp, and the brightness of this filament is varied by alteration of the current passing through it until the filament disappears owing to the identity of its brightness with that of the surrounding image In this pyrometer measurements are always made with a red glass. The objection to the use of a red glass for a standard instrument is that the "effective" wave-number necessarily varies slightly with the spectral distribution of the sources compared through it; for, since its transmission is considerable over an appreciable range of frequencies, the higher frequencies within this range receive more weight as the temperature of the source rises, *i e*, the "effective wave-number" is gradually increased. For a glass of usual type this change may amount to $\delta\nu = 80$ cm^{-1} ($\delta\lambda = 3$ mμ) in the range from 1,600° to 2,700° K Methods of determining the effective wave-number cannot be dealt with here, but for these the original papers or a book dealing with pyrometry should be consulted [48]

The difficulty involved in measuring the temperature of a black body to the accuracy necessary for its establishment as a satisfactory standard of light has led to the suggestion to adopt the melting point of platinum as the fixed temperature of the black body [49], thus avoiding the necessity for an absolute measurement of temperature. H E Ives has realised this form of standard [50] by using as a black body a platinum tube heated by means of an electric current passing through it The brightness of a small aperture in the side of the tube is measured continuously until the tube melts, and the final value thus found is taken as the brightness of a total radiator at the melting point of platinum The value found for this brightness is 55 40 candles per sq cm.

Electric Glow Lamps as Standards.—The above description of the more successful of the various standards that have been proposed from time to time will be sufficient to show that an accuracy of 1 per cent is the most that can be obtained as regards either reproducibility or constancy. Now the precision of photometric measurement, without colour difference, is at least 0 2 per cent. It follows that lamps can be compared with one another to an

accuracy which is at least five times as great as the reproducibility of the standards of candle-power.

This position is so anomalous that in 1921 the International Commission on Illumination decided to base the unit of luminous intensity, not on any of the so-called primary standards, but on the agreement of a number of electric incandescent filament lamps, which have been proved to be of very great constancy ([51]), and which have been compared with one another to the highest accuracy possible in photometry at the present time There is thus no existing lamp which has a luminous intensity of exactly 1 international candle, but the electric lamps at the chief national standardising laboratories preserve, among them, the value of this unit to the same accuracy as that obtainable when they are used for the preparation of sub-standards The definition of the international candle given on p 86 is the result of this international agreement ([52]), but it is recognised that the position, however convenient in practice, is objectionable in theory, and there is no doubt that the search for a real primary standard will go on, and when it is found the value assigned to the unit will be the same as the mean value given by the present incandescent lamp standards Probably the most promising line of attack is that indicated in the preceding section Improved accuracy in the measurement of temperature will automatically bring with it a black-body standard of light.

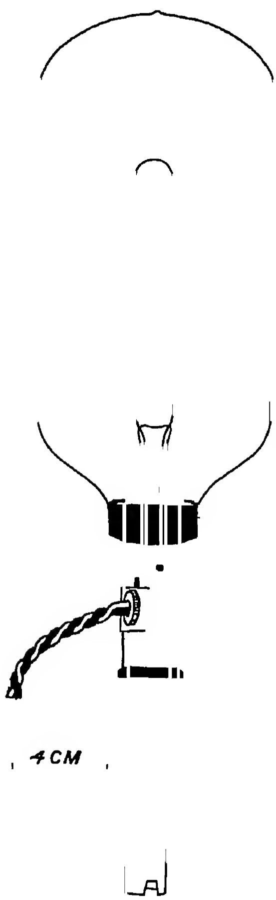

Fig 70—A Fleming-Ediswan Standard Lamp

Meanwhile the electric incandescent standard lamps remain as the custodians of the unit of light One of them, kept at the National Physical Laboratory, is shown in Fig 70; it consists of a single "hairpin" loop of carbon enclosed in a cylindrical bulb of 100 mm. diameter When in use the lamp is set up so that the leading-in wires are at the bottom, and the plane of the filament is at right-angles to the direction in which the candle-power measurements are made Distances are measured from this plane, but the lamps are always used at a certain fixed distance from the photometer, so that measurements are always made at an illumination of 10 metre-candles This use of a fixed distance has the effect of eliminating the departure from the exact inverse square law owing either to any lack of centering of the image of the filament formed by the walls of the bulb or to lens action by the curved glass walls. The filament hairpin is about 100 mm long, and as its distance from the photometer is approximately 1,250 mm (the candle-power of these lamps being about 15), the departure from the inverse square law

due to size of source is less than 0·2 per cent. (see Fig 52, p 103). Other standard lamps of a different construction are preserved at the Bureau of Standards, Washington, the Laboratoire Central d'Electricité, Paris, and the Physikalisch-Technische Reichsanstalt, Charlottenburg, Berlin Although the 1909 agreement referred to in the definition of the international candle was based on comparisons of carbon filament lamps operating at about 3 lumens per watt (tungsten basis), subsequent comparisons have also been made by means of tungsten filament lamps (see Fig 71), operating at an efficiency of about 6 7 lumens per watt Thus practical international agreement has been arrived at as regards the unit of candle-power at both efficiencies ([53]).

The rate of decrease of candle-power in carbon lamps of the type described above, and in tungsten filament lamps such as those described in the next section, is probably less than 3 per cent per 100 hours of actual burning if the lamps are carefully aged and used with every precaution ([54]) The decrease is therefore quite inappreciable in ten years if each lamp be run, on the average, for not longer than fifteen minutes per annum

Sub-Standards of Candle-Power.—The above sections have dealt with the primary standard of light, but, as in the case of standards of all kinds, it is necessary for the purposes of everyday measurement to use other lamps, known as sub-standards, which have been carefully measured by the primary standards and are compared with them at intervals as may be found necessary

Until the constancy of the electric lamp as a source of light had been fully proved, one of the flame standards described in this chapter was generally employed as a sub-standard, its candle-power having been certified by a standardising laboratory to be in accordance with that of the similar primary standard The use of flame lamps as sub-standards, however, has now been abandoned almost entirely in favour of the more convenient electric glow lamp

For work of the highest degree of accuracy it is desirable to use a specially constructed lamp, such as that shown in Fig 71, which represents the type employed at the National Physical Laboratory ([55]) The tungsten filament is mounted in a single plane so that this may be used for defining the zero of the distance measurements When the lamp is in use this plane is arranged to be perpendicular to the axis of the photometer bench (see next chapter). The lamps are carefully "aged" before being standardised, *i e*, they are run for at least 100 hours at an efficiency equal to or greater than that at which they are to be used in practice Their rate of candle-power fall over the latter half of this period should not exceed about 3 to 4 per cent per 100 hours They may be operated either at a specified current or at a specified potential In the latter case the potential is measured at the ends of leads which are soldered to the contact plates, and so form a permanent part of the lamp ([56]). The process of candle-power measurement, and of electrical adjustment, will be described in Chapter VI One of the most important precautions in using a sub-standard lamp is to ensure that no potential which is even slightly in excess of that at which the lamp is standardised can ever be applied to it, even for a few seconds If this should happen inadvertently, the lamp should be re-standardised.

There are many precautions which have to be observed in the manufacture of standard lamps. The joint by which the filament is connected to the leading-in wires should be welded, and not simply "pinched," so that any possibility of uncertain contact may be avoided Further, hooks and supports which are loose, so that they sometimes touch the filament and sometimes do not, cause a variation in candle-power due to local cooling of the filament by conduction The glass bulbs are considerably larger than those that would be used for lamps of the same candle-power designed for ordinary work This is in order to reduce the fall of candle-power due to "blackening," *i e.*, the fine deposit on the glass which takes place gradually in all electric glow lamps, and which produces a marked increase in the absorption factor of the bulb (57)

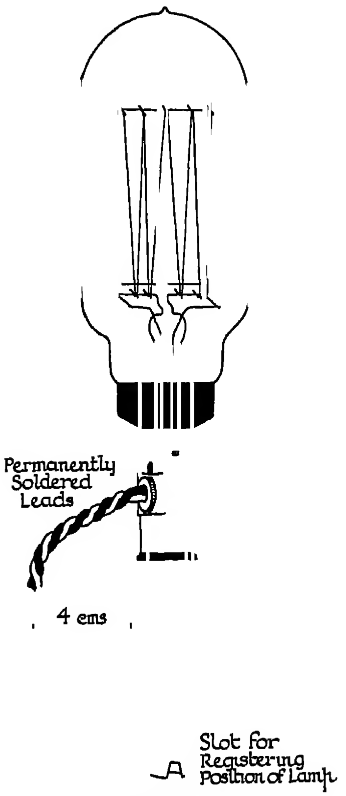

FIG 71 —Tungsten Filament Standard or Sub-standard Lamp

It is, naturally, of the first importance that sub-standard lamps should be used so that the light which reaches the photometer from them is that emitted in a certain direction, for not only does the candle-power of such lamps vary in the manner described in the last chapter, where the emission of a radiating cylinder was being considered (p 108), but the inevitable slight variations of thickness in the glass bulb give rise to lens effects which produce small local variations of candle-power If these variations be present in a marked degree, so as to produce noticeable "streaks" or "blotches" on a featureless white surface held in the path of the light, the lamp, though perhaps perfect in other respects, is valueless as a sub-standard

The "registering" of a lamp, so as to ensure that its position with respect to the photometer is always the same, is most conveniently achieved by mounting the lamp permanently in a holder provided with a stem which fits into the carriage of the photometer bench, and registers in its correct position in that holder by means of a small slot, such as that shown at the bottom of the stem of the lamp illustrated in Fig. 71 A further advantage of a permanent mounting of this kind for sub-standard lamps is that the potential can be measured at the ends of terminals which are permanent and fixed points in the lamp circuit. All uncertainty of contact at the lamp terminals, which is so frequently a source of unsteadiness when lamp sockets are used, is thus completely avoided, and, further, if the lamp be carefully "lined up" when it is mounted, so that the plane of the filament passes through the axis of the stem, no further

Fig 72 —Standard or Sub-standard Lamps

[*To face p* 138

lining up is necessary when the lamp is used on the bench. When it is impossible to use a self-registering mounting for a sub-standard, the best method of "lining up" is to etch on opposite sides of the bulb two small circles, so placed that when the lamp is in the correct position the line passing through the centres of these circles also passes through the photometer head. In other words, this line defines the direction in which the sub-standard has the measured candle-power When the lamp is set up for use on the bench it is first placed with its axis vertical (either upright or pendent), and it is then turned about this axis until the two circles are in line with the photometer head. It is very desirable that a sub-standard should be used in the position in which it has been standardised, *viz.*, either always upright or always pendent, as the case may be

Another possible constructional defect in sub-standard lamps arises from the fact that a real image of the filament is formed by reflection in the curved surface of the bulb. If the filament be centrally placed, the image, the brightness of which is approximately 8 per cent of that of the real filament, will lie very nearly in the same plane as the filament, and so will not produce any appreciable shift in the position of the effective light centre A badly placed filament may, however, give an image which is considerably displaced, and which therefore gives rise to unsuspected errors, particularly if the lamp be used at short distances from the photometer (see p 420) ([58])

For work where the highest degree of precision is either unnecessary or impossible, the ordinary type of electric lamp in which the filaments are disposed cylindrically (squirrel-cage type) may be used as a sub-standard In this case the necessity for accurate "lining-up" is still greater than with the planar type of filament. It is often desirable for low intensity measurements to use a sub-standard of low candle-power (1 to 2 candles), in which the filament consists of a single "hairpin" of tungsten or carbon

The direct measurement of the mean spherical candle-power of light sources by means of some form of integrating photometer (see Chapter VII) is now becoming more and more general For this purpose some form of sub-standard of known mean spherical candle-power must be employed, and it is frequently desirable to use a gas-filled lamp for the purpose in order to avoid colour difference. The mean spherical candle-power of a vacuum lamp of suitable type is first measured by means of the apparatus described on p 204, using the ordinary standards of candle-power This lamp is then compared in the integrating photometer with other (vacuum or gas-filled) lamps which are to serve as sub-standards The precautions chiefly necessary in preparing sub-standards of this kind are those which ensure constancy of light flux, *viz*, (*a*) ageing, (*b*) certainty of contact and accurate potential (or current) measurement, (*c*) constancy of position (upright or pendent), particularly in the case of gas-filled lamps Filament form, lens effects in the glass, and the position of the filament image are unimportant

The Acetylene Sub-Standard.—For approximate work in laboratories where a steady source of electric supply is not available, a screened acetylene flame of special form has been found useful as a sub-standard ([59]). This form of standard, known as the Eastman-

Kodak standard, is shown in section in Fig 73 The burner is a Bray air-mixing burner, consuming about ¼ cub ft per hour and giving a cylindrical flame about 3 mm. in diameter and 50 mm high. The flame is surrounded with a black cylindrical hood of metal having at one side a small horizontal opening C provided with a wedge-shaped diaphragm. The centre of the opening is adjusted in height, according to the burner used, until the part of the flame visible through C is that which is least affected in brightness by small changes in gas pressure. The height of C above the burner tip is then usually between 18 and 20 mm. The pressure of the gas worked with is 9 cm of water. Each burner requires separate standardisation by comparison with a tungsten lamp, which it matches in colour when the efficiency of the lamp is about 9 l p.w The effective position of the source being that of the inner opening in C, this opening may conveniently be arranged to be over the centre of the bench carriage.

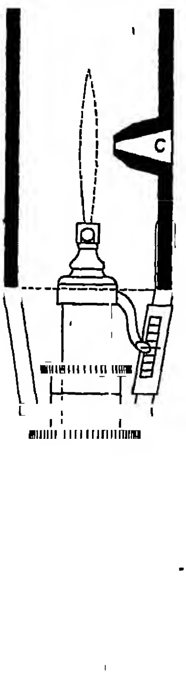

FIG 73 —The Eastman-Kodak Acetylene Sub-standard.

Comparison Lamps.—Even sub-standards should not be used more than is absolutely necessary, and where much photometric work has to be done it is usual to employ a lamp, known as a comparison lamp, for the actual measurements, and to determine the value of this lamp at the beginning of each day's work by means of the sub-standards. This comparison lamp may be of any type so long as its candle-power can be relied upon to remain constant for the period of use In the substitution method of photometry, described in the next chapter, the use of a comparison lamp is always necessary (see p 161) Many different types of lamps have been used as comparison lamps at different times [60], but an electric incandescent lamp, either with or without a colour filter, is now employed almost universally on account of its steadiness and general convenience

It is sometimes convenient to be able to use the same comparison lamp for measuring lamps of widely different candle-powers This can be arranged by placing a lamp of high candle-power in a whitened box having, on the side facing the photometer, a ground glass window fitted with a variable diaphragm The candle-power of the window can then be adjusted to a suitable value by altering the size of the diaphragm opening [61]

For continuous work with gas mantles a comparison source of similar type is sometimes used [62]. A large upright mantle is surrounded with a cylindrical screen containing a small aperture of convenient (often adjustable) size, so arranged that only the central part of the mantle is visible from the photometer If the gas pressure be well regulated, such a comparison source will maintain its candle-power reasonably constant for a period of several hours.

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(46) This simplification is justifiable since Wien's formula gives the same results as Planck's to within 0 1 per cent so long as (ν/T) is not less than 4 8 cm $^{-1}$ deg $^{-1}$, *i e*, for $\nu = 15{,}000$ so long as $T < 3{,}000$

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See also A G Worthing and W. E Forsythe, Phys Rev, 4, *1914*, p 163, Nela Bull, 1, *1917*, p 140, W E Forsythe Opt Soc Am, J, 5, *1921*, p 494
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(48) M. Pirani Deut Phys Gesell, Verh, 15, *1913*, p 826
M Pirani and W W Loebe Deut Phys Gesell, Verh, 17, *1915*, p 47
E P Hyde, F E Cady and W E Forsythe Astrophys J, 42, *1915*, p 294, Phys. Rev, 6, *1915*, p 70, Nela Bull, 1, *1917*, p 133
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F Henning Z f Phys, 30, *1924*, p 285, P T R, Wiss Abh, 8, *1924*, p 109
E Griffiths "Methods of Measuring Temperature," p 140, art in "Dict Appl Phys," vol 1, p 651
F Henning and W Heuse Z f Phys, 32, *1925*, p 799
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Alternatively, measurements may be made with a spectrophotometer See F Hoffmann, Z f Phys, 27, *1924*, p 285, Z f I, 43, *1923*, p 96

(49) C W. Waidner and G K. Burgess. El. World, 52, *1908*, p 625, J G W, 52, *1909*, p 287

H E Ives Phys Rev, 8, *1916*, p 250, El World, 68, *1916*, p 679, Rev Gén de l'El, 1, *1917*, p 52, Lighting J, 4, *1916*, p 150, Frank Inst, J., 186, *1918*, p 122

See also J Violle, C R, 141, *1905*, p 1188

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(50) Frank Inst, J, 197, *1924*, pp 147 and 359, C I E, Proc, 6, *1924*, p 77, Gas J, 167, *1924*, p 642 and 171, *1925*, p 110, Rev d'Optique, 3, *1924*, p 505, E u M (Lichttechnik), 42, *1924*, p 249

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P Fleury. Soc Franç des Elect, Bull, 5, *1925*, p 700

(52) Germany has not yet accepted the international candle as defined on p 86, but still retains the Hefner unit (HK) based on the amyl-acetate lamp It is to be hoped, however, that this anomalous position of the co-existence of two units of luminous intensity will disappear in due course, and that the Hefner unit will be replaced by the international candle which now rests on a far more accurate and scientific basis

(53) C C Paterson and B P Dudding Phys Soc, Proc, 27, *1915*, p 263, Phil Mag, 30, *1915*, p 63, Electrician, 75, *1915*, p 251, J of Gas Lighting, 130, *1915*, p 448, N P L, Coll Res, 12, *1915*, p 81

(54) This figure is based on experience at the National Physical Laboratory See also H E Ives and L R Woodhull, Bureau of Standards, Bull, 5, *1909*, p 555, Electrician, 63, *1909*, p 992

(55) C C Paterson and B P Dudding Phys Soc, Proc, 27, *1915*, p. 263, N P L, Coll Res, 12, *1915*, p 81, Phil Mag, 30, *1915*, p 63, Electrician, 75, *1915*, p 251

(56) P Good Electrician, 55, *1905*, p 430

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(58) B F Thomas Int El Congress, Chicago, *1893*, Proc, p 198

(59) C E K Mees and S E Sheppard Phot J, 50, *1910*, p 287, Congrès Int. de Photog, 5, *1910*, p 287

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(60) W M. Stine "Photometrical Measurements," p 193

(61) H E Ives and L R Woodhull Bureau of Standards, Bull, 5, *1909*, p 555, Electrician, 63, *1909*, p 992

(62) C Carpenter and J W Helps C I E, Proc, 1, *1903*, p 143, J of Gas Lighting, 82, *1903*, p 960, Inst Gas Eng, Trans, *1906*, p 300, Am Gas Inst, Proc 6 (2), *1911*, p 223, J G W, 46, *1903*, p 1031, Gas World, 39, *1903*, p. 14

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CHAPTER VI

MEASUREMENT OF CANDLE-POWER IN A SINGLE DIRECTION

PHOTOMETRY may conveniently be divided into two branches, according to the nature of the information which it is desired to obtain "Candle-power" photometry has for its object the measurement of the output of light sources as expressed either in candle-power or in luminous flux, while "illumination photometry" is only incidentally concerned with the sources, and has for its principal object the measurement of the illumination which they produce at a given point. Illumination photometry will be dealt with in a later chapter of this book (Chapter XII), this and the following chapters being devoted to a description of the apparatus and methods of candle-power photometry The measurement of candle-power in a single direction will be dealt with in the pages immediately following, while the determination of the candle-power distribution of a source and its total flux output, and the apparatus special to that particular branch of the subject, will be described in Chapter VII

General Considerations.—It has been said already that the eye is the ultimate judge in all photometry, since the sensation of light is essentially a psycho-physiological phenomenon inseparable from that organ of special sense Nevertheless, methods of photometry have been devised in which purely physical apparatus is used to measure radiant energy in such a manner that the energy at any given frequency is weighted according to the luminosity of light of that frequency (see p 64), so that the physical apparatus becomes, in reality, a representative of the *average* human eye These methods are classed together under the heading of "Physical Photometry," and are described in Chapter XI , in them it is the energy reaching a sensitive surface, *i e* , illumination, that is measured While the same is true of visual photometry in that the ultimate measurement depends on the illumination of a sensitive surface, the retina, in this case the illumination is due to the brightness of the surface looked at, so that in effect visual photometry may be said to depend on the measurement of brightness

In common with all the other organs of sense, the eye cannot *measure* with any degree of accuracy (1) , in fact, its power of adaptation (p 54) is so great that it is probably the worst of all the sense organs in this respect Measurements must therefore depend on the judgment of equality (2) Under the most favourable conditions a difference of brightness of about 1 6 per cent (see pp 8, 52) can be detected, and it is found that by a practised judgment of the midway point between the first appearance of lack of equality in either direction a measurement accurate to about 0 2 per cent can be obtained by taking the mean of a large number of readings This, then, represents the limit of accuracy of visual photometry The art of photometry and the design of photometric apparatus have

for their object the attainment of this limit by enabling the eye to be used under the most favourable conditions ([3]). It may be said, in fact, that every photometer is a combination of two principal parts, *viz* : (*a*) some device for enabling the eye to compare, as accurately as possible, the brightness of two surfaces, one of which is illuminated by the source to be measured, and (*b*) means for varying the brightness of the other surface according to some known law The only exceptions to this general rule are the so-called "absolute" photometers, which depend on such physiological phenomena as (i.) the relation between the retinal illumination and the diameter of the pupil (see p 55), (ii.) the amount of reduction required to bring the measured light just to the threshold of visibility ("extinction" photometers, see p 2), (iii.) the relation between visual acuity and brightness ("acuity" photometers, see p. 236), or (iv.) critical frequency (see p 249)

Classification of Photometers.—It is convenient to divide apparatus for visual photometry into several classes according to the method used for obtaining the variation of brightness of one of the surfaces to be compared. By far the most important of these classes is that in which the inverse square law of illumination is used In the other classes various other laws are employed, such as the tangent-squared law of polarisation, Talbot's law of the transmission factor of a sector disc, the law of transmission of an absorbing screen, the cosine law of illumination, *etc* This classification is by no means a rigid one, for apparatus belonging to two or more classes may be, and frequently are, used together, but it will be useful for the purposes of this chapter, and will be adopted in what follows

Photometry by the Inverse Square Method.—The simplest, and at the same time most commonly used, form of photometer depending upon the inverse square law of illumination consists of two essential parts, *viz* .—

(i.) A specially designed piece of apparatus termed a "photometer head" (sometimes, for brevity, a "photometer" ([4])), the function of which is to enable the eye to judge when equality of brightness is attained between the two comparison surfaces within it, each of these surfaces being illuminated by one of the two sources of light to be compared

(ii.) A graduated bench upon which the photometer head and the sources may be mounted in such a manner that the distances of one or both of the sources from the head may be varied and measured readily and accurately

If, then, I_1 and I_2 be the candle-powers of the sources, while ρ_1

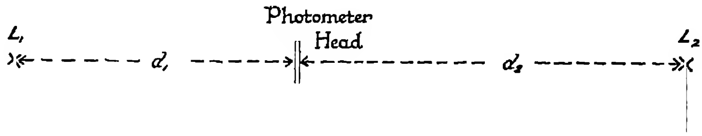

FIG 74 —Photometry by the Inverse Square Method

nd ρ_2 are the reflection factors of the surfaces, and d_1 and d_2 their istances from the sources, it follows that when the two surfaces

are equally bright $\rho_1 I_1/d_1^2 = \rho_2 I_2/d_2^2$ Hence, if ρ_1 and ρ_2 be known or equal, I_1/I_2 can be found from a measurement of d_1 and d_2 The simplest arrangement is that in which the photometer head and the sources are in one straight line, as shown in Fig 74, one or both of the distances d_1 and d_2 being variable at will by moving one of the sources (L_1, L_2) or the photometer head, or both, along the "photometer bench"

The Photometer Bench.—From the equation given above it is clear that the distances d_1 and d_2 must be measured to an accuracy superior to that of the photometric comparison. Since the distance enters as a square, an accuracy of 0 2 per cent. in the photometric measurement demands an accuracy of at least 0 1 per cent in distance, and since it is desirable that the total of the various errors involved may not be much in excess of the 0·2 per cent theoretically attainable, it is clear that the distance measurement should be accurate to at least 0 05 per cent, *i e.*, half a millimetre in 1,000 mm The photometer bench, then, must be graduated accurately in millimetres, and should allow of the use of distances in excess of 1 metre on either side of the photometer head. This is further necessary on account of the dimensions of the sources to be measured and the possible range of values of I_1/I_2

The bench may consist of a simple vertical wooden beam carrying movable saddles, on which are mounted the light sources and the photometer head It is, however, necessary to have the movements of these saddles as smooth and easy as possible, so as to enable the observer to pass through the balance point quickly and without much manual effort [5], and also to avoid any vibration of the sources Rigidity is essential in order that the true distance between each source and the comparison surface in the photometer may be accurately measured on the bench These requirements are more fully met in a bench such as that shown in Figs 75 and 130 (p 221) [6] The particular pattern there illustrated was designed by Messrs Alexander Wright & Co., of Westminster, in co-operation with the National Physical Laboratory It is a modification of the bench made by Messrs Franz Schmidt and Haensch, of Berlin [7] The bars B, B are parallel steel rods of 32 mm. diameter, placed at a distance of 178 mm. between centres. These bars are supported at four or five points, according to the length of the bench Close to one bar a broad brass strip bearing a scale of millimetres is mounted at an angle of 45° with the vertical The figuring of this scale is from a left-hand zero [8], and is marked at every 10 mm, the dimensions of the graduations being shown to half scale in Fig 76. The length of the bench may be from 3 to 5 metres. The brass strip sometimes bears a second, "squared," scale, graduated in such a way as to indicate the square of the distance from the zero point The 1,000 mm mark of the millimetre scale is marked 10 on the squared scale, so that if the standard illumination be 10 metre-candles, and a distance of 1,000 mm. therefore correspond to a candle-power of 10, the candle-power of any source may be read directly on the squared scale when this source is giving the standard illumination at the photometer head. The necessity for squaring the reading of the millimetre scale is thus avoided [9].

The Photometer Bench Carriage.—The carriages C, C which travel

on the bars of the bench and bear the standard lamps, photometer head, *etc*, are all similar in general design. One of them is shown in Fig 77 The primary essentials of these carriages are, as has been said, lightness and ease of motion, combined with rigidity and

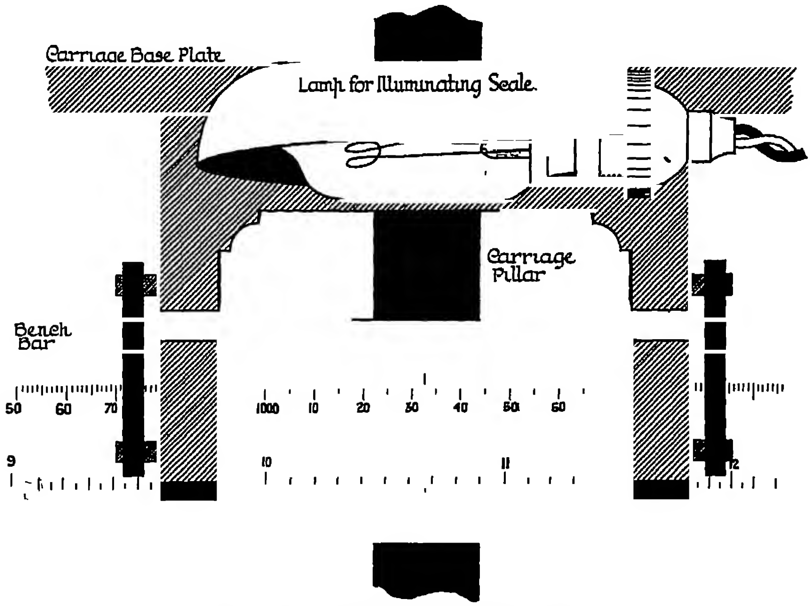

FIG 76 —The Photometer Bench Scales

steadiness Upon the ease and rapidity with which a carriage can be moved depends, to a very great extent, the accuracy of the photometric measurement which can be made by means of the movement of this carriage For inside a region of about 1·8 per cent., where contrast is unperceived by the eye, photometric measurement depends on the judgment of the half-way position between the just perceptible limits on either side The accuracy of this judgment naturally depends on the rapidity with which the limit on each side can be presented to the eye. The less the physical effort involved in this operation (down to a limit well below that ordinarily attainable in photometric apparatus), the more accurate will be the mid-point judgment ([10]). The necessity for rigidity and steadiness have been mentioned already

The secondary requirements of a carriage will be best understood from the following description of the design actually employed on the bench already described A broad aluminium base plate *P* (Fig 77) runs on the photometer bench by means of three wheels *W*, which are spool-shaped so as to ride easily on the bars *B*, *B* (Fig. 75) This plate carries at its centre a vertical pillar *V*, into which fits the tubular stem of a circular table *T*. The pillar *V* is capable of a vertical motion of about 130 mm. by means of a diagonal rack and pinion *R*, while the table *T* is capable of rotation about a vertical axis within the pillar *V*. Each of these motions is provided with a clamp which, in the case of the table *T*, takes the form of a small

split-collar S bearing a key-piece, which can be tightened on to T in any desired position and which fits down into a similarly shaped slot in the upper edge of the pillar V The table T is graduated in degrees round its outer edge, while the pillar V carries an arm A

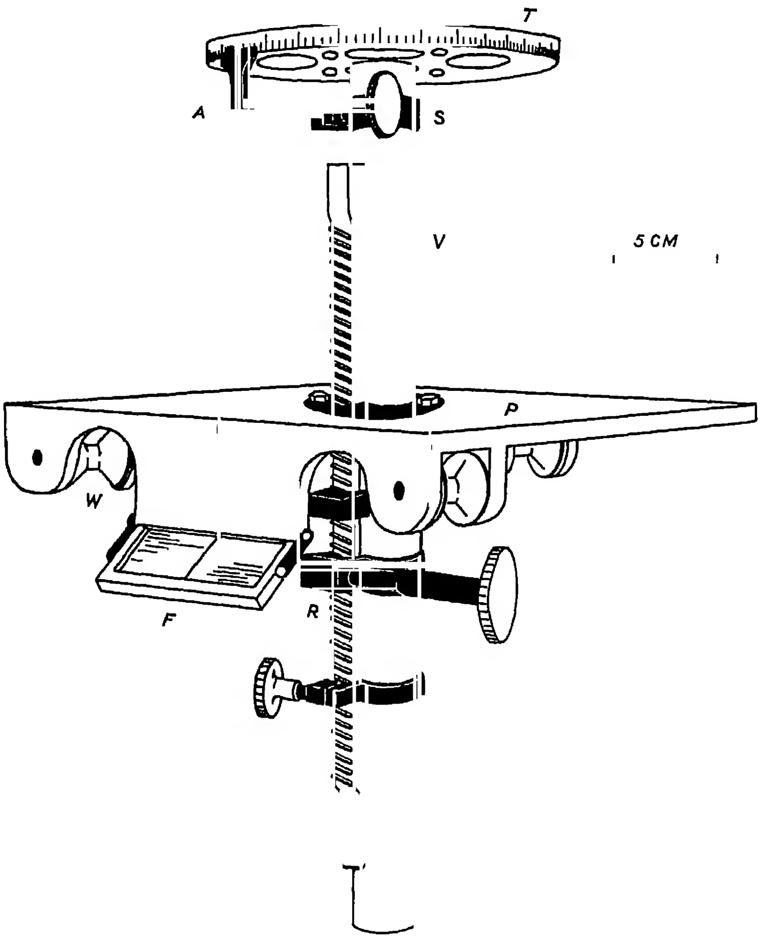

FIG 77 —The Photometer Bench Carriage

with a mark on it which is in the same vertical plane with the axis of the pillar and the fiducial mark on the framework F which moves over the brass scale of the photometer bench The base plate P also carries (i) a clamp for clamping the carriage to the bench at any desired position, (ii) a short pillar, which grips the wire Z (Fig. 75) by which the carriage is moved when in use, and (iii) a second pillar, which carries a split-ring for clamping the carriage to a brass tube used for fixing two carriages at a definite distance apart so that they can be moved as one unit.

The same pattern of carriage is used for mounting the various pieces of apparatus which have, for different purposes, to be used on the bench. Standard lamps, mounted as shown in Fig 71

of emission. It is desirable that the brightnesses of the surfaces should not be greatly affected by a small alteration in the angular position of the photometer head. For this reason they should be arranged as nearly as possible perpendicular to the incident light, for $\cos\theta$ is equal to unity (to an accuracy of 0·1 per cent) up to a value of 2 5° and hence, if the light be incident normally, the photometer head may be turned through angles smaller than this without affecting the accuracy of measurement as long as the surfaces are matt. If, however, the light be incident at an angle of from 45° to 20°, as in the case of the Ritchie wedge or its modifications ([13]), a change of 2° in the position of the head produces an alteration of from 1 to 3 per cent in the illumination of the photometer surface, and this alteration is in opposite directions on the two sides, so that the total error is twice as great ([14])

The Bunsen Photometer Head.—The first really accurate photometer head to be devised was that of Bunsen ([15]) In this head, which is still in common use, a piece of thin opaque white paper, with a translucent spot obtained by treating the paper locally with oil or wax, is mounted between the lamps to be compared, and at right angles to the line joining them (Fig 74, p 147) Then if the illumination of the left-hand side of the paper be E_L, while that of the other side is E_R, it follows that the brightness of the opaque part of the Bunsen disc on the left is $\rho E_L/\pi$, while the brightness of the translucent part is $(\rho' E_L + \tau' E_R)/\pi$, where ρ, ρ' and τ' are respectively the reflection factor of the opaque part, and the reflection and transmission factors of the translucent part of the disc.

There are several methods of using this photometer In one (the substitution method) E_R is kept constant by means of a subsidiary source, and the candle-powers of two other sources, I_S and I_T, say, are then compared by finding the respective distances, d_S and d_T, at which these sources must be placed from the photometer in order that the translucent spot may disappear ([16]) When disappearance takes place $\rho E_L = \rho' E_L + \tau' E_R$, so that E_L has a constant value, and therefore $I_S/d_S^2 = I_T/d_T^2$

In the second method the two sources to be compared are placed one on each side of the photometer, and the points of disappearance of the translucent spot on each side are noted In this case $\rho E_L = \rho' E_L + \tau' E_R$, and $\rho E'_R = \rho' E'_R + \tau' E'_L$, if ρ and ρ' are *the same for both sides of the Bunsen disc.* Hence

$$E_L/E_R = \tau'/(\rho - \rho') = E_R'/E_L'$$

and hence $$(I_S/d_S^2)(d_T^2/I_T) = (I_T/d_T'^2)(d_S'^2/I_S)$$

or $$I_S/I_T = d_S d_S'/d_1 d_T'.$$

If absolute symmetry of the photometer head cannot be assumed, then the head must be reversed and the same process gone through again It is easy to show that the true value of I_S/I_T is the geometric mean of the values obtained with the photometer (*a*) direct and (*b*) reversed ([17]).

The third method of using the Bunsen photometer is that of comparing the contrast between the translucent and opaque parts on the two sides where these are viewed simultaneously. This can easily be achieved by placing the disc in a box containing two

mirrors M, M (Fig 78) slightly inclined towards the disc S, so that images of the two sides are seen in close juxtaposition by an observer at O [18]. When there is equality of contrast on the two sides it follows that $\rho E_L/(\rho' E_L + \tau' E_R) = \rho E_R/(\rho' E_R + \tau' E_L)$, or $E_L = E_R$ If symmetry of the disc and mirrors cannot be assumed, the photometer head is reversed and the process repeated, the geometric mean of the two results again giving the true value The separation of the two fields, inevitable when mirrors are used, may be avoided by the use of the prism system, shown in Fig. 79 [19]

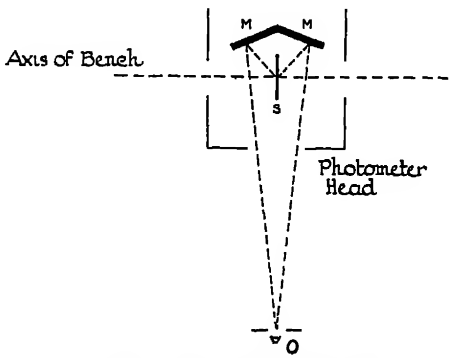

FIG 78 —The Bunsen Photometer

It will be noticed that in this method of using the Bunsen screen the criterion is equality of *contrast* instead of equality of *brightness*, and it has been found that in favourable circumstances the eye is capable of appreciating contrast equality even more accurately than it can appreciate brightness equality [20]. This principle has, therefore, been adopted in the accurate form of photometer head to be described in a later section of this chapter (see p. 157).

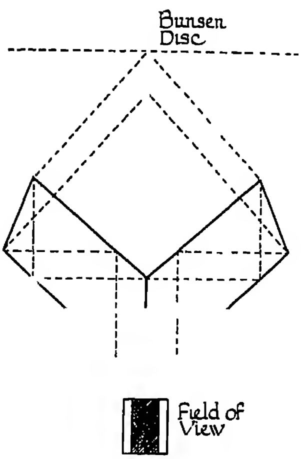

FIG 79 —Prism System for Viewing the Bunsen Disc

Since the values of ρ, ρ' and τ' vary considerably with the angle of emergence of the light, it is essential that, whatever method be followed in using the Bunsen disc, the line of sight should always make the same angle with the normal to the disc.

The theory of the Bunsen disc has been worked out very completely by Weber and others [21]. It has frequently been described quite wrongly in text-books [22], it being stated that, with the sources to be compared one on each side of the photometer head, the balance point is the position of disappearance of the translucent spot *when the disc is viewed from one side* A simple experiment will serve to demonstrate that the points of disappearance on the two sides are separated by a distance which is far from negligible even in rough photometric work [23]

Methods of preparing the Bunsen disc have been described by many writers [24], and the translucent spot has been given many different forms, including a circular disc, a star, and a vertical band. In any case, the dimensions of the translucent part should not be large. When an actual "grease-spot" is used, a sheet of suitable white paper [25] is stretched on a board, and a disc of brass of the form and dimensions desired for the spot is heated, plunged into

molten paraffin wax, and, after draining, is placed on the sheet of paper and then removed The superfluous wax is absorbed with a sheet of blotting paper and an iron, moderately heated so as not to spoil the edges of the spot The Leeson disc consists of a sheet of white paper in which a star-shaped hole has been cut with the sharpest possible edge A sheet of thin translucent paper is then pressed on to each surface of this sheet, and a Bunsen disc with a very fine line of demarcation is obtained Topler's disc is similar, but has a circular spot

It is important that any Bunsen disc in which a paper surface is used should be kept in a dustproof and dark enclosure when not actually in use, otherwise dirt and the discoloration due to exposure to light will gradually produce a lack of equality between the two comparison surfaces ([26])

The Disadvantage of Mixed Light on the Comparison Surfaces.—It will be noticed that there is one defect in the Bunsen photometer head. This is the fact that the brightness of the translucent portion of the field is due partly to transmitted and partly to reflected light, *i e*, each comparison surface receives light from *both* sources, with a consequent reduction of sensitivity. When the photometric setting is obtained, the brightness of this part of the field is proportional to $\tau' E_R + \rho' E_L$, while the brightness of the opaque part of the field is ρE_L. It follows that if the photometer head be displaced by a small distance x to the left of its balanced position, the percentage *increase* of brightness of the opaque part will be $200x/d_L$, while the percentage *decrease* of brightness of the translucent part will be $200x(\tau' E_R/d_R - \rho' E_L/d_L) \div (\tau' E_R + \rho' E_L)$.

Dividing through by $\tau' E_R$ and putting $E_L/E_R \equiv m$, where m is very nearly unity, this becomes

$$200x\left(\frac{1}{d_R} - \frac{\rho'}{\tau'}\,\frac{m}{d_L}\right) - \left(1 + \frac{\rho' m}{\tau'}\right).$$

Clearly the accuracy with which the photometric setting can be made increases with the percentage change of contrast for a given movement of the photometer head, *i e*, with increase in the value of the expression written above Hence, since the ratio of d_R to d_L is governed entirely by the ratio of the candle-powers of the two sources, while m is very nearly unity, it follows that the sensitivity increases as ρ'/τ' diminishes, and attains its limiting value when $\rho' = o$, *i e*, when the translucent part of the field derives its light from one source only.

This is clearly a particular case of a more general principle that the sensitivity of a photometer is reduced if either comparison field receive light from both the sources being compared. For suppose one field A receives the whole of its light from a source L_A, while the other field B receives a fraction p of its light from the same source L_A, and the rest from a second source L_B Then any movement from the position of balance which produces an increase of x per cent in the illumination of A produces a simultaneous increase of px per cent in the illumination of B If this same movement produce a decrease of y per cent in the illumination of B due to L_B, the aggregate contrast produced by the movement is $(y - px)$ per cent, and this clearly increases as p decreases. The principle

just proved shows once again the necessity for a sharp boundary between the two comparison fields, for the presence of an intermediate region owing its brightness to both sources, and therefore less affected by any given movement of the photometer head, causes an undesirable separation between those portions of the surfaces which alone the eye should compare in making its equality setting

Thus the following may be laid down as the conditions to be fulfilled in a sensitive photometer head —

(a) The light should be incident normally at the comparison surfaces

(b) The surfaces should either be perfect diffusers or else be viewed from a fixed direction

(c) The surfaces should be presented to the eye with the sharpest possible boundary between them. There should be no overlapping nor any appreciable separation ([27])

(d) Each surface should receive its light from one only of the sources to be compared

The Lummer-Brodhun Photometer Head.—The conditions above laid down are best met in the form of photometer head due to O Lummer and E Brodhun ([28]) The principle of this photometer depends on the use of a so-called "cube" made up of two right-angled glass prisms, as shown in Fig. 80 In one of the prisms the

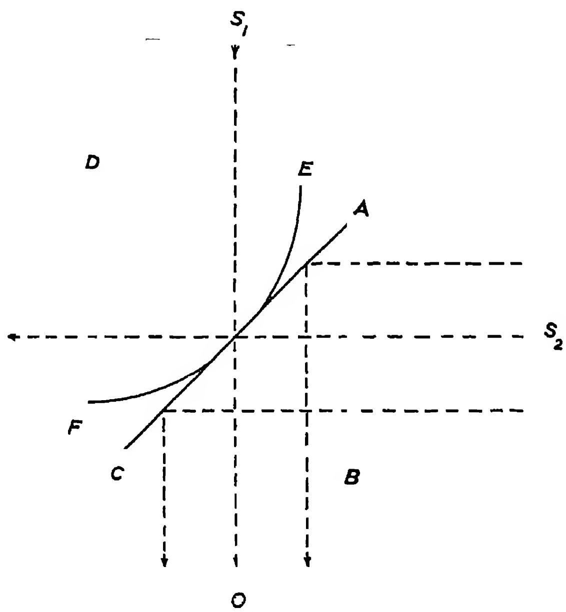

FIG 80 —The Lummer-Brodhun Cube

principal surface is spherical instead of flat, but it has a small region at the centre which is flat, and which makes optical contact with the central portion of the flat surface of the other prism, while the outer parts of the surfaces are separated by air It follows that light entering the prism system at the surface AB passes undeviated

through the central portion in optical contact, while over the outer portion it is totally reflected, and emerges at *BC* On the other hand, light entering *DE* is transmitted through the central portion and emerges at *BC*. Hence, if S_1 and S_2 be two comparison surfaces, the central part of S_1 is seen directly by an eye placed at *O*, while the outer part of S_2 is seen by total reflection at the surface *AC*, and the two together form a ring and disc field with a very fine and

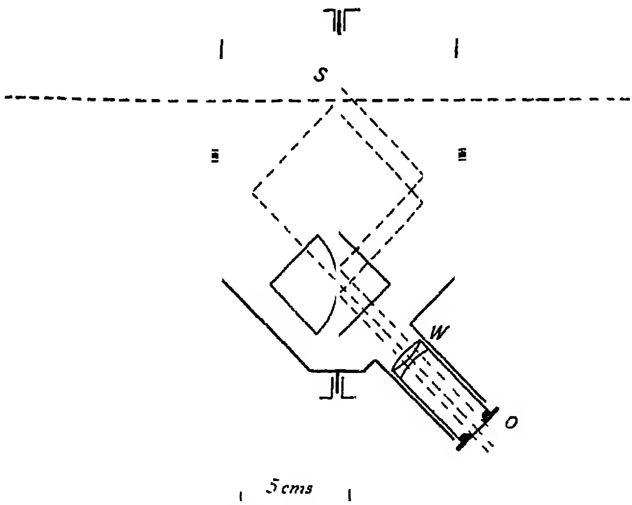

FIG 81 —The Lummer-Brodhun Head (Equality-of-Brightness Type).

sharp line of demarcation if the prism *DEF* be skilfully constructed (29) For photometric purposes the cube and comparison surfaces are arranged as shown in Fig 81. *S* is a sheet, about 4 mm thick, of plaster of Paris or some other white diffusing substance, held in a brass framework, the plaster surface being circular and about 52 mm in diameter The two sides of this disc are illuminated by the light from the two sources whose candle-powers are being compared, and the light from them is brought to the two prisms of the cube by means of silvered glass mirrors or auxiliary total reflection prisms, as shown in the figure The field of view seen by the eye at *O* is then as shown in Fig 82 The eyepiece is provided with a lens at *W*, so that the surface of the cube may be brought to a focus and the necessary sharpness of the boundary between the two parts of the field obtained The plaster screen, mirrors, cube, and eyepiece are mounted rigidly inside a brass box provided with two openings by which the light from the sources may reach *S* (see Fig 89). These windows are provided with brass cover-plates, which are used to close the windows when the photometer is not in use, and thus prevent, as far as possible, the entrance of dust This, by settling on the glass surfaces, produces dark specks on the

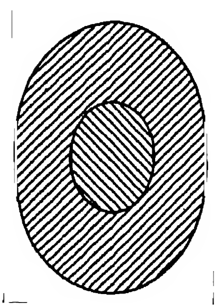

FIG 82 —The Lummer-Brodhun Equality-of-Brightness Field

field which are very annoying to the eye when it is endeavouring to make a photometric balance The interior of the photometer box is lined with black velvet or optical black [30] in order to absorb stray light due to reflection from the glass surfaces. The screen S is removable, so that it can be reversed or taken out altogether for the purpose of testing the screening, *etc* (see p 170). The whole photometer box is pivoted about its axis by means of two steel bearings working in a solid brass semi-rectangular framework, so that it can be completely reversed It is also provided with a degree scale, which works under a clamp and pointer attached to the framework, so that the photometer may be used at any desired angle. The framework has at the bottom a short stem which fits into a tubular holder, and is thus mounted on a carriage travelling on the photometer bench

A slight modification, due to H Kruss [31] and shown diagrammatically in Fig 83, is the introduction of a reflection prism, by

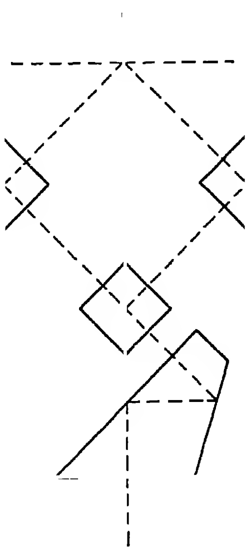

FIG 83 —Prism System for Direct-Vision Lummer-Brodhun Head

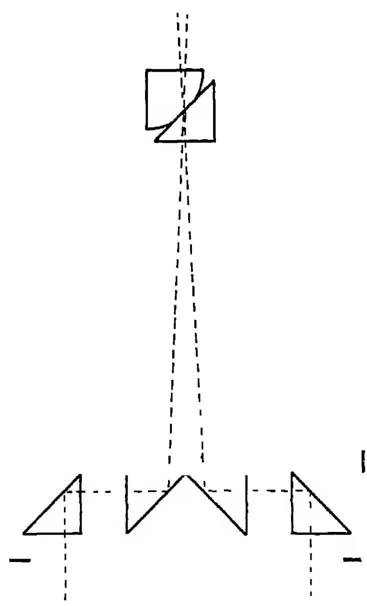

FIG 84 —Binocular Vision Lummer-Brodhun Head

means of which the light is redirected in such a way that the eyepiece can be situated either along or parallel to the axis of the photometer box. Further, by means of the prism system shown in Fig. 84, binocular vision of the photometer field may be obtained [32]

The Lummer-Brodhun Photometer (Contrast Type).—In the photometer above described the equality of brightness of the two comparison fields at the position of balance causes the boundary between these fields practically to disappear when this setting is made, provided the condition (*c*) set out in the section above be fulfilled, and the lights compared be of the same colour Disappearance of the boundary, however, is not that condition which enables the eye to judge most accurately of equality between two fields, and

for this reason, as already remarked with regard to the third method of using the Bunsen disc, the contrast photometer, in which equality of contrast is the criterion instead of equality of brightness, possesses a greater sensitivity The Lummer-Brodhun cube may be adapted for use in this way by altering the form of the surface of contact of the two prisms ([33]) The faces of both prisms are flat, and that of the prism *ABC* (see Fig 85) is sand blasted or etched with the pattern shown shaded in Fig 86 The result is that when the prisms

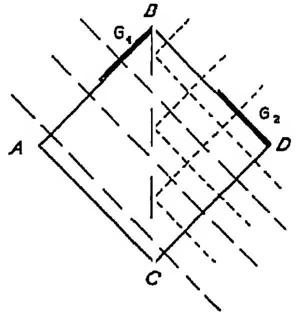

Fig. 85 —The Lummer-Brodhun Contrast Cube

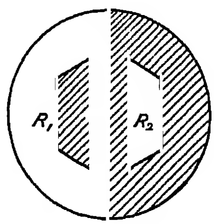

Fig 86 —The Lummer-Brodhun Contrast Field

are pressed together, either with or without balsam, so that the smooth parts are in optical contact, the light passes straight on in the region shown unshaded in Fig 86, while the light entering at *BD* is totally reflected over the region shown shaded in that figure It follows that if the cube be mounted as in Fig 87 the brightness of the unshaded region is due to the left-hand side of *S*, while the brightness of the shaded region is due to the right-hand side of *S* If thin sheets of glass G_1, G_2 be added as in Fig 85, then owing to reflection at the two additional glass surfaces thus introduced, the brightness of the trapezoidal patch R_1 in Fig 86 will be about 8 per cent less than that of the background to R_2, while the brightness of R_2 will similarly be 8 per cent less than that of the background to R_1 Thus at the position of balance the contrast between trapezoid and background will be 8 per cent on both sides of the field of view, and this contrast will be increased on one side and diminished on the other as the photometer is moved away from the position of balance Thus the criterion is equality of contrast ([34]). The most suitable degree of contrast for maximum sensitivity has been investigated by Lummer and Brodhun ([35]), who used, instead of the single glass plates G_1 and G_2, two double plates *KLM* and *K'L'M'* (see Fig 88), which could be rotated through equal angles about the vertical axes *L* and *L'* Since the transmission of light by a glass plate varies with the angle of incidence (see p 113), the contrast could be varied, and, indeed, reversed, by altering the ratio between the angles *ALM* and *BLK* It was found that the sensitivity was a maximum (0 2 per cent.) with a contrast of about 3 to 4 per cent , while with a contrast of 8 per cent it was only half as great (0 4 per cent), but the superior simplicity of the two fixed glass plates has led to their general adoption in place of the more complicated system shown in Fig 88. The correct degree of contrast could

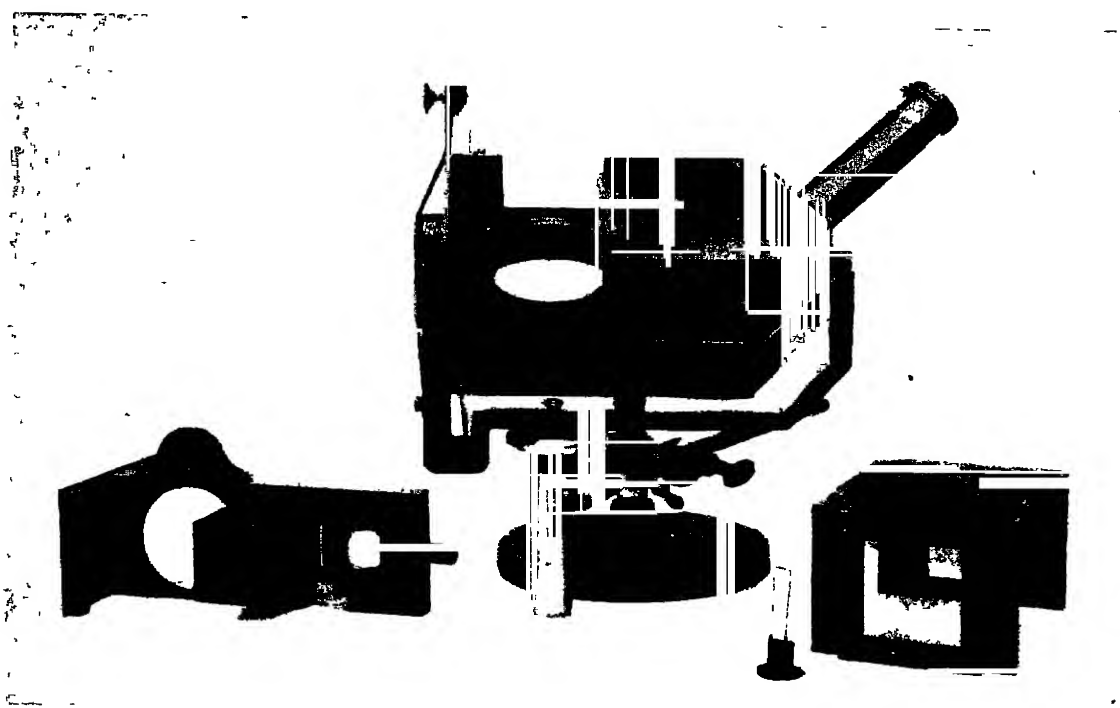

FIG 89.—Details of the Lummer-Brodhun Contrast Head

[*To face p.* 159

e readily obtained by substituting for the plain glasses G_1, G_2 two eutral glasses having an absorption factor of 4 per cent (apart from

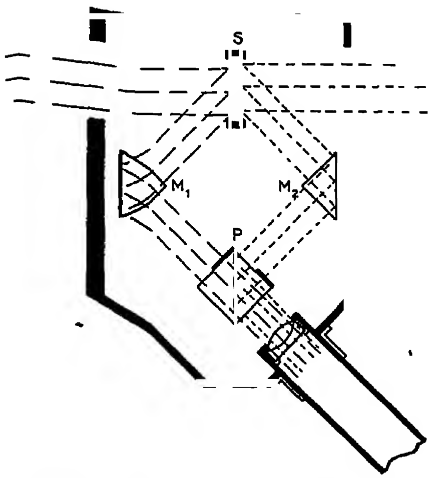

Fig. 87 —The Lummer-Brodhun Contrast Head

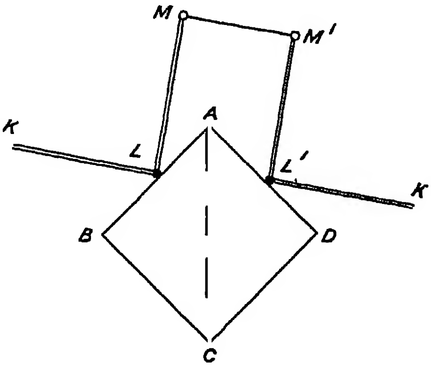

Fig 88 —The Lummer-Brodhun Cube with Variable Contrast

ırface losses) and cementing these to the cube [36] It has also been roposed to use a field in which the contrast is graduated, increasing ·om below upwards on one side, and ·om above downwards on the other [37]. 'he position of balance is then found by djusting to equality of contrast at the ιiddle parts of the two trapezoidal atches

The component parts of the Lumιer-Brodhun contrast head are shown ι Fig. 89

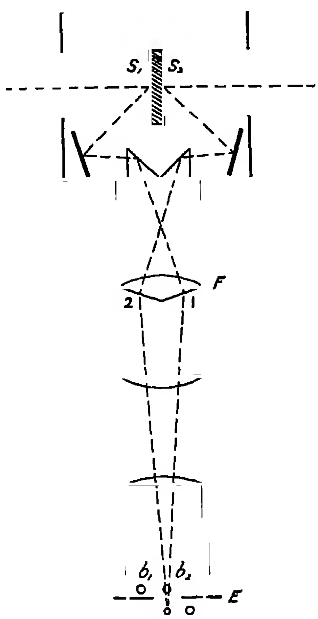

Fig 90 —The Martens Photometer

The Martens Photometer.—There are ›veral other forms of photometer head hich are used on the bench in the same ay as the Bunsen or Lummer-Brodhun ›rm. One of these, designed by F. F ·artens [38], is shown diagrammatically ι Fig. 90 Light from each side of the hotometer screen passes through a series : lenses and a Fresnel biprism F, so that vo images of each surface are formed ι the plane of the exit pupil of the eye-.ece E The positions of these images ·e shown in the diagram, those of surce S_1 being a_1 and a_2, formed respecvely by light from the halves 1 and 2 the biprism Similarly, the images of 2 are b_1 and b_2, and the angle of the prism is so related to the separation of the total reflection ·isms and to the distance FE that a_1 and b_2 coincide; a_2 and

b_1 are stopped by a diaphragm, while an eye placed at E sees the two halves 1 and 2 of F bright by reason of the light from S_1 and S_2 respectively. The dividing line, formed by the edge of the biprism, can be made very sharp The use of a biprism for photometry was first suggested by M. v. Frey and J v. Kries (39), and was adopted by König in his spectrophotometer (see p. 281) The arrangement can be adapted to give a contrast field (40) A somewhat similar arrangement is the Hufner rhomb, shown in Fig 91

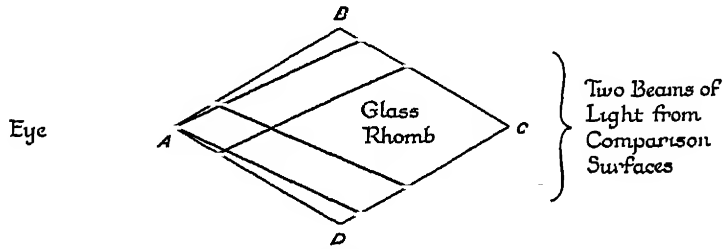

FIG 91 —The Hufner Rhomb

This consists of a glass rhomb $ABCD$, the angles of which are such that two beams of light, one from each of the surfaces to be compared, are caused to emerge at A in juxtaposition, the line of demarcation being the fine edge of the rhomb

The Joly Block Photometer.—A modification of the Bunsen head, which is very simple in construction and therefore frequently used for work of medium accuracy, consists of two equal thin blocks of some translucent material (41), preferably opal glass, separated by a thin sheet of silver foil or other opaque material, as shown in Fig 92 (42) The light reaching the outer surfaces of the blocks is diffused internally, and gives the sides of the blocks seen by the eye a certain brightness The photometric setting is made by obtaining equality of brightness on both sides of the dividing line. Disadvantages of this photometer are (a) the absorption of light in the blocks, which makes it unsuitable for comparing sources of low candle-power, and (b) the uncertain position of the surface from which the lamp distance is to be measured In the case of a highly diffusing medium, this surface is probably very close to the outer surface of the block. The allowance for screen thickness should, therefore, always be made as described below (p. 163)

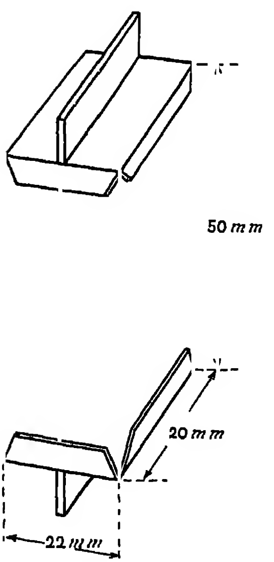

FIG 92 —The Joly Block Photometer

Use of the Photometer Bench.—The preceding sections of this chapter have been devoted to a description of the photometer bench and its accessories, and to an account of the various forms of photometer head which have been designed for use with it The most

obvious method of applying the inverse square law to the comparison of the candle-powers of two sources is that of placing the sources on carriages fixed at any convenient positions on the bench, and then moving the photometer head to and fro between them until the position of balance is found. This simple method is, however, open to various objections In the first place, it assumes absolute symmetry in the photometer head Equality of brightness of the comparison field results from equality of illumination of the two surfaces exposed to the light from the sources only if these surfaces have equal reflection factors, and if the light from them is equally treated as regards reflection, transmission, *etc* , before it reaches the eye

Another objection, of less importance, is the amount of calculation necessitated by this method, for if one source be fixed at the zero mark on the bench, while the other is at a distance d from it, then, if the photometer setting be x, the ratio of the candle-powers is $(d - x)^2/x^2$

To overcome the first objection it is usual to employ the substitution method, so often used in accurate physical measurement [43]. In this a third source, whose candle-power need not be known, is used as a comparison lamp on one side of the photometer head, while the two sources to be compared are placed in turn on the other side of the photometer. A photometric balance with the comparison lamp is made in each case Clearly, if the candle-power of the comparison lamp be assumed to be I_C, then the required ratio of the candle-powers of the other two sources $I_1/I_2 = (I_1/I_C)/(I_2/I_C)$, and this is quite independent of the value of I_C, and therefore of any symmetry in the photometer head, for this must affect equally both of the ratios I_1/I_C and I_2/I_C

The second difficulty is overcome by fixing the distance between the photometer and the comparison lamp, so that the brightness of one comparison surface is a constant If, then, photometric balance be obtained with the photometer head at distances d_1, d_2, d_3 respectively from a number of other sources in turn, it follows that $I_1/d_1^2 = I_2/d_2^2 = I_3/d_3^2 =$ Much calculation is therefore avoided when a number of measurements have to be made in succession, especially if each of the expressions I/d^2 be made equal to some convenient figure, say 10^{-5}, with d in millimetres, so that $I_n = 10^{-5}d_n^2$. This method is often described as the "fixed distance" method

There are some cases in which it is impossible or inconvenient to adopt the fixed distance method, as, for instance, when working with flame sources [44] The substitution method must still be employed, in order to avoid errors due to asymmetry in the photometer head [45], but the sources are fixed and the head is moved between them (Fig 93, *a*) The sub-standard and test lamp are placed in turn at the zero of the bench, while the comparison lamp is fixed at a distance d If the positions of balance of the head be d_1 and d_2 for two lamps whose candle-powers are respectively I_1 and I_2, it follows that $I_1/I_2 = d_1^2(d - d_2)^2/d_2^2(d - d_1)^2$ . . [46]

In most cases, however, it is possible to fix the photometer head either with respect to the comparison lamp or in relation to the test lamp and sub-standard. The latter arrangement is by far the more

convenient when the candle-power distribution of a source is being measured by means of mirror apparatus, or when the source is enclosed in an integrating sphere (see Chapter VII) The carriage holding the photometer head is then clamped at the zero of the bench (Fig. 93, *b*), and the comparison lamp is moved by means of the cord and pulley arrangement previously described (*Z* in Fig 75) By this method $I_1/I_2 = d_2^2/d_1^2$, where d_1 and d_2 are respectively the positions of the comparison lamp when sources of candle-powers I_1 and I_2 are in the test lamp position.

For simplicity in calculation the method first described, *viz.*, that

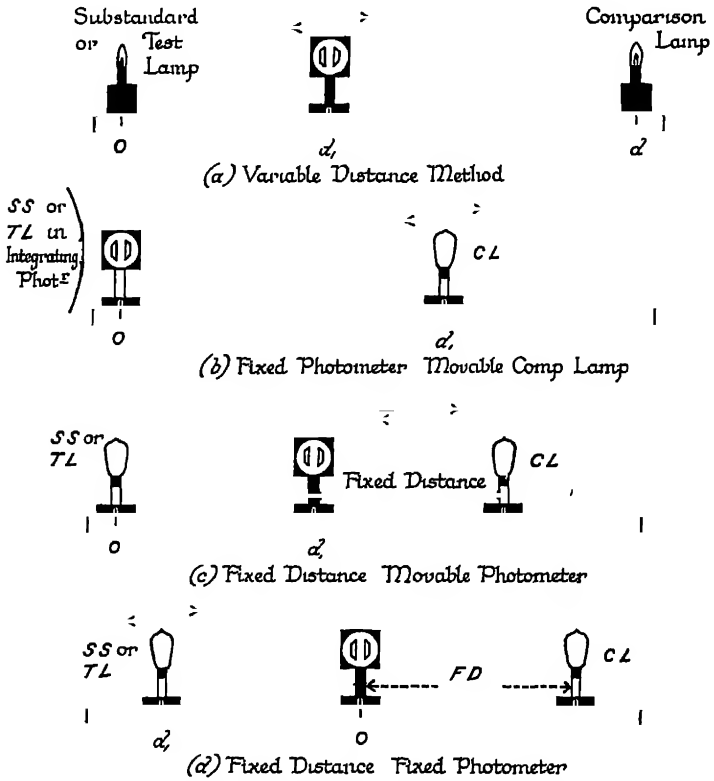

FIG 93 —Methods of Bench Photometry

in which the photometer is fixed with respect to the comparison lamp, is much to be preferred. Either of two methods may be used, according to circumstances When the zero of the bench is near its left-hand extremity, the carriages bearing the photometer head and the comparison lamp are clamped together by means of a stout bar so that they move together as a single unit (Fig 93, *c*). The sub-standard and the test lamp are placed at the *zero* of the bench, and the fixed distance between the photometer head and comparison lamp is so adjusted by means of the bar that $I_S/d_S^2 = 10^{-5}$, d_S being the position of balance of the photometer head for a sub-standard of candle-power I_S For approximate work a "squared" scale

(see p. 148) may be used The photometer is fixed at the mark I_S on this scale and the comparison lamp is moved until a balance is obtained in the photometer head The comparison lamp is then clamped to the photometer at this distance and a test lamp is substituted for the sub-standard. The position to which the photometer has then to be moved in order to restore the balance in the head, gives I_T at once on the " squared " scale The disadvantages of this arrangement are (i) the observer has to move his head to and fro with the photometer in making a setting, and (ii) the unit to be moved is heavy, since it consists of *two* carriages and the connecting bar

These disadvantages may be avoided by fixing the photometer and the comparison lamp to the bench and moving the sub-standard or test lamp To avoid complicating the calculations, it is desirable to have the zero mark at or near the centre of the bench and to scale in both directions from this zero (Fig 93, *d*) The disadvantage of this method is that the centre zero implies the waste of a considerable part of the bench length when any illumination higher than normal has to be used at the photometer head, as, for instance, in measuring sources of high candle-power, when the use of a sector disc or absorbing medium is not desirable. The various bench methods just described are shown diagrammatically in Fig 93

It is usually found that the method in which the photometer head and comparison lamp are moved together is the most generally useful The necessity for moving the observer's head is not found to cause any noticeable inconvenience The movement, in any case, is a slight one The weight to be moved may be reduced by using aluminium in place of brass wherever possible in the construction of the carriages and photometer head This method will, therefore, be described in detail, and the modifications necessary if either of the other methods be employed can easily be inferred First, however, two sources of error common to all methods of bench photometry must be considered

Separation of Photometric Comparison Surfaces : Thickness of Photometer Screen.—It will be seen that as long as the substitution method is employed, symmetry of the photometer head, never completely attainable in practice, need not even be aimed at The assumption is made, however, that the vertical plane of each comparison surface in the head passes through the fiducial mark on the photometer carriage This condition is frequently not fulfilled For example, in the case of the ordinary Lummer-Brodhun head the plaster screen is 4 mm thick, so that each comparison surface is 2 mm right or left of the carriage mark It follows that with this instrument the actual distance between each source and the surface of the screen which it illuminates is $(d - 2)$ mm when the distance between the source and the centre of the photometer as measured on the bench is d mm Hence the true illumination is $E = I/(d - 2)^2$.

If the left-hand comparison surface be t_l mm to the left of the fiducial mark, while the right-hand surface is t_r mm to the right of this mark, the allowance for " screen thickness " may be made as follows, according to the method of working chosen —

(*a*) *Variable Distance Method*—The sub-standard or test lamp is placed t_l mm. to the *left* of the bench zero, while the comparison

lamp is placed at the position ($d + t_r$), although the value d is used in the calculations.

(*b*) *Comparison Lamp alone Moved*.—The photometer head is placed with its fiducial mark t_r mm to the left of the bench zero If any calculations be based on the distance between the sub-standard (or test lamp) and the photometer, the distance ($t_l + t_r$) mm must be subtracted in each case

(*c*) *Photometer and Comparison Lamp moved together*.—The sub-standard or test lamp is placed t_l mm. to the left of the bench zero. The effective distance between the comparison lamp and the photometer is t_r mm. less than the fixed distance as read on the bench, but this distance usually does not enter into the calculations

(*d*) *Centre Zero*.—The photometer head is placed t_l mm to the right of the bench zero, assuming that the comparison lamp is on the right The effective distance of the comparison lamp from the photometer is ($t_r + t_l$) mm less than the distance as read on the bench.

The magnitude of the error involved in the neglect of this correction clearly increases as the ratio I_S/I_T departs from unity; for, taking the fixed distance method as an example, the illumination due to the sub-standard is $I_S/(d_S - t_l)^2$, and this is equal to the illumination due to the test lamp $I_T/(d_T - t_l)^2$

The true value of I_T/I_S is $(d_T - t_l)^2/(d_S - t_l)^2$, and this equals $(d_T/d_S)^2[1 + 2t_l(1/d_S - 1/d_T)]$ approximately Since the value of the expression enclosed within the square brackets does not differ from 1 by more than 0 1 per cent so long as $2t_l(d_S - d_T)/d_Sd_T < 10^{-3}$ numerically, it follows that if, when d_S and d_T are of the order of 1,500 mm, the difference between them does not exceed about 500 mm, *i e*, the ratio between the two candle-powers is not greater than 2, then the correction need not be made so long as t_l does not much exceed 2 mm, *i e*, in the case of the Lummer-Brodhun head In some other photometer heads, however, t_l is much in excess of this value [47], so that the correction for screen thickness is always necessary in such cases The extra work involved in making it is, moreover, so small that there is no justification for neglecting it in work having any pretensions whatever to accuracy.

Importance of Exact Positioning of the Photometer Head.—Owing to the fact that no surface is perfectly diffusing, there is a source of error in all the ordinarily used forms of photometer head, if proper care be not taken to ensure that the comparison surfaces are perpendicular to the incident light. Fig 94, which is exaggerated for the sake of clearness, shows the effect of twisting the photometer head through a small angle. The light which reaches the eye from the left-hand side of the disc S leaves the surface of the disc in the direction SA, while that from the other surface leaves it in the direction SB. It follows that, if the surface be not perfectly diffusing, the reflection factor will be different in the two cases (see pp 114, 343), and in the case of most surfaces, when both sides of S are equally illuminated, the right-hand side will appear the brighter in the photometer. This effect may amount to as much as 0 4 per cent for an angle of twist of 2° in the case of a plaster screen Although this error is avoided in the substitution method of photometry if the twist of the head be not altered between the readings on the

different lamps, it is clearly desirable to remove all possibility of error from this cause by adjusting the screen to be exactly perpendicular to the bench axis and then clamping the registering collar on the pillar attached to the photometer framework The adjustment may be made quite accurately by placing a small piece of mirror on the top of the photometer so that it is exactly in the plane of the screen S The image, in this mirror, of the lamp L will be in line with the lamp itself when viewed from a point on the axis of the bench, if S be properly adjusted. A slight vertical tilt of the head is clearly less important, owing to the fact that the plane of SA and SB is perpendicular to the plane of S

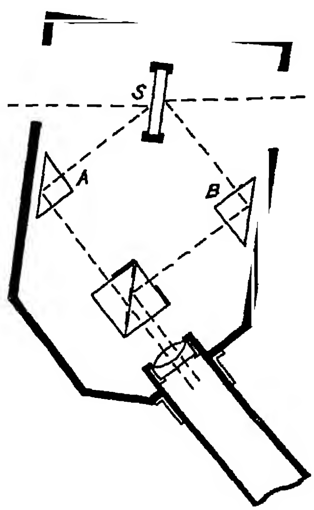

FIG 94 —The Effect of Twisting the Photometer Head

Procedure in making Measurements on the Bench, using the Substitution Method and a Fixed Distance between the Photometer and the Comparison Lamp.—In the following detailed description of the method used in measuring candle-powers by the fixed distance method, the source to be measured will be referred to as the "test lamp" (candle-power I_T), while the lamps used as the basis of the measurement will be termed "sub-standards" Each such sub-standard will be assumed to be an electric lamp of the type illustrated in Fig 71, p 138, and its candle-power, as determined by comparison with a standard at one of the national standardising laboratories, will be taken as I_S For the sake of convenience in description, it will be assumed that the test source is also an electric lamp, and that it is to be measured in a single direction. The modifications in procedure necessary in the testing of other sources will be indicated later in this section. The photometer head used will be assumed to be the Lummer-Brodhun contrast head, with a screen 4 mm thick.

The test may be divided into four parts, as follows —

(i) A comparison lamp L_C is placed in the carriage on the right of the photometer This lamp is preferably of the same type as the sub-standard, but it may be of any convenient type so long as its position can be maintained quite rigidly in the carriage mounting and there is no possibility of even slight changes of candle-power over a burning period of a few hours The electrical connections may be as described in Chapter XVI The function of this lamp is to maintain a constant illumination of the comparison surface on the right-hand side of the photometer head

(ii) A sub-standard L_S is placed in the carriage on the left of the photometer head, and its leads are connected as shown on p. 440. The mean plane of the filaments is adjusted to be accurately over a line 2 mm. to the left of the zero mark of the bench (48) For the

purpose of this adjustment it is convenient to have a plumb line hanging at a short distance in front of the zero mark, as shown in Fig 95 The suspension of this plumb line should be capable of a small movement in the direction of the bench axis When the carriage is placed with its fiducial mark 2 mm left of the zero of the bench, the plumb line may be adjusted by sight so that it lies in the plane passing through this — 2 mm mark and the axis of the carriage pillar The lamp filaments should then be also in this plane If they are not, the carriage must be moved until, with the eye placed so that the plumb line still covers the —2 mm. mark on the bench, the filaments are seen to be directly behind the line. During this adjustment a low potential should be applied to the lamp terminals so that the filament is just glowing, although not sufficiently to dazzle the eye and prevent clear vision of the plumb line and the bench scale When the position of the lamp has been correctly fixed, the carriage is firmly clamped to the photometer bench.

FIG 95 —Arrangement for setting Lamps on the Bench Zero.

(iii) The lamps L_S and L_C are now switched on, at first with their adjusting resistances fully in, so that there is no chance of even a momentary excess of potential above that at which these lamps are to run, especially in the case of the substandard. The potentials on the lamps are then gradually raised to their correct values, and the photometer carriage is clamped at the position on the bench given by the relation $I_S/d_S^2 = 10^{-5}$, supposing that 10 metre-candles is being taken as the working illumination The distance d_C between the photometer carriage and the comparison lamp carriage is then adjusted so that as good a balance as possible is obtained in the photometer, and these carriages are then clamped together by means of the brass bar previously referred to. The distance d_C is noted, and the photometer carriage is then unclamped from the bench.*

A series of photometric settings is now made with the lamps carefully maintained at a constant potential Supposing two observers to be working together, each one in turn makes five or ten settings without looking at the readings he obtains, while the other observer notes down these readings to the nearest half millimetre

* It is a useful practice to move the combined carriages after unclamping and to check the distance between them in order to make sure that there is no possibility of their moving relatively to each other during subsequent work.

and watches the instruments which indicate the potentials (or currents) on the comparison lamp and the sub-standard circuits. It is to be noticed that the observer does not see his own readings until after the completion of the set, so that he cannot be unconsciously biassed in either direction [49] The mean reading for observer A may be denoted by ${}_Ad_S$, and that for observer B by ${}_Bd_S$. It will generally be found that these mean distances are not exactly equal to each other or to $\sqrt{10^5} \times I_S$, which may be denoted by d_S. The usual procedure is to find the alteration which would be necessary in d_C in order to bring these quantities into agreement. Since $I_S/{}_Ad_S^2 = I_C/d_C^2$, it follows that $I_S/d_S^2 = I/_C(d_C + x)^2$, where $x = (d_C/{}_Ad_S)(d_S - {}_Ad_S)$, so that, since ${}_Ad_S$ is nearly equal to d_S, the "correction" for A is $d_C(d_S - {}_Ad_S)/d_S$, and similarly that for B is $d_C(d_S - {}_Bd_S)/d_S$

Several more sub-standards, L_2, L_3, L_4, *etc* , are now inserted in turn in place of L_S and the different corrections to d_C found by A and B are tabulated as follows ·—

Date Comp lamp No at volts.

$d_C = 1{,}739{\cdot}_0$ mm.

Observer (1)	Sub standard (2)	d_S (true) (3)	d_S (observed) (4)	(3) — (4) (5)	$(d_C/d_S) \times$ (5) (6)
	L_S	$1{,}364{\cdot}_3$	$1{,}362_6$	$+1{\cdot}_7$	$+2{\cdot}2$
A	L_2	$1{,}326_4$	$1{,}326{\cdot}_2$	$+0_2$	$+0\ 2$
	L_3	$1{,}358_0$	$1{,}359_0$	-1_0	$-1\ 3$
	L_4	$1{,}359_2$	$1{,}357_7$	$+1_5$	$+1\ 9$
			Mean correction for A		$+0\ 7$
	L_S	$1{,}364_3$	$1{,}364_8$	-0_5	$-0\ 6$
B	L_2	$1{,}326_4$	$1{,}332_0$	-5_6	$-7\ 3$
	L_3	$1{,}358_0$	$1{,}361_6$	-3_6	$-4\ 8$
	L_4	$1{,}359_2$	$1{,}359_4$	$-0{\cdot}_2$	$-0\ 2$
			Mean correction for B		$-3{\cdot}2$

The figures in the extreme right-hand column should show about the same degree of consistency for each observer as that shown in the above example If they do not, more sub-standards should be taken. If the inconsistencies are too large to be accounted for by personal errors, defects in the comparison lamp or in the electrical circuits should be looked for.

The correction to the fixed distance d_C is generally made according to the results obtained by one observer Subsequent measurements of test lamps made by this observer (A, say) with this corrected fixed distance do not then require any correction The results obtained by B require correcting, however, to bring them to the values which that observer would have obtained if his value had

been adopted for the fixed distance The amount of this correction is clearly xd/d_C, where x is the difference between the fixed distance actually used and the fixed distance found by B, *i.e.*, $x = ({}_Bd_C - d_C)$. In the example given above $x = -3\ 9$ mm with d_C reset to suit A.

(iv) Test lamps may now be put in the position formerly occupied by the sub-standards, each being lined up so that its " photometric centre of symmetry " (*i.e* , the centre line of the light-giving system) is in the plane perpendicular to the axis of the bench and passes through the -2 mm. mark on the scale. Photometric measurements are then made on each lamp by both observers as before. After the correction to the distances found by B has been made as described at the end of the last paragraph ([50]), the mean of the results of both observers is found for any one test lamp, and the candle-power I_T found by squaring this distance (in mm) and dividing by 10^5. A convenient method of booking the readings is shown in the following scheme —

Date Comp. lamp No at volts
Fixed distance, 1,739 7 mm.

Test Lamp (1)	Observer (2)	d_T (observed) (3)	d_T (corrected) (4)	Mean (5)	I_T (6)
T_1	A	$1{,}493_{\ 2}$	$1{,}493_{\ 2}$	$1{,}492_{\ 7}$	$22{\cdot}2_8$
	B	$1{,}495{\cdot}_6$	$1{,}492{\cdot}_2$ *		
T_2	A	$832{\cdot}_7$	$832{\cdot}_7$	$833_{\ 0}$	$6{\cdot}9_4$
	B	$834_{\ 5}$	$833_{\ 2}$		

$$* \ 1{,}492{\cdot}_2 = 1{,}495_{\ 6} - (d_T/d_C) \times 3\ 9$$

For approximate work one observer is often considered to be sufficient. In this case the fixed distance is set to suit this observer, and no correction is necessary The candle-powers of the test lamps are found, either by direct reading on a " squared " or candle-power scale, or by squaring the reading on an ordinary millimetre scale.

In the above description of photometric procedure it has been assumed that what is required is the candle-power of a lamp at a given potential or current. Sometimes it is required to determine the potential or current at which a lamp has a given candle-power I_T This is obtained by fixing the test lamp at the -2 mm. mark, and the photometer head at the point d_T $(= \sqrt{I_T \times 10^5})$ The electrical conditions are then adjusted to give a fairly close photometric balance in the photometer head Two sets of candle-power measurements are made in the ordinary manner, with the test lamps at potentials respectively about one-half of 1 per cent above and below the potential thus obtained. These measurements enable the correct value of potential or current at the specified candle-power to be obtained at once by interpolation (see Appendix X). For approximate work it is sometimes sufficient to set the photometer head at the distance which corresponds with the definite candle-

power, and to obtain the photometric balance by altering the potential on the test lamp. This is done by means of a resistance inserted in the test lamp circuit, and placed so as to be conveniently under the control of the observer at the photometer head ([51]).

When the source to be measured is not electrical, the photometric procedure is exactly the same for operations (i.), (ii.) and (iii.). For (iv.) the lining-up of the test lamp is also similar, but the control conditions necessarily depend on the particular nature of the source. In the case of a gas flame, mantle, or similar source, the pressure of the gas must be controlled by some form of regulator ([52]). The rate of consumption must be adjusted to the scheduled value, or to the value at which the maximum candle-power is obtained. This rate should be measured while the photometric measurements are in progress. Adjustment of the air inlet may also be required. No photometric measurement should be made until the lamp has been burning for at least thirty minutes, and a final slight adjustment of the gas and air inlets may be required at the end of this period before the observations are begun. Other factors which affect the candle-power of gas lamps are the calorific value of the gas used ([53]) and, to a less extent, the humidity and pressure of the surrounding atmosphere ([54]). The centre plane of a flat flame, and the axis of a gas mantle, are usually adopted for lining-up over the zero mark of the bench. It should, however, be remembered that a gas mantle is very opaque to the light emitted from the opposite side of the mantle, so that the true photometric centre is at a distance of about $r\sqrt{3}/2$ from the centre towards the photometer head, if the mantle be assumed to be a cylindrical, perfectly opaque and diffuse radiator of radius r. The same remark does not apply to a flame which is usually so transparent that the mean plane of a duplex flame may be taken as the plane midway between the two individual flames.

When a candle-power measurement is made in a single direction, unless this direction is specified beforehand (as, for example, by the position of the leading-in wires in an electric lamp), it is usual to choose a direction in which the illumination is even and as free as possible from bright or dark lines or spots. A piece of white paper placed in front of the photometer head will generally show considerable unevenness of illumination in the case of an ordinary commercial electric lamp, for instance. As the lamp is rotated in the carriage pillar light and dark vertical lines will be seen to move across the paper owing to images of the filaments in the bulb and slight lens effects due to vertical striations in the walls of the bulb. If these be included in the field covered by the photometer head the candle-power may be found very sensitive to exact positioning of the lamp.

To define the direction of candle-power measurement it is generally sufficient in approximate work to mark an arrow on the lamp cap on the side of the lamp facing the photometer. For more accurate work the procedure described in connection with the preparation of sub-standards (see p. 139) may be followed.

Measurement of Sub-standards.—The process by which the candle-power value is assigned to a sub-standard at a standardising laboratory is very similar to that described above. This sub-standard now becomes the "test lamp," while the standardising laboratory's master sub-standards are used for the comparison. The only

important difference is, in fact, the number of measurements made on each lamp, the final value assigned to it being the mean of the values obtained on at least three separate occasions by two or more observers The master sub-standards used at the standardising laboratory are compared in a similar manner with the international standards, a large number of observers making measurements on many separate occasions, so that their mean values are probably correct to at least 0·1 per cent.

For approximate work in laboratories where the amount of photometric work done is considerable, as, for example, in a lamp factory, it is usual for a number of lamps to be compared at the laboratory with sub-standards obtained from a standardising laboratory Such lamps then become working sub-standards, and, if compared at frequent intervals with the real sub-standards, they may be used until they fracture or develop some other defect

When a sub-standard is measured, the current taken by it at the standard potential is usually measured by means of a potentiometer and standard resistance (see p 438). This current value will then serve as a check on the constancy of the lamp in subsequent work. The correction to be applied to the measured current when the potentiometer is in the sub-standard circuit (p 439) should be noted. It is quite satisfactory to use under-run tungsten filament lamps for the measurement of carbon lamps, provided the candle-power at the under-running voltage be sufficient to give the necessary illumination at the photometer head (55).

Screening.—A very important precaution to be observed in all photometry is that of preventing stray light from reaching the photometer screen (56) Stray light may be regarded as any light which reaches the photometer otherwise than directly from the source being measured It may be due either to the other sources of light in the room or, more frequently, to reflections, by objects near the bench, of light from either of the lamps being compared

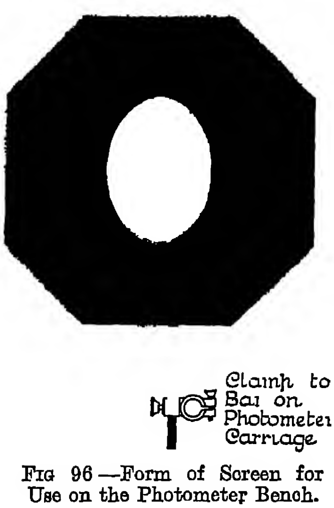

FIG 96—Form of Screen for Use on the Photometer Bench.

There are several methods of avoiding stray light, though all depend on the same principle, *viz*, the interposition of opaque black screens which completely shield the photometer except in the direction of the lamp. These screens may be of the general form shown in Fig. 96, with apertures of various sizes They are supported at intervals along a light aluminium tube which is clamped to the carriage holding the photometer. The distances between the screens are adjusted in relation to the sizes of the apertures, as shown diagrammatically in Fig 97, so that no light can reach the photometer except from the region *AB* The lamp is situated in the centre of this region, and some 50 cm behind it is placed a large screen

covered with clean black velvet,* so that in effect the photometer and lamp are enclosed in a light-tight box. Sometimes, as an additional precaution, black curtains are hung along the bench on either side to prevent too much light from reaching the screens, and

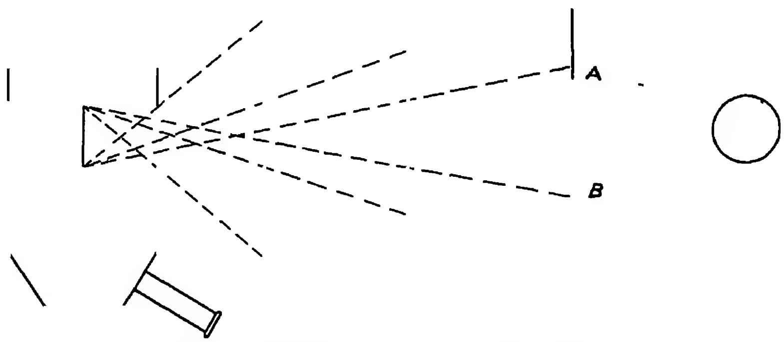

FIG 97 —Arrangement of Screens on the Bench

so finding its way by reflection to the photometer, for the reflection factor of even a matt-black paint may be from 3 to 5 per cent.

In a slightly different method of screening a light-tight box is actually used. Two rectangular frameworks, one near the photometer, and the other near the lamp, are connected by a square tube of black material which, being pleated like an accordion bellows, elongates or contracts as the photometer moves away from or towards the lamp Here, again, a black velvet screen is placed at a sufficient distance behind each lamp.

In order to test the effectiveness of the screening arrangements, the photometer disc should be removed and each lamp looked at from the window on the opposite side of the photometer. Nothing should be visible in any direction except the lamp itself, the remainder of the field of view should be completely occupied by black surfaces.

Sometimes trouble is experienced from light reflected by the polished metal bars of the photometer bench itself In the direction of specular reflection this light, unless completely stopped by the screens, may cause a considerable error in the photometric measurements. It can be avoided completely by covering the bars with a piece of black velvet at the region half-way between the lamp and the photometer.

Too much emphasis cannot be placed on the importance of adequate screening in accurate photometry It is no exaggeration to say that half the inconsistencies in the photometric measurements made in an ordinary laboratory are due to imperfect screening.

Transportable Photometers.—Many different types of so-called "portable" photometers have been designed for use outside the laboratory or testing room ([57]). These consist, generally, of a short bench of simplified form in the centre of which is fixed a photometer

* It should be noted that velvet readily collects dust and may, when sufficiently coated, reflect an appreciable amount of light.

head A sub-standard is used on one side, and the lamp under test is moved along the bench on the other side of the photometer

The accuracy of instruments of this kind is not great, and for most purposes for which a portable photometer is required it is preferable to abandon the bench altogether and use one of the illumination photometers described in Chapter XII The test surface of the photometer is placed at one end of a blackened tube, and the test lamp at a certain fixed distance d from it. If E be the illumination measured by means of the photometer, $I = d^2E$

The Weber Photometer.—A very convenient form of portable photometer in which the inverse square law is employed without the use of a bench is that first designed by L. Weber in 1883 [58], and subsequently modified by the introduction of a Lummer-Brodhun cube, and still later by the substitution of an electric lamp for the benzine comparison lamp originally used. This photometer, since the scale employed for the distance measurement is short compared with that of a photometer bench, is most useful for work of moderate accuracy and in places where a bench is not available. As now constructed by Messrs Franz Schmidt and Haensch, it is shown in sectional elevation in Fig 98 T_2 is a stout wide

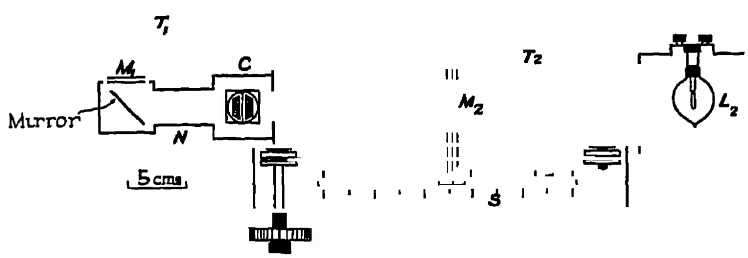

FIG 98 —The Weber Photometer

horizontal tube along which moves an opal glass plate M_2 This is illuminated by a small electric glow lamp L_2 contained in a square box at the end of T_2. The distance of M_2 from this lamp is measured by means of a pointer moving over a scale S The current through L_2 is maintained constant by means of an external rheostat and battery At the other end of the instrument is a smaller tube T_1 containing a second opal glass plate M_1, fixed in position M_1 is illuminated by the lamp L_1 (not shown) which is to be measured. The brightnesses of M_1 and M_2 are compared by means of the mirror system and the Lummer-Brodhun cube C The photometric balance is obtained by adjustment of the position of M_2, and if the distance of M_2 from L_2 be d_2 when a balance is obtained, while the constant distance between L_1 and M_1 is d_1, then $I_1/d_1^2 = \kappa I_2/d_2^2$, where κ is a constant of the instrument [59] κI_2 is found by standardising with sub-standards in the usual way ; in fact, it plays the same part as the candle-power of the comparison lamp in the substitution method of photometry. For increasing the range of the photometer, absorbing glasses are introduced in front of M_1 to increase the value of κ (see p. 182).

Polarisation Photometers.—In certain problems of photometry,

notably the comparison of the candle-power of two sources in homogeneous light (spectrophotometry, see Chapter IX), it is not generally practicable to vary the distance between the source and the photometer head, so that the inverse square law cannot be applied For such problems recourse is had to one of the other laws governing the brightness of a surface illuminated by a source, notably the squared tangent law of polarisation, referred to in Chapter II. (p 30)

The principle of a polarisation photometer is, briefly, the production of contiguous (or, occasionally, superposed) images of the two comparison surfaces by means of an optical train which includes a device for plane polarising the light which forms one or both of these images. The polarising device may consist of a pile of glass plates upon which the light is incident at or near the polarising angle (see p 113), or, more conveniently, it may consist of a doubly-refracting prism, such as a Nicol or Wollaston (see p. 29) [60] If the beams of light forming the two images be polarised in mutually perpendicular planes, then the interposition of a second, or analysing, Nicol prism reduces the intensity of one image by the factor $\cos^2 \theta$, and that of the other image by the factor $\sin^2 \theta$, where θ is the angle between the optic axis of the Nicol prism and the plane of polarisation of the light forming the first image (see p. 29)

If, therefore, the analysing Nicol be capable of rotation and photometric balance be obtained at the angle θ, it follows that the ratio between the brightnesses of the two images, supposing no Nicol interposed, would be $\tan^2 \theta$

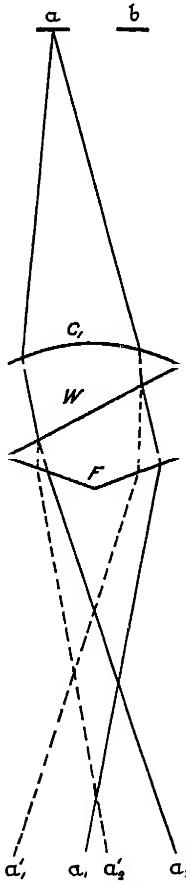

FIG 99 —The Martens Polarisation Photometer (general principle)

The Martens Polarisation Photometer.—The methods of producing the two contiguous images with the finest possible dividing line are different in the various polarisation photometers that have been designed from time to time. In the Martens polarisation photometer [61] a Fresnel biprism is used, as in the ordinary form of Martens photometer already described (see p 159) The polarisation instrument is shown diagrammatically in Fig 99 Here a and b are the two comparison surfaces The light from a passes through a plano-convex lens C_1 into a Wollaston prism W Here it is split up into two beams, one, shown by the full line, being polarised in the plane of the paper, while the other, shown by the broken line, is polarised in a plane perpendicular to this. Each of these two beams passes through the biprism F and, in consequence, is again split into two parts, so that the light from a is now divided into four beams which, on passing through a suitable lens system, form four separate images of a in the plane of the entrance pupil of the eyepiece. Of these four images two, *viz.*, a_1 and a_2, are polarised in the plane of the paper, while

the other two, a'_1 and a'_2, are polarised in the perpendicular plane The light from b is treated similarly, and the angle of the biprism is designed in relation to the relative positions of object and image, and to the separation of a and b, so that a'_1 and b_2 coincide at the eye, while the remaining six images are stopped by a diaphragm It follows that one half of the biprism F appears bright due to the light from a, which is polarised in the plane of the paper, while the other half appears bright due to light from b, which is polarised in a perpendicular plane. The interposition of a Nicol prism in a

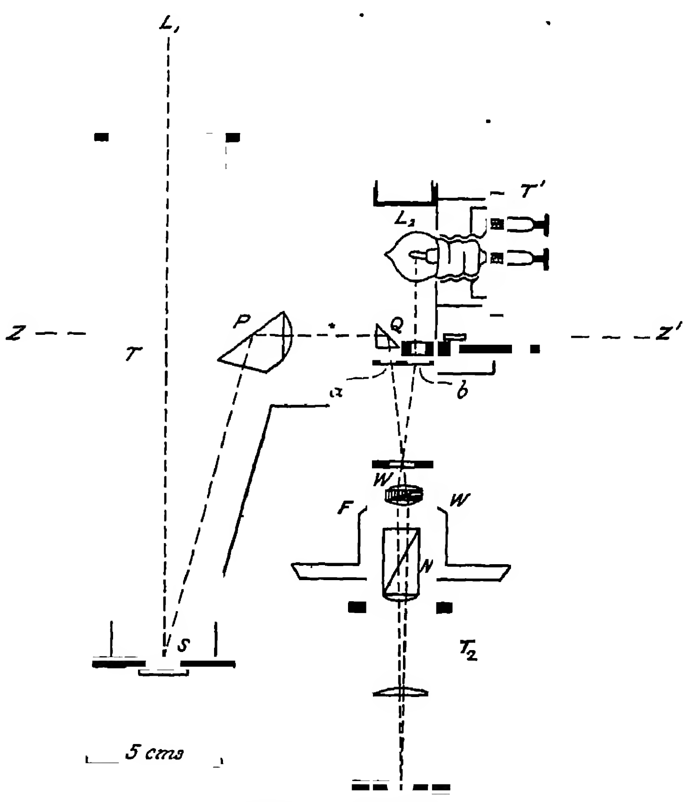

Fig 100.—The Martens Polarisation Photometer

holder capable of rotation about the axis of the instrument therefore gives the complete photometer, which in its actual form is shown in Fig 100

T is a hollow tube, open at one end and closed at the other with a small cap holding a white plaster screen S This tube is capable of rotation about the horizontal axis of the instrument ZZ', while the instrument itself is mounted on a vertical pillar capable of rotation in azimuth, so that the tube T can be moved to such a position that the light from the lamp to be measured, L_1, passes along the tube and is incident normally at S The light from S is deflected by the totally reflecting prisms P and Q, and a lens cemented to P forms an image of S at a position a, which is close to the second

surface of Q The comparison lamp used for the measurements, L_2, is placed at one end of a horizontal tube T', and illuminates a small sheet of translucent opal glass b. The image a and the surface b form the two comparison surfaces, and the remainder of the instrument is seen to be arranged as just described Now the brightness B_a of image a'_1 is proportional to the illumination of S (E_s, say), while that of b_2, B_b, is a constant, so that if θ be the angle on the scale of the Nicol prism, $E_s = \kappa \tan^2 \theta$, where κ is a constant of the instrument. κ is obtained by calibration with a sub-standard of candle-power I placed at a distance d from S. If, then, α be the value of θ with this arrangement, $I/d^2 = \kappa \tan^2 \alpha$, so that $\kappa = I \cot^2 \alpha/d^2$.

A modification of the Weber photometer (see p. 172) has been designed ([62]) in which two Nicol prisms, crossed at a measurable angle, are employed instead of the movement of the plate M_2

Since the polarisation photometer makes use of the squared tangent law, it follows that the scale of the instrument becomes very inaccurate when the ratio of brightness of the comparison surfaces B_a/B_b is far removed from unity, for $d(\tan^2 \theta)/\tan^2 \theta \, d\theta = 4 \operatorname{cosec} 2\theta$, which becomes infinite when $\theta = 0°$ or $90°$, and has its minimum value when $\theta = 45°$ ($B_a/B_b = 1$) The angular movement for a given contrast is reduced to one-half of its maximum value when B_a/B_b is equal to about 15 The maximum absolute accuracy is 1 per cent for a movement of 0·1 mm. on an angular scale of 80 mm diameter, so that it is clear that the accuracy attainable with this form of photometer, using a scale of practicable dimensions, does not approach that obtainable with a photometer bench of even 3 metres total length. Further, the instrument possesses the two errors common to all apparatus involving accurate measurement of angular rotation, *viz.*, (*a*) error of centering of the Nicol prism in relation to the axis of rotation, and (*b*) zero error due to lack of exact coincidence of the optic axis with the plane of polarisation of the ordinary ray transmitted by the Wollaston prism when $\theta = 0$. The first error is overcome by taking readings on the scale in opposite quadrants The second error is eliminated by taking as the true value of θ the mean of the angles measured on each side of the zero line

It is clear that in this form of photometer the light from the comparison surfaces must be quite free from polarisation ([63]). Since specular reflection produces polarisation, it follows that the comparison surfaces must be as matt as possible, and the direction of specular reflection must be carefully avoided

The Martens photometer may be used for determining the plane of polarisation of partially polarised light as well as the ratio between the intensities of the polarised and unpolarised parts The tube T_2 is removed from the rest of the instrument and closed at the end by a diaphragm with a single hole admitting unpolarised light from an exterior surface The balance point of the analyser N is found, and it is clamped in this position The tube T_2 is then directed towards the surface giving partially polarised light, and is rotated *as a whole* to find the four positions of photometric balance The plane of polarisation of the light bisects the angle between the lines joining the pairs of mutually opposite positions found, and the

correct bisector is determined by the direction in which the photometric balance is disturbed as the tube is slightly rotated from the balance point.

The tube T_2 is then placed at an angle of 45° with the plane of polarisation thus found, and the Nicol N is unclamped and rotated until a balance is obtained, at an angle α, say The percentage of polarised light is then 100 sin 2α. For the unpolarised part of the light U can be resolved into two components, each of magnitude $\frac{1}{2}U$, and polarised respectively in and perpendicular to the plane of polarisation of the polarised part P. Since the analyser makes an angle $(45° + \alpha)$ with the plane of polarisation of P, at the position of balance

$$(\tfrac{1}{2}U + P)/\tfrac{1}{2}U = \tan^2(45 + \alpha),$$

whence $P/(U + P) = \sin 2\alpha$.

Photometers depending upon Talbot's Law.—By Talbot's law (see p. 58), if a disc with an aperture in it, such as that shown in Fig. 101, be set rotating between a lamp and a photometer head so that the light from the lamp only reaches the photometer for a certain fraction of the whole time, and if the rotation be so fast that all sense of flicker is lost when the eye looks into the photometer, the effective candle-power of the lamp is reduced in ratio of the time of exposure to the total time (64), *i.e.*, if the aperture in the disc has the form of a sector of angle $n°$, the effective candle-power is $I(n/360)$. It was long thought that Talbot's law did not apply when n *was small* (65), but it is probable that the deviations found were due to small inaccuracies in the measurement of n (which naturally became more important as n was reduced), since Hyde, in a very careful series of measurements (66), found that the deviations lay within the limits of experimental error (0 3 per cent.) for values of n as low as 10°.

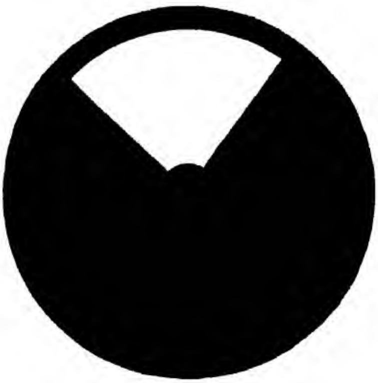

FIG 101.—The Simple Sector Disc

The older forms of sector disc had fixed openings (67), but in later instruments the angle of opening could be adjusted while the disc was in rotation (68)

The Napoli-Abney form (*l.c. note* (68)) is illustrated in Fig. 102 The shaft H carries near one end a grooved pulley, which may be driven at any desired speed by an electric motor. At the other end is a disc A, of which two (or sometimes three) equal sectors have been removed, except by the shaft and the rim A second, exactly similar, disc is placed behind this one, and is rigidly attached to a sleeve which slides on the shaft. Fixed to this sleeve is a pin, which engages in a spiral groove cut in the shaft so that the longitudinal position of the sleeve along the axis of the shaft controls the relative positions of the two discs. The width of the sector openings is thus capable of control by means of a wheel W attached to the sleeve

and acted upon by a groove in a lever L, which moves over a divided scale S.

The form of sector disc with variable opening may be used as an approximate form of photometer, the variation of angular opening

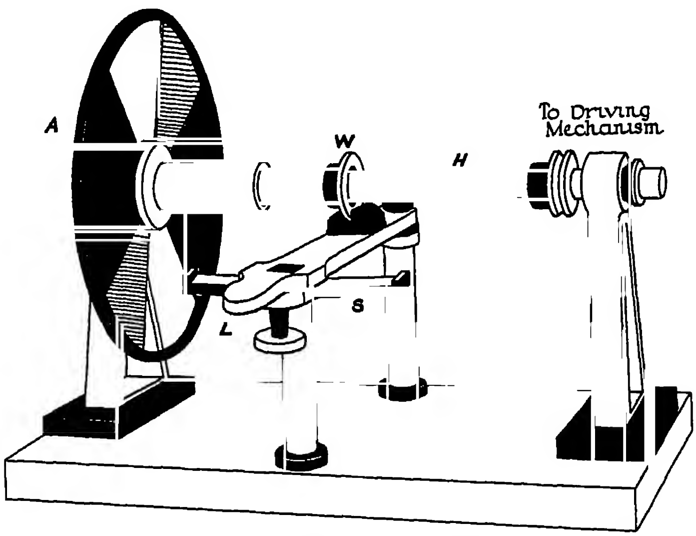

FIG. 102.—The Napoli-Abney Sector Disc

being employed instead of distance variation in order to control the illumination at the photometer head It is, however, very difficult to obtain a sufficiently slow change of aperture for accurate work, and, even supposing this done and all "lost motion" eliminated, the angle must be determined to a fifth of a degree (with an opening ratio of one-tenth) in order that an accuracy of 1 per cent. may be obtained. Further, the rotation of the disc has to be stopped for every reading unless some stroboscopic method be employed for viewing the scale on the disc while the latter is in rotation For this purpose a neon tube may be used for illuminating the scale, the current through the tube being made intermittently by means of a contact on the shaft or on the periphery of the disc ([69]). Alternatively, the method of illumination shown diagrammatically in Fig. 103 may be adopted ([70]) A narrow slit S is cut in the sector disc about $\frac{3}{4}$ inch above the fiducial mark K. S is intermittently illuminated as it passes in front of a fixed lamp L, and an accurately-focussed image of it is formed by means of the concave mirror M on the mark K and the neighbouring parts of the graduated scale. These illuminated parts are viewed by the eye at E through the lens L', and when the sector is in rotation this scale appears quite

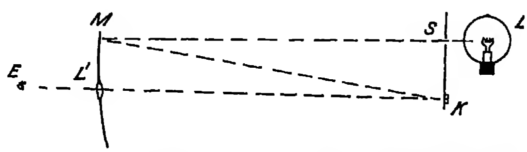

FIG 103—Intermittent Illumination of the Rotating Sector Disc

stationary if S be sufficiently narrow. Another form of disc has been described in which the angle of opening is measured electrically (71)

A different form of apparatus, designed for use as a photometer, is that in which the sector disc remains stationary while the light from the lamp is caused to rotate by means of the double total-reflection prism arrangement shown in Fig 104 (69) This prism system is mounted on a double disc which rotates about the axis of the stationary sector disc S The light follows the path shown by the broken line, so that the illumination of the screen P is proportional to the opening It is to be noticed that in this device the Fresnel prisms must not act as a stop on the beam of light, i e., the whole of the source must be clearly visible from every part of P when the sector disc is not acting as an obstruction The chief advantage of this device, apart from ease of construction, is the accuracy and rapidity with which the opening of S can be adjusted so that it can be employed as the actual mode of brightness variation in making a photometric balance It is so employed in a form of photometer (72) which resembles the Weber photometer (p 172), except that the variation of illumination of one of the comparison surfaces is produced in this way instead of by movement of the plate M_2 towards or away from the source L_2.

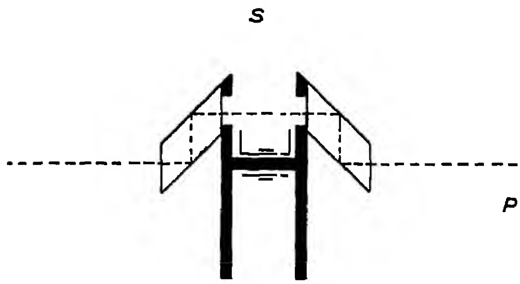

FIG 104 —The Brodhun Sector Disc

Although the sector disc is convenient in cases where a photometer bench cannot be used, and where an accuracy of 1 or 2 per cent is sufficient, its chief use in precision photometry is in combination with the bench, where its function is to reduce in a given ratio the illumination from sources of very high candle-power without the need for an absorbing wedge (73)

The best form of disc for this purpose is one with a fixed opening, such as that designed by Hyde for his work on Talbot's law, mentioned above (74). Each disc consists of a sheet of metal in which three or more apertures are cut, as shown in Fig 105. The edges of the apertures are very carefully cut and filed down so as to be exactly radial, particularly in the cases where n is small. The

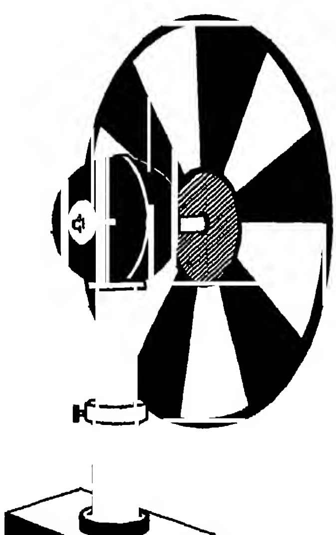

FIG 105 —The Sector Disc with Fixed Apertures

edge of the metal is bevelled on the side facing the photometer in order to avoid possible reflection from a flat edge Although the remainder of the disc is painted black, the paint is not carried over the edge, as otherwise the angle of opening is diminished A convenient set of discs is one containing the nominal ratios 1/5, 1/10, 1/20 and 1/50 The exact ratio for each disc is best obtained by direct calibration on the photometer bench, a lamp of convenient candle-power (say 100) being balanced against a comparison lamp both with and without the disc. If the distances of the lamp from the photometer in the two cases be d_1 and d_2 respectively, it is clear that the disc ratio is equal to $(d_1/d_2)^2$ For ratios below 1/10 it is generally best to calibrate "in cascade," using, say, the known 1/5 or 1/10 disc in the first photometric balance, and then substituting the disc of unknown ratio ([75]) It is necessary that the edges of the apertures should be strictly radial, otherwise the disc ratio will vary with the distance of the disc axis from the photometer bench axis.

The disc required for any particular test is clamped on to the end of a shaft driven by a small electric motor, as shown in Fig 105 Care must be taken to arrange the disc between the lamp and photometer so that the whole of the lamp may be seen from the photometer when an aperture passes, while, on the other hand, no part of the lamp is visible except through an aperture in the disc

It should be noted that a sector disc cannot, in general, be used in flicker photometry (see p 262).

Photometers depending on the use of Absorbing Media.—Yet another method of varying the illumination of a photometer comparison surface is to insert between it and the source of light a piece

FIG 106 —The Double-wedge Absorbing Filter

of transparent medium the transmission of which can be varied in a known manner ([76]) The simplest form of apparatus for achieving this consists of two thin wedges of neutral glass, or of lampblack in gelatine enclosed between glass plates, arranged as in Fig 106, so that one wedge can be moved over the other. The transmission factor thus remains uniform over the whole surface of overlap, but changes as one wedge is moved over the other, so that the thickness of the combination is altered It is clear that, if the angle ϕ of each wedge be very small, then a movement δl of one wedge over the other produces a change of $\phi\delta l$ in the thickness t If α be the specific absorption of the wedge material, the rate of change of transmission is $d\tau/dt = -\alpha\tau$ (see p 116) Hence $(1/\tau)(d\tau/dl) = \phi\alpha$

If α be 1 mm^{-1} (2·3 mm for 10 per cent), then $\phi\delta l = \delta\tau/\tau$, so that for a movement of 1 mm to produce a 1 per cent change in τ, ϕ must be 0·01 radian, or about half a degree Clearly, since $(1/\tau)(d\tau/dl)$ is a constant, the scale connecting l and τ is logarithmic, *i.e.*, it gives the same percentage accuracy throughout It must be remembered, however, that in practice the absolute accuracy will depend on (a) the constancy of α throughout the body of the filter, and (b) the constancy of ϕ, which is extremely difficult of attainment in such a very small angle. In practice it is generally found better

to calibrate any given pair of wedges by other photometric methods, and not to rely on the theoretical relation given above.

In cases where the accuracy required is not great, or the breadth of the beam of light which traverses the wedge is small, a single wedge of neutral glass may be used in combination with a wedge of clear glass, for in this case the variation of transmission over the length of wedge occupied by the beam may be neglected

Instead of neutral glass, a Goldberg wedge, made of gelatine containing a very fine lampblack in suspension, is often used [77]. A small amount of diffusion cannot be avoided in the case of this material [78], but it is more truly neutral than the most "neutral" glass at present obtainable [79]

Instead of a wedge, a strip of photographic plate may be used. The gradation of transmission may be obtained (i.) by exposing the plate through a Goldberg wedge, (ii.) by grading the time of exposure from one end to the other, or (iii.) by using a wedge-shaped diaphragm over a diffusing source of light, so as to produce a gradation of illumination from one end of the plate to the other [80]. The objection to this form of wedge is the diffusion of light inseparable from an exposed photographic film.

A layer of liquid, adjustable in depth, has also been used as a variable absorbing filter for photometric purposes [81].

A device in which the difficulties due to diffusion or to lack of neutrality are avoided altogether is that shown in Fig 107 [82] Two exactly similar gratings having, say, sixty lines to the inch, are clamped face to face at a distance apart approximately equal to the breadth of a line. The lines are opposite to each other, so that the transmission factor of the double grating for light passing through it normally is 50 per cent., if the lines and spaces are equal in breadth (neglecting the losses by reflection at the glass surfaces) As the double grating is rotated about an axis parallel to the lines the transmission factor decreases to zero, and by calibration the relation between angle and transmission may be accurately determined. The great advantage of this form of filter is that it is strictly neutral as regards colour of the transmitted light, and that, once calibrated, it may be relied upon to retain its calibration The dimensions given above may be varied in any direction, subject to the limitations that (a) the lines must not be so fine as to produce any noticeable diffraction; (b) they must not be so coarse as to produce any noticeable pattern on the photometer screen It will be seen that increasing the separation of the gratings gives a closer scale, *i.e*, $d\theta/d\tau$ becomes less This filter is a modification of an earlier form, in which one of the gratings was moved across the other so that the lines of one grating gradually overlapped the spaces of the other [83].

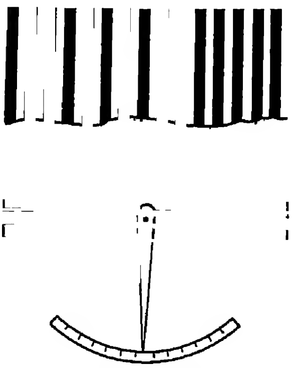

FIG. 107—The Variable Neutral Absorbing Filter

Even the best variable absorbing filter is but a poor substitute for a photometer bench, and, just as in the case of the sector disc,

the chief field of usefulness for an absorbing filter in photometry is in conjunction with a photometer bench, where a filter of fixed and accurately known transmission may be used to reduce the candle-power of a high intensity source so as to bring it within a range convenient for comparison with the sub-standards available ([84]). A coloured filter may be used to alter the spectral distribution of the light from the source, but this will be dealt with later in Chapter VIII. For the purposes considered in this chapter a filter should be as neutral as possible, *i e*, its transmission factor should be the same for light of all frequencies. It should also be truly transparent, *i e.*, it should not diffuse any of the light passing through it Neither of these conditions is fulfilled in practice by any homogeneous absorbing filter. So-called "neutral" glasses nearly always have a higher transmission factor in the extreme red than elsewhere in the spectrum. This is clearly seen when several are placed together for the purpose of looking at a bright source, such as the sun.

Instead of reduction by transmission, reduction by reflection may be used, the light from the high intensity source being reflected once or several times from a polished surface of black glass ([85])

The effect of diffusion in an absorbing filter may be minimised by placing the filter close to and at a fixed distance from either the photometer or the lamp It should *not* be so placed between the photometer and the lamp that its distance from either or both is liable to change, since the light which is diffused by the filter acts as if it emanated from the filter as a source The necessity for the fulfilment of these conditions may be shown theoretically as follows. Let τ and τ' be respectively the "regular" and "diffuse" transmission factors of the filter, and let the distances of the filter and of the photometer surface from the source be respectively x and d The illumination at the filter is I/x^2 That at the photometer is $\tau I/d^2+(I/x^2)\tau' f/\pi(d-x)^2$, where f is the area of the filter This equals

$$(I/d^2)[\tau + \tau' f d^2/\pi x^2 (d - x)^2].$$

Now if $(d - x)$ be small compared with d, x is nearly equal to d, and the right-hand term of the expression in the brackets reduces to $\tau' f/\pi(d - x)^2$, which is constant as long as $(d - x)$ is constant. Similarly, if x be small compared with d, this term becomes $\tau' f/\pi x^2$, which is constant as long as x is constant Thus constancy of effective transmission can only be obtained when the filter is close to and at an invariable distance from either the source or the surface it illuminates

Absorbing filters may be used in two ways A single plane filter may be used to reduce the illumination from a source of very high candle-power so that it may conveniently be balanced against a comparison lamp of normal candle-power, and it may also be used on the comparison lamp side of the photometer when very small sources are being measured In any case, the transmission factor must be known for light of the same spectral distribution as that given by the source with which it is to be used (see p 249) It is convenient in the photometric laboratory to have a set of neutral filters of this kind with transmission factors of the order of 10, 5 and 2 per cent. These will enable sources up to about 2,000 candles to be measured with a bench distance of 2 metres when the standard

illumination at the photometer is 10 metre-candles. The transmission factors of such filters must be accurately determined, either at a standardising laboratory or by using a high candle-power source of the same kind as that with which the filters are to be used For instance, if they are required for work with gas-filled lamps operating at an efficiency of about 15 lumens per watt, then such a lamp, with a candle-power of about 250 candles, may be used on the test lamp end of the photometer bench, while a similar lamp of about 30 candles will serve as a comparison lamp, and, if used at a distance of about 1 metre from the photometer, will give a comparison illumination of about 30 m c It will be noticed that the exact value of this illumination does not matter. With no filter on the test lamp side, the reading of the photometer at the balance point will be d, about 3 metres. With the 10 per cent. filter in place the reading d' will be about 1 metre. The transmission factor of the filter will clearly be $(d'/d)^2$ The other filters may now be measured in a similar way, either directly or "in cascade," by using the already determined factor of the 10 per cent filter, and placing this filter on the comparison lamp side, when the filter being measured is on the test lamp side. The correction for the thickness of the photometer screen should be made as described on p. 163 The range of use of the Weber photometer, and of many portable illumination photometers, is increased by inserting neutral filters in such a position that the apparent brightness of M_2 is reduced and a higher illumination can be balanced with the same comparison lamp The instrumental constant must be determined with each glass or set of glasses in position

It is usually safe to assume that a lampblack-gelatine neutral filter is sufficiently neutral for its transmission factor to remain quite constant over the whole range of colour given by electric lamps, from ordinary vacuum lamp efficiency (6·7 lumens per watt) upwards The value determined with such sources should not, however, be assumed to hold with great accuracy when the filter is being used with such sources as the electric arc, incandescent gas mantles, the mercury vapour lamp, or the yellower flame sources The transmission factor of filters for use with sources of different colours will be referred to again in Chapter VIII

Whenever filters of appreciable thickness are used in photometry, it should be remembered that, owing to refraction, the effective distance of the light source from the photometer screen is reduced by the quantity $(n - 1)t/n$, where t is the thickness of the screen and n its refractive index (see p 23) Thus for a screen of glass ($n = 1{\cdot}5$) of 3 mm thickness the measured distances must be reduced by 1 mm. in order to find the true candle-power of the source

Other Methods of Varying the Photometer Illumination.—Methods of photometry have at various times been based on a large number of devices, other than those enumerated above, for altering the brightness of the photometer comparison surfaces according to a known law [86] In some forms of photometer the variation of brightness according to the cosine law of illumination has been used [87] This method is still employed in certain portable photometers for measuring illumination (see p. 350) Since for every

known surface the reflection factor varies with the direction of the incident light, the cosine law cannot be assumed to hold accurately, and the instrument therefore requires calibration by a more fundamental form of photometer

Another method, depending on the use of a diaphragm of variable opening, is also used in some portable illumination photometers (see p 353) A source of light is placed behind a translucent screen, which illuminates one surface of the photometer head The illumination is directly proportional to the area of a diaphragm which may be placed (*a*) directly in front of a translucent surface acting as the effective source ([88]), (*b*) close to a lens which forms an image on the photometer surface ([89]), or (*c*) close to a lens which serves as

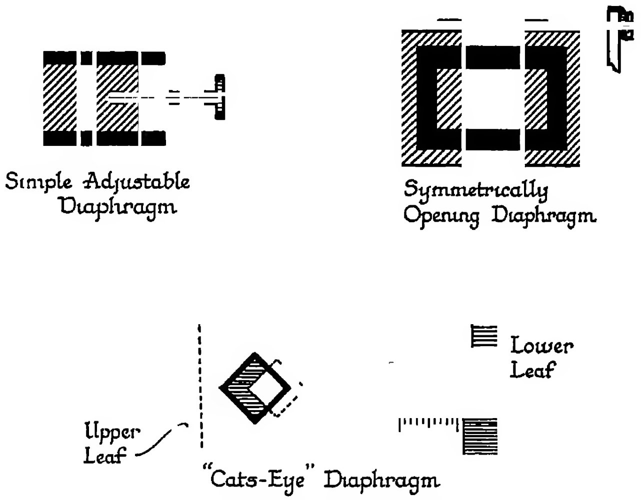

FIG 108 —Forms of Variable Diaphragm

an objective to an ocular system with a small artificial pupil ([90]), so that the brightness of the image of the translucent surface seen by the eye varies as the diaphragm area (see p 109) Forms of diaphragm which have been used are shown in Fig. 108

A negative lens may be used to reduce the illumination of a screen placed behind it, for if L be the source (Fig 109), P the lens of focal length f, and S the screen, the light flux, which without the lens would occupy an area on the screen equal to $\pi h^2(d + l)^2/d^2$, actually occupies an area $\pi h^2(x + l)^2/x^2$ where $1/x - 1/d = 1/f$ (see p 22), so that the ratio of reduction of illumination is

$$\left[\frac{(d + l)x}{d(x + l)}\right]^2 = \left[1 + \frac{dl}{f(d + l)}\right]^{-2}$$

on account of lens action This ratio must be further reduced on account of losses due to reflection at the glass surfaces and absorption within the substance of the lens ([91]). This effect has been the basis of a so-called " dispersion " photometer in which the brightness of the screen is varied by changing the position of a negative lens placed between it and the source to be measured ([92]).

Photometers have been designed in which the light from the two comparison surfaces is caused to traverse two prisms of glass separated at the interface by a very thin air film (see p 26). The interference bands formed by the transmitted light from one comparison surface are superposed on those formed by the reflected light from the other surface, so that the light bands of one set coincide

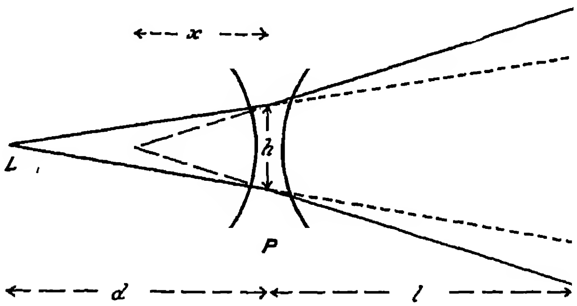

FIG 109 —The Reduction of Illumination by a Negative Lens

with the dark bands of the other set. When both sets are of equal intensity the field of view appears uniformly bright ([93])

An optical effect which it has lately been proposed to apply to photometric measurement is the apparent curvature of the path of an object which is really moving in a straight line, but which is unequally illuminated on the two sides. The effect is due to the relation between the brightness of an object and the lag in the visual impression produced by that object ([94]) Instruments based on this effect have been described by C. Pulfrich ([95]).

Instantaneous Candle-Power.—A special problem in photometry is the measurement of candle-power at any given instant in the case of a source which is undergoing rapid periodic or continuous changes, as, for example, an electric incandescent filament lamp supplied with alternating current of comparatively low periodicity, or the same lamp during the period immediately following the starting or stopping of the current.

In the case of a periodic change of which the frequency is above the critical frequency of the eye (see p. 62), a sector disc synchronised with the electric supply may be interposed between the test lamp and the photometer ([96]). Photometric readings are then taken in the ordinary way, allowance being made for the transmission of the sector disc Since the disc is synchronised with the candle-power fluctuations, it follows that light reaches the photometer from the test lamp only during a brief interval of time, and always at the same part of a cycle. By changing the angular position of the disc opening, the candle-power can be measured at each point on the cycle

In the case of frequencies below that at which flicker appears, it is necessary either to use a similar disc on both sides of the photometer or to place the disc between the photometer and the observer's eye. In the latter case a rapid observation of the photometer field is made at each exposure, and the position of balance is attained by

a process of successive approximation. A similar method may be used in the case of a continuous change, a shutter being placed in front of the photometer so that the field is visible only at the required instant in the course of the change ([97]). This type of measurement is necessarily tedious, since the whole change has to be gone through for every attempt required to obtain a single observation.

When the changes of candle-power are not too rapid, approximate measurements may be made with an illumination gauge such as that described on p 354, or any convenient modification of it ([98])

Relative Merits of Photometers.—A discussion of the advantages and disadvantages of the many different types of photometers that have been devised at various times would serve no useful purpose in this book

Several investigations have been made with the object of finding the relative sensitivity of different types of photometer head ([99]). It seems to be the general conclusion that for photometry in which there is little or no colour difference between the sources being compared the Lummer-Brodhun contrast field gives the best results, although the Bunsen, if properly designed and used, may be almost as good When there is a marked colour difference, however, the results are less definite (see p. 262) The degree of illumination of the field of view has a considerable influence on the sensitivity of the eye to brightness contrast (see p. 52), and it is therefore only to be expected that it will also affect the precision of photometric measurements The curve of Fig. 110 shows the variation of precision with

FIG 110 —The Dependence of Photometric Precision on Brightness (Uppenborn).

field illumination found by F Uppenborn ([100]) In cases where the comparison field is very small (of the order of 2° to 4°) the use of an evenly illuminated background having approximately the same brightness as the comparison field has been found to improve the accuracy of the settings ([101]) Other factors affecting the precision of photometric measurement are mentioned in Chapter XI

It has been intended in the descriptions given above to indicate briefly the particular merits of each instrument and the scope of the work for which it is best suited. Extreme accuracy can never

be combined with great simplicity of construction, with portability, or with cheapness. On the other hand, one or more of these latter characteristics may, in some problems, be more important than the last ¼ per cent in accuracy. For work of the highest precision a photometer bench, Lummer-Brodhun contrast type head, substandards obtained from a standardising laboratory, and the best possible electrical equipment must be used Nearly all the so-called "portable" photometers have to be calibrated with such an arrangement, but if carefully treated after calibration the best can be relied upon, generally, to 1 or 2 per cent., and are often convenient for use in positions where it would be impossible to set up a bench.

The peculiarities of observers and the effects of experience, personal health, *etc.*, which are unavoidable in physiological photometry ([102]), will be mentioned in a later chapter of this book (see p 315) Here all that can be said is that progress in photometric accuracy seems to lie in the direction of using some criterion of equality of brightness other than absence of contrast or equality of opposite contrasts Probably some combination of physical with physiological photometry will prove the best arrangement if an accuracy superior to 0·1 per cent. is to be achieved

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(13) See p. 2

(14) L Wild and others Electrician, 57, *1906*, pp 529 *et seq* The above statement assumes that the substance forming the photometer surface behaves as a perfect diffuser, but A P Trotter has pointed out (Electrician, 57, *1906*, p 627, "Illumination, etc," p 95) that in the case of dulled Bristol board, owing to the peculiar nature of the diffusion curve for this surface, the angle error in a Ritchie wedge on which the light is incident at an angle of 35° is less than one half of 1 per cent per degree tilt The error is avoided altogether in a form of photometer described by P Yvon (C R 75, *1872*, p 1102, J of Gas Lighting, 22, *1873*, p 60, Pogg Ann, 148, *1873*, p 334) in which the lamps are placed so that each illuminates its respective side of the wedge perpendicularly This arrangement, however, clearly cannot be adopted on a straight bench

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(29) Instead of a double prism a sheet of glass silvered over portions of its surface has been used. See L Wild, Illum Eng., 3, *1910*, p 303, and H P Gage, Phys Rev, 32, *1911*, p 627; Illum Eng Soc N Y., Trans, 19, *1924*, p 508 See also H. W Dove, Berlin Monatsber, *1861*, p 483; Pogg Ann., 114, *1861*, p 145, Verein z Beförderung d Gewerbefleisses in Preuss, Verh, 6, *1861*, p 171, Dingler's Polytechn J, 162, *1861*, p 154, Z f Chem u Pharm, 5, *1862*, p 64, Z f d gesamte Naturwiss, 19, *1862*, p 453, Am J Sci, 33, *1862*, p 269, Presse Scientifique, *1862* (I), p. 224, Phil. Mag, 25, *1863*, p 14, F Géraldy, Lum El, 1, *1879*, p 64 Also A H Pfund, Johns Hopkins Univ Circular No 186, *1906*, p 20, and C S M Pouillet, C R, 35, *1852*, p 373, L'Institut, 20, *1852*, p 301, Pogg Ann, 87, *1852*, p 490, Cosmos, 1, *1852*, p 546

(30) This consists of a matt surface of black powder dusted on to a coating of black enamel One such optical black is manufactured by Messrs Johnson and Sons, of Cross Street, Finsbury, London

(31) J G W., 37, *1894*, p. 61, and 39, *1896*, p 265, L'Industrie El, 20, *1911*, p 272

(32) H Krüss Z f I, 30, *1910*, p 329, Z f Bel, 18, *1912*, p 437

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(33) O. Lummer and E Brodhun Z f I, 9, *1889*, p 461, J G W, 32, *1889*, p 767

(34) A simple form of contrast photometer has been designed by H Krüss, J G W, 54, *1911*, p 121, Z f I, 31, *1911*, p 203, Z f. Bel, 18, *1912*, p 437

(35) Z f I, 9, *1889*, p 461.

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(36) E P Hyde and F E Cady Opt Soc Am, J, 6, *1922*, p 615

(37) A H Pfund Phys Rev, 4, *1914*, p 477, Z f I, 35, *1915*, p 84

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(38) Deut Phys Gesell, Verh, 1, *1899*, p 278, J G W, 43, *1900*, p 250.

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(40) E Liebenthal "Prakt Phot," p 191

(41) See W M Stine, "Photometrical Measurements," p 61

(42) J Joly. Roy Dublin Soc, Proc, 4, *1884*, p 345, Nature, 31, *1885*, p 330, and 32, *1885*, p 537, B A Report, *1885*, p 917, Engineering, 41, *1886*, p 530, Nature, 34, *1886*, p 48; J of Gas Lighting, 46, *1885*, p 837, Phil. Mag, 26, *1888*, p. 26, Lum El, 29, *1888*, p 238, Nature, 48, *1893*, p 269

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(43) See, for example, E T Z, 18, *1897*, p 473, Electrician, 41, *1898*, p 426; L Weber, Wied Ann, 31, *1887*, p 676, A J Rowland, Frank Inst., J, 148, *1899*, p 376, Electrician, 44, *1899*, p 110, Der Mechaniker, 8, *1900*, p 2

(44) J G Clark J of Gas Lighting, 92, *1905*, p 826

(45) In the rare cases when a substitution method cannot be used, lack of symmetry must be compensated by taking the geometric mean of the values given by two sets of readings, one with the photometer head in its normal position and one with it reversed (see p 152)

(46) For a graph of the function $x^2/(d-x)^2$ see Appendix XI, p 475

(47) See, *e g.*, the description of the Joly head, p 160, also E W Lehmann, Wied. Ann, 49, *1893*, p 672, Z f I, 14, *1894*, p 101, J G W, 37, *1894*, p 106, E T.Z, 14, *1893*, p 407; Lum El, 49, *1893*, p 385, El World, 22, *1893*, p. 118, J de Phys, 3, *1894*, p 186, Z f. Psychol u Physiol d. Sinnesorgane, 5, *1893*, p. 347

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(48) See p 164

(49) Alternatively, a recording device may be attached to the photometer carriage so that at the conclusion of each setting the observer himself may, by depressing a key, cause an electrically operated pen to mark a dot on a paper scale carried on a cylinder attached to the bench See A P Trotter, "Illumination, etc," p 132, C P Matthews, Phys Rev, 7, *1898*, p 239, Am I E E, Trans, 15, *1898*, p 579, E B Rosa and G. W. Middlekauff, Am I E E, Proc, 29, *1910*, p 1191, Bureau of Standards, Bull, 7, *1911*, p 11, El World, 56, *1910*, p 152, Phys Rev, 31, *1911*, p 311, Z f Bel, 17, *1911*, p 163

(50) If a candle-power scale be used on the bench instead of the ordinary millimetre scale, this correction may conveniently be made by means of a device described by G. W Middlekauff (El World, 56, *1910*, p 152, Bureau of Standards, Bull, 7, *1911*, p 11, Phys Rev, 31, *1910*, p 311, E B Rosa and G W Middlekauff, Am I E E, Proc., 29, *1910*, p 1191)

(51) C H Sharp "Lectures on Illum Engineering" (Johns Hopkins Univ Press, *1911*), p 454

(52) For the effect of pressure on the c p of a flame, see, *e g*, L W Wild, J of Gas Lighting, 98, *1907*, p 24

For the effect on a mantle, see p 444 and references in notes (20) and (21), p 456

(53) For the effect on a flame, see, *e g*, L W Wild, J of Gas Lighting, 111, *1910*, p 254

For the effect on a mantle, see p 444 and references in note (22), p 456

(54) A temperature of 75° F. has been recommended as a standard by a committee of the American Gas Institute (Proc, 2, *1907*, p 454) C O Bond, Am Gas Inst, Proc, 7, *1912*, p. 291, Am Gas Light J, 97, *1912*, pp 418 and 420, J of Gas Lighting, 121, *1913*, p 190) has found that increase of humidity above normal (taken as 8 litres of water vapour per cubic metre of air) results in a decrease of candle-power as follows —

Inverted lamp, inner cylinder	0 785 per cent per litre
Upright lamp with straight chimney . .	0 597 ,, ,,
Upright lamp with air-hole . .	0 407 ,, ,,

(55) H E Ives and L R Woodhull Bureau of Standards, Bull, 5, *1909*, p 555

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(57) See, for example, A Poppe, Frankfort Phys Verein, Jahresber, *1857–8*, p 74; Polytechn Centralbl, 25, *1859*, col 600, Dingler's Polytechn J., 151, *1859*, p 432

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C Deshler and A. S McAllister El World, 33, *1899*, p 386, Brit Patent No. 17,945 (*1900*)

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(58) Schlesische Gesell f vaterl Cultur, Jahresber, 62, *1884*, p 241, Z f I, 4, *1884*, p 257, Wied Ann, 20, *1883*, p 326, and 26, *1885*, p 374, J G W, 28, *1885*, p 267, Dingler's Polytechn J, 254, *1884*, p 122, Central-Ztg f Opt u Mech, 4, *1883*, pp 181 and 194, E T Z, 5, *1884*, p 166, Lum El, 12, *1884*, p 468, J de Phys, 3, *1884*, p 143, J of Gas Lighting, 45, *1885*, p 766, and 46, *1885*, p 238, D R Patent No. 26,196 (*1883*), Z f Elektrot, 4, *1886*, p 118

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(60) For references to the many different forms of polarisation photometers, see notes (21) to (33), p 11

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(72) E Brodhun Z f I, 14, *1894*, p 310, 15, *1895*, p 337, 27, *1907*, p 8

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(73) F Exner Der Gastechniker, 7, *1887*, p 49, Central-Ztg f Opt u Mech, 7, *1886*, p 266

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(The name in square brackets is that of the photometer found to give the greatest precision *at colour match*)

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workers [4] It has the great disadvantage that the light is not incident normally on the photometer screen, so that a very slight inaccuracy in the setting of P produces large errors in the measured candle-power (see p 164).

Another method is that involving the use of a movable mirror system. When only one mirror is used, it may be arranged to occupy

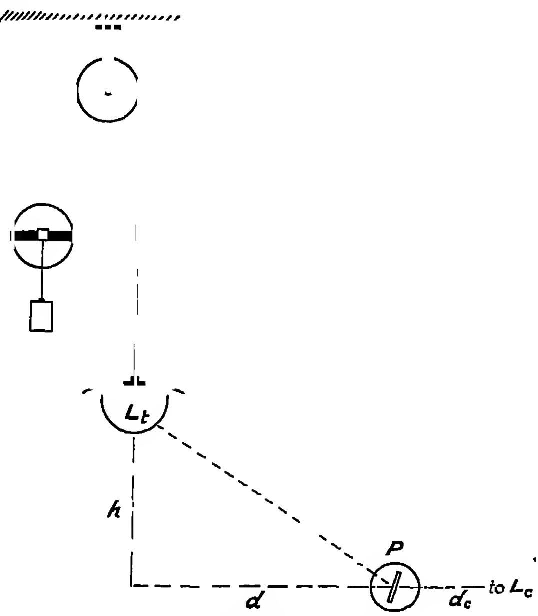

FIG. 112 —Hartley's Photometer.

the position of P in Fig 112, and to move about a horizontal axis so as to deflect the light from the source along the axis of the photometer bench [5]. The reflection factor of the mirror cannot, however, be assumed to be constant at all angles of incidence [6], and a better scheme is that shown diagrammatically in Fig. 113 [7] The source to be measured remains fixed at L_T, while the mirror M is carried on a rigid arm by means of which it can be moved round a horizontal axis passing through the centre of L_T This axis coincides with the axis of the photometer bench, passing through the photometer and the comparison lamp The inclination of M to this axis is capable of adjustment so that the light from L_T can be directed to the photometer when the latter is at different distances. Very frequently two mirrors are used, one on each side of L_T [8].

For sources of small dimensions apparatus which can be accommodated on a portable framework, or even on a photometer bench, has been designed [9].

The chief disadvantage of this method is the fact that the light from M reaches the photometer obliquely, so that as M is moved round the error mentioned in Chapter VI (p 164) is introduced This defect may be avoided by causing the light to suffer two [10] or three [11] reflections before reaching the photometer (see Fig 114), but since each mirror must be larger than the light source to be

measured, apparatus of this kind becomes very heavy and cumbersome in the case of large sources

The most satisfactory device is that in which the source and mirror of Fig 113 are interchanged, so that the source moves in a

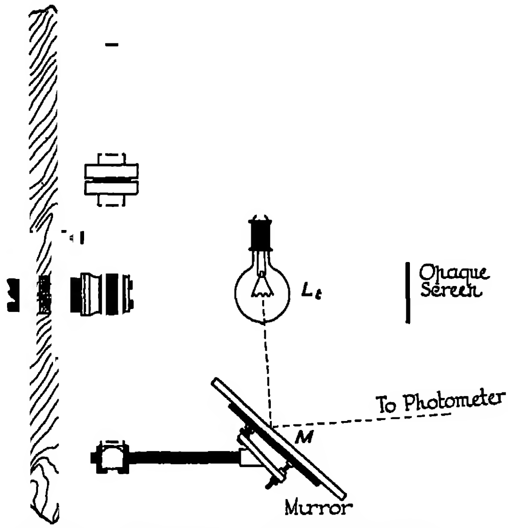

Fig 113 —Radial Mirror for Polar Curve Measurement

vertical circle around the axis of the photometer bench, while the mirror, fixed at an angle of 45° to this axis, rotates so as always to face the source [12]. The light is then incident on the mirror at a constant angle of 45°, and is reflected along the axis of the bench.

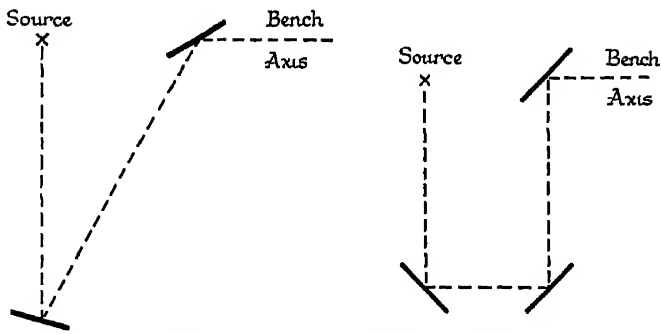

Fig 114 —Mirror Apparatus for Polar Curve Measurement

The apparatus may take the form shown in Fig 115, which gives a view from the position of the photometer Fixed to a strong upright is a toothed wheel W_1, and one race of a ball and roller bearing B of $3\frac{1}{2}$ inches diameter The other race of this bearing carries a brass disc D of 14 inches diameter, to which is rigidly attached the steel tube G and a fitting for the mirror M At one end of G is a collar for the axle of a second toothed wheel W_2, of the same size as W_1 and in gear with it by means of a third wheel W_3 (or a chain

may be used instead) The forward part of this axle carries a swan-neck bracket, on which is supported the light source ([13]) The counterweight Z can be adjusted on G so that the whole system ZGW_2 can be moved about B with very little effort It can be clamped at any angle which is an even multiple of 5° by means of a pin engaging in one of the outer ring of holes seen near the outer edge of the brass disc D. The pin is carried on a long rigid arm (not

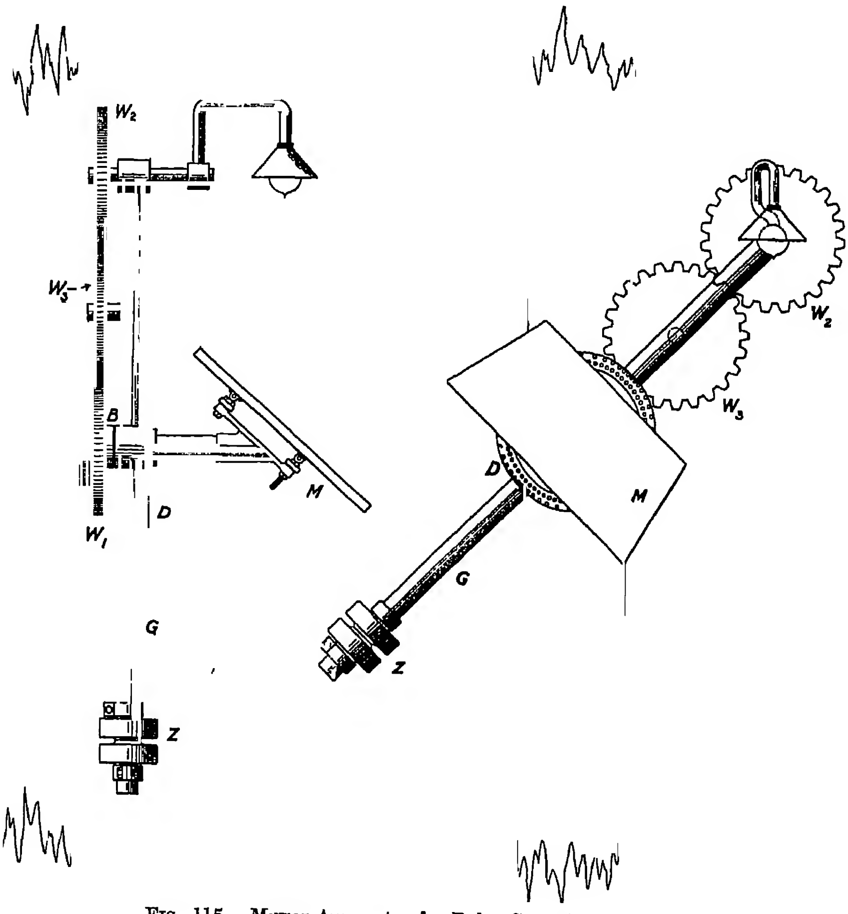

FIG. 115 —Mirror Apparatus for Polar Curve Measurement

shown) which swings from a fixed point above the apparatus A second row of holes in D enables G to be set at either of the Russell angles (see p 93) The angle which M makes with the axis of W_1 (which is also the axis of the photometer bench) is normally 45°, but it is capable of adjustment to allow for cases in which the source projects unusually far in front of G. The dimensions of M must be sufficiently great to accommodate the largest source for which the apparatus is likely to be required, and it is clear that when measurements are being made no part of the effective light source, including any reflector or other surface contributing to the total light, may

be cut off by the mirror. This is tested by viewing the image of the source in the mirror from the position of the comparison-surface in the photometer For sources such as street lighting fittings or large semi-indirect bowls with reflectors, a mirror measuring 36 × 24 inches is required.

Where sources of high candle-power have to be measured it is necessary to place the photometer at a considerable distance from the source, and on account of the large range of candle-powers met with in practice it is convenient to have, instead of a very long photometer bench, a short bench of about 2,500 mm in length mounted on a table provided with small wheels which move along a track in the floor A pointer on the table leg, and a series of marks at 1 metre intervals on the floor, enable the bench to be fixed at any convenient position for the particular magnitude of candle-power being measured at any time.

The candle-power range covered by a single lamp in different directions is often considerable, and in order to avoid the necessity for moving the table too frequently it is often convenient to clamp the photometer at the zero of the bench and move the comparison lamp If the fixed distance of the comparison lamp, found by standardisation in the usual manner (see p. 166), be d_C, while the distance of the photometer from the light source *viâ* the mirror is d_T, then the candle-power corresponding to a position x of the comparison lamp is $(d_T^2/\rho)(d_C/x)^2 \times 10^{-5}$, where ρ is the reflection factor of M [14] ρ may be determined most conveniently either by placing G in the horizontal position and using a lamp of which the horizontal candle-power in the direction facing the mirror is known, or by making a measurement of the horizontal candle-power of the source in the ordinary way, the azimuth being the same as that facing M when the distribution was determined In either case ρ is equal to the ratio (candle-power measured *viâ* M)/(true candle-power)

The constant distance d_T is measured by means of a steel tape or otherwise, as follows one observer places his eye in the position of the photometer head and directs a second observer to make a mark on the mirror M in the position of the photometric centre of the lamp image which he sees. The distances from this mark to the centre of the photometer and to the lamp centre are added together, and this sum, increased by 1 5 times the thickness of the mirror glass, is the value of d_T required [15] For a determination of the candle-power distribution in a vertical plane, measurements are made with the mirror M at angular intervals of 10° for most sources of fairly uniform distribution In some sources rapid changes of candle-power may take place near the axis (0° or 180°) or in the horizontal direction It is then necessary to reduce the intervals between the readings in these regions

While measurements are being made the photometer is screened from the direct light from the source by means of a large black screen in the form of an annulus, the centre aperture in which is sufficiently large to avoid any interference with the light reaching the photometer from M All metal and wooden parts in the apparatus itself must either be painted a dull black or covered with black cloth while measurements are in progress

It will have been noticed that one set of measurements only gives

the distribution curve in a single vertical plane. Very often this is sufficient to give the information required, since for many sources the distribution in every vertical plane may be assumed to be the same to the order of accuracy desired A vacuum electric glow lamp may be rotated about a vertical axis so that the mean distribution curve may be obtained with one set of measurements, and in this case a small electric motor is fixed to the fitting carried on W_2, while the lamp is fixed in a rotator similar to one of those described later in this chapter (p. 202) Most sources, however, cannot be rotated (16), and a really accurate knowledge of the candle-power distribution in all directions in space can only be obtained by means of some eighteen sets of measurements with the source at every 10° of azimuth This is the most fundamental method of obtaining the mean spherical candle-power (m s c.p) of a source (17).

Methods Involving the Use of Physical Photometers or Illumination Photometers.—It is clear that candle-power distribution can be measured without mirrors by means of some form of physical photometer, such as the photo-electric cell (see Chapter XI , especially note (90), p 339 The possibilities of this method have not yet been investigated thoroughly.

For approximate work a portable illumination photometer with detached test plate may be used (see Chapter XII). The plate is mounted on a light arm so that it can be moved in a circular track having the source as centre The illumination of the test plate at any position clearly gives the corresponding candle-power of the source (18) When this method is used it is necessary to have the arm which carries the test plate of such a length that the inverse square law may safely be applied to calculate the candle-power of the source from the illumination of the plate (see pp 102 *et seq*).

Candle-Power Distribution : Measurement of Unsteady Sources.—In the above description it has been assumed that the source remains constant in candle-power, to the accuracy of the experiment, during the whole set of measurements In the case of some sources, such as the electric arc, however, this assumption cannot be made, and it is therefore desirable to have some means of comparing the candle-power in any direction with that in a fixed reference direction In this way a true representation of the distribution can be obtained while the absolute value of candle-power is variable from minute to minute

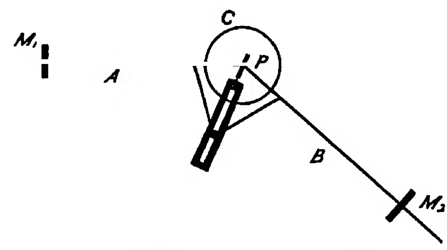

FIG 116 —Rousseau's Apparatus for Measuring Unsteady Sources

One form of apparatus for measurement of this kind is shown in Fig. 116, where M_1 and M_2 are two exactly similar mirrors, movable along the arms A and B. A remains horizontal, while B moves round in a vertical plane. A link motion maintains the photometer screen P in the plane bisecting the angle between A and B. C is a graduated circle, behind which is placed the source of light at a distance which is small compared with M_1P and M_2P Photometric balance is obtained with M_2 at any angle,

by moving M_2 along B Thus $I_\theta/(M_2P)^2 = I_H/(M_1P)^2$, where I_θ is the candle-power of the source in the direction PM_2 and I_H is the horizontal candle-power at the same instant [19]

Fleming has used the apparatus shown diagrammatically in Fig 117 [20]. The light from L in the direction θ reaches the photometer P by way of the pair of mirrors M_1 and M_2, which can be moved round the axis LP, and the stationary mirror M_3. The light

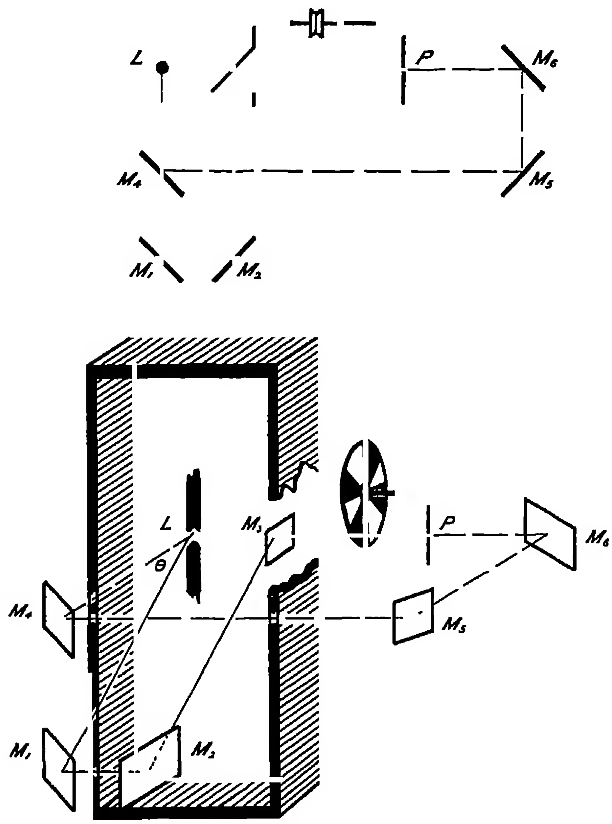

FIG 117 —Polar Curve Measurements on Unsteady Sources

from L in the horizontal direction reaches P by way of M_4, M_5 and M_6 The mirrors are so arranged that the length of path traversed by the light is the same in both cases. The photometric balance may be obtained by means of a sector disc or variable absorbing filter placed in the path of one beam, or comparison may be made with a polarisation photometer, if the polarisation due to the reflections in the mirrors be destroyed I_θ/I_H may thus be obtained.

Mean Candle-Power Determinations.—Where the actual distribution of candle-power is known, or is of little importance owing to the use of reflecting devices, the performance of a source may be judged from the candle-power in a certain specified direction, or from the

total output of light In the former case, either the horizontal or the downward vertical direction is generally chosen [21] The fitting shown in Fig. 118 may be employed for the vertical (axial candle-power) measurement The mirror M rotates about a line passing through its silvered surface, and the scales A and B are graduated from this line as zero, so that by sighting the lamp filament between the scales its exact distance from the mirror can be found

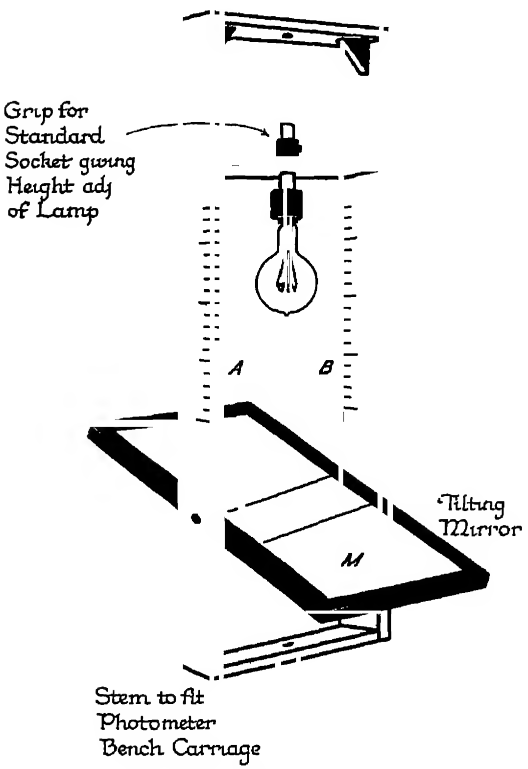

FIG 118 —The Measurement of Axial Candle-power

When the horizontal candle-power basis is adopted, the mean value of the luminous intensity in all directions lying in the horizontal plane is usually measured, rather than the value in a single direction [22] This mean value, termed the "mean horizontal candle-power," is easily found from the horizontal polar curve, determined by the point to point method described on p 194 [23]. In the case of sources such as a vacuum electric lamp, however, it can be more simply obtained from a single measurement made with the lamp rotating about its axis sufficiently fast to avoid flicker in the photometer head [24]. A speed of about 120 to 180 revolutions per minute is usually sufficient [25] The form of rotator designed by C C Paterson, and used at the National Physical Laboratory [26], is shown in section in Fig. 119. It consists essentially of two parts, one fixed rigidly to the stand, the other capable of rotation and carrying the lamp A is an ebonite disc bearing two concentric copper rings, which are respectively connected to the terminals T, T. From these terminals flexible leads are conveyed inside the vertical tube B to the lamp holder Rigidly attached to this tube is a solid brass disc D, which is friction-driven by a small wheel driven through a shaft and pulley system from a small electric motor C is a second circular ebonite block bearing two annular grooves concentric with the copper rings of A, and half filled with mercury, so that these rings can rotate freely with about $\frac{1}{4}$ inch of their lower edges dipping into the mercury. The mercury in these two grooves is connected to four terminals S, S, to which are attached the current supply leads and voltage measuring leads from the bench With this

apparatus, if the mercury troughs and copper rings be kept scrupulously clean, there is generally no difficulty in maintaining the current through the lamp perfectly steady, always a somewhat difficult problem when measurements have to be made through moving contacts * If the efficiency of the contacts be suspected, the values of current and potential obtained with the lamp in the rotator are compared with those found when the lamp is in a standard socket (see p 438) Differences between the values obtained with the lamp rotating and the lamp stationary are sometimes found to be due to imperfections in the internal construction of the lamp, such as a very slight looseness of contact at the pinch where the filament is joined to the leading-in wire It is clear that, instead of rotating the lamp, it is possible to use a pair of mirrors arranged as in Fig. 120, and rotated about the lamp axis ([27]). This method avoids any difficulties due to moving contacts or to distortion of the filament by centrifugal force ([28]), but the source is not measured in its normal burning position

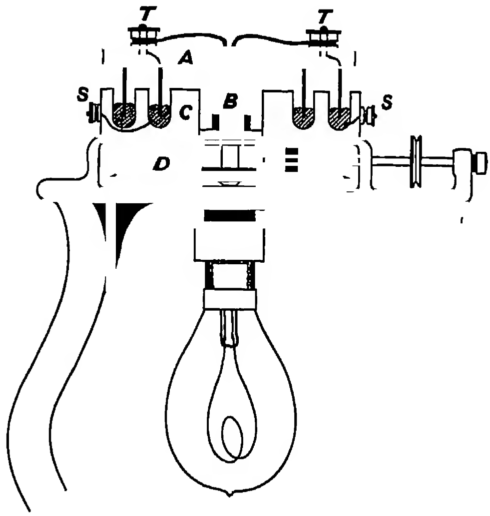

FIG 119 —The Paterson Lamp Rotator

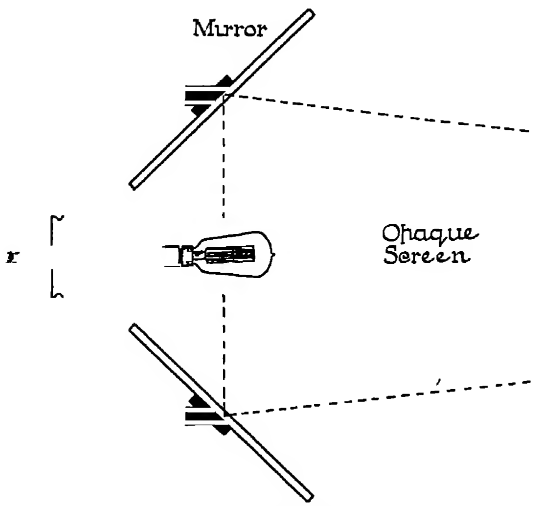

FIG 120 —The Measurement of Mean Horizontal Candle-power

Total Flux Measurement.—The modern development of light sources in which the distribution of candle-power conforms to no particular standard, and the increasing use of reflectors and other auxiliary devices to modify the distribution of the light as emitted by the actual

* Although the description given above applies to a rotator designed for the measurement of lamps in the pendent position, it is clear that an exactly similar instrument may be used for lamps measured in the upright position

source, have led to the increasing disuse of the mean horizontal candle-power as a basis for the rating of illuminants. Instead, it is now becoming customary to rate all sources in terms of their total output of luminous flux or, what comes to the same thing, the average value of the candle-power when measured in all directions in space ([29]), for (see p 87) this average candle-power, often called the " mean spherical candle-power," is numerically equal to the total flux from the source (expressed in lumens) divided by 4π This is, moreover, the only possible basis for rating very unsymmetrical sources ([30]) For certain special purposes the average value of the candle-power in the upper or lower hemisphere, or within a specified zonal region, may be required It has been shown in Chapter IV. (p 91) how this information can be obtained from the polar curve of light distribution in a vertical plane, assuming the distribution to be the same in every such plane ([31])

In cases where the light source can be rotated this assumption as to symmetry of distribution about the axis need not be made, for, by the use of apparatus in which a rotating device is mounted on a tilting framework of the kind illustrated in Fig 111, the mean polar curve can be obtained from a single set of measurements covering 180° only ([32]). Alternatively, the rotating device may be mounted with its axis horizontal, and this axis may then be moved in the horizontal plane ([33]). In either case it is desirable that the part of the rotating device which is actually fixed to the axial rod bearing the lamp shall be as small as possible, so as not to obscure light from the lamp in nearly axial (cap end) directions. Ideally the pulley, *etc*, used for the purpose should not be larger than the lamp cap. Sharp has described ([34]) a form of apparatus in which the lamp is rotated about a vertical axis, and is measured by means of a double three-mirror system carried on a light framework which is movable about the horizontal axis (Fig 121)

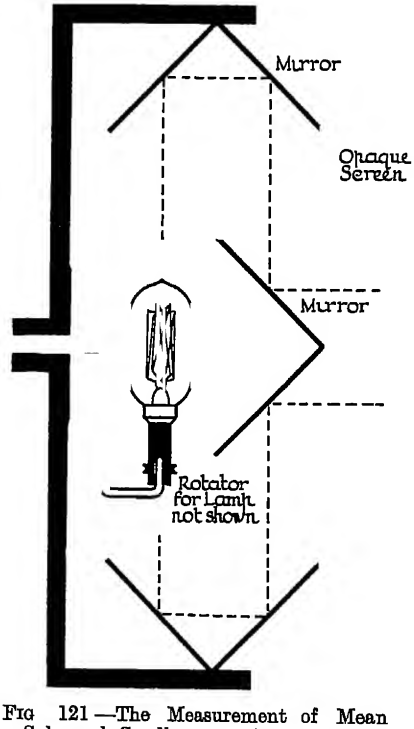

FIG 121 —The Measurement of Mean Spherical Candle-power (point-to-point method)

Lumenmeters.—It has already been pointed out that rotation of the source can only be done with safety in the case of a few sources of light, and even then the accurate determination of a polar curve necessitates at least twenty measurements It is not surprising, therefore, that repeated attempts have been made to devise a means for finding the mean spherical candle-

power of a source by a single measurement, or even a small number of measurements.

The first apparatus of this kind was the lumenmeter [35], shown in plan in Fig. 122 The source S was placed at the centre of a

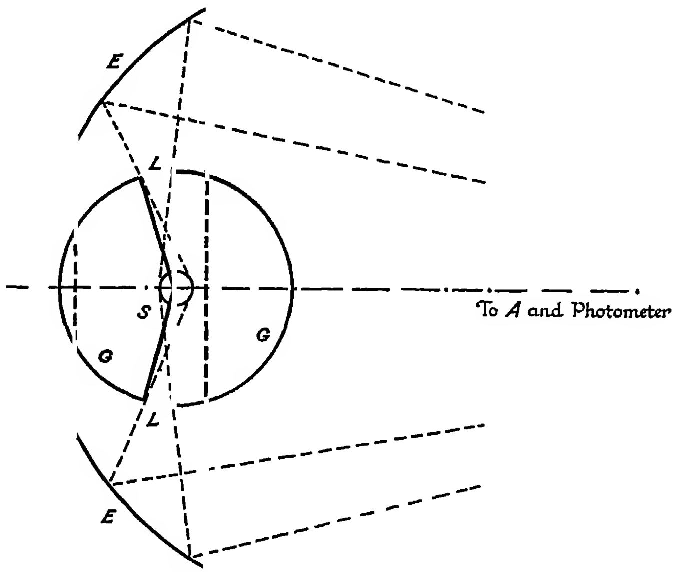

FIG 122 —Blondel's Lumenmeter

spherical blackened globe G, G This globe had cut out of it two lunar apertures L, L, and the light from S passed through these and was reflected from an ellipsoidal mirror E to a sheet of translucent material A placed at or near its second focus (S being at one focus of E). The light transmitted through A was diffused, and illuminated the photometer. In a modification of this instrument E was replaced by an annular diffusing ring, and A was removed. If the angular openings of the sphere were 18°, the mean of the readings obtained with the lamp in ten positions 18° apart gave the mean spherical candle-power, supposing the apparatus to have been calibrated by means of a source of known m s c p

Other apparatus used extensively before the advent of the integrating sphere depended on the use of a large number of mirrors placed round the source, so that the beams of light emitted from it in various directions were all reflected to a single photometer surface Of this kind were the mesophotometer [36] and the instruments of Matthews, Dyke, Léonard, and others [37].

The Integrating Sphere.—The apparatus most commonly used at the present time for the measurement of mean spherical candle-power depends on a principle first enunciated by Sumpner in 1892 in connection with work on the reflection factors of various surfaces [38] He showed that, if a source of light were placed inside a hollow sphere coated internally with a perfectly diffusing material, the brightness of any part of the surface due to light reflected from the remainder of the sphere was the same, and was proportional to

the total flux emitted by the source It has been mentioned already (see p. 104) that the flux received per unit area by one part of the surface of a sphere from a given area of any other part which has a given brightness is the same, whatever be the relative positions of the two parts on the sphere. Thus the flux reflected from each part of the spherical surface is equally distributed over the other parts, and, conversely, the flux received at each part of the surface by reflection from the remainder is everywhere the same and bears a fixed relation to the total flux received by the whole sphere The theoretical expression for this relation in terms of the reflection factor of the spherical surface is very easily obtained; for if the brightness of any portion of the sphere surface be B, and the reflection factor, assumed uniform over the whole surface, be ρ, the flux received at any element of area δs of the sphere due to a first reflection from the remainder of the sphere is equal to $(\delta s/4r^2)\int B ds$, the summation being made over the whole surface of the sphere. Now the brightness B of any area is equal to the flux reaching that area multiplied by ρ and divided by π, so that, if the total flux given by the source be F, $\int B ds = \rho F/\pi$, and hence the flux received by $\delta s = \rho F \delta s/4\pi r^2$ or $\rho F/4\pi r^2$ per unit area Thus the brightness of any part of the surface due to the direct light and light which has suffered one reflection is $B + \rho^2 F/4\pi^2 r^2$. The flux reaching δs by the first and second reflections is, therefore, $(\delta s/4r^2)\int(B + \rho^2 F/4\pi^2 r^2)ds$, which is equal to $\rho \delta s F/4\pi r^2 + \rho^2 \delta s F/4\pi r^2$, so that the flux received per unit area by one and two reflections is $(F/4\pi r^2)(\rho + \rho^2)$ Similarly, it may be shown that the flux received per unit area due to any number of reflections is $(F/4\pi r^2)\{\rho + \rho^2 + \rho^3 + \ldots$ to infinity$\}$, so that the total flux received per unit area by reflection is $F\rho/4\pi r^2(1 - \rho)$, which, since $F/4\pi$ is the mean spherical candle-power I_0, is equal to $I_0\rho/r^2(1 - \rho)$ i e., the illumination of the sphere by reflected light is equal to the average illumination by direct light multiplied by the factor $\rho/(1 - \rho)$, which for $\rho = 0\ 8$ becomes equal to 4 ([39]).

It will be seen that this expression for the reflected flux is independent of the position of the source and of the distribution of the light from it, and this fact is the theoretical basis underlying the use of the integrating sphere, for it follows that a measurement of the illumination *due to reflected light* of any part of the sphere wall gives at once a measure of the total flux from the source, irrespective of its distribution. The proposal to use the sphere in this way as a photometer was first made by Ulbricht in 1900 ([40]), and much work, both practical and theoretical, has been devoted to the problem since that time ([41]).

Practical considerations make it impossible to use the expression found above for the relation between the total flux emitted by a source inside a sphere and the illumination of the sphere surface due to reflected light. In the first place, it is supposed not only that ρ is constant for the whole surface of the sphere, but also that this surface is perfectly diffusing. Further, it is assumed in the above theoretical treatment that the sphere is perfectly empty, so

that the reflected flux is entirely undisturbed by the presence objects within the sphere Clearly, the very presence of a light source of finite dimensions is a violation of this condition, while the necessary provision of an aperture or window in the sphere wall for the purpose of measuring the illumination due to reflected flux, and the introduction of a disc to prevent direct light from the source from being included in this measurement, are further departures from the ideal conditions.

All these elements of uncertainty naturally put absolute measurements of mean spherical candle-power by means of the sphere quite out of the question, but they do not prevent its use for finding the relative candle-powers of two sources if suitable precautions be taken. This, after all, is the sole function of any photometric apparatus used for making measurements by the substitution method, and it does not, therefore, imply any important restriction of the usefulness of the integrating sphere The precautions to be taken arise from the necessity for ensuring that the departures from theory above mentioned affect equally the measurements on the two sources

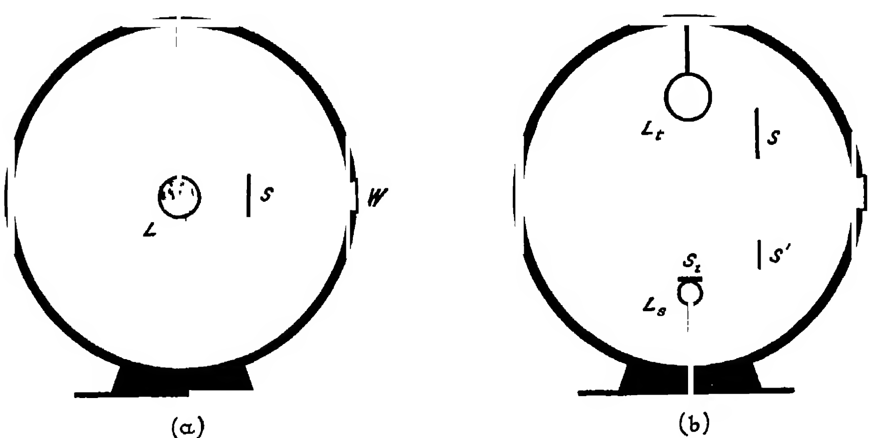

FIG 123 —Methods of using the Sphere
(a) The Substitution Method (b) The Simultaneous Method

to be compared. These precautions will be dealt with in some detail in later sections of this chapter, but first it will be convenient to give a brief account of the two methods of using the sphere most commonly adopted

The Substitution Method.—In the simpler of these, which may be called the true "substitution" method, a sub-standard lamp of known mean spherical candle-power is placed in the sphere, and the reflected flux incident at some part of the sphere wall is measured. This measurement may be made with some form of illumination photometer sighted through an aperture in the sphere wall at the opposite surface The more usual method, however, is to measure the brightness of the outer surface of a small translucent diffusing window (*W*, Fig 123) provided at some point in the sphere wall, this brightness being assumed proportional to the reflected flux received by the inner surface of the window The test lamp is then substituted for the sub-standard and the measurements are repeated.

If B_S and B_T be respectively the values of brightness found in the two cases, the candle-power of the test lamp $I_T = I_S(B_T/B_S)$

As will be shown later, the best position for the source in making measurements by this method is the centre of the sphere (see p. 216). Direct light from the source must be prevented from reaching the window (or the area viewed by the illumination photometer), since only the reflected flux is to be measured Between the source L and the window W, therefore, is placed a screen S in the form of a disc, just large enough to prevent the window from receiving any but reflected light. The best position for the screen will be discussed later (see p 213).

The Simultaneous Method.—In the other method of using the sphere, termed for convenience the "simultaneous" method, both sub-standard and test lamp are in the sphere throughout the measurements, as shown diagrammatically in Fig. 123, (*b*) B_S is first measured with only L_S alight; L_S is then extinguished and B_T is measured with only L_T alight. As before, $I_T = I_S(B_T/B_S)$ This method is generally used in the measurement of arc lamps or other sources which necessitate the presence of auxiliary apparatus of considerable bulk in the sphere. The disturbance of the reflected flux distribution by this apparatus is then approximately the same for the measurements of both B_S and B_T. The extent of this disturbance may be reduced considerably by whitening the external surface of the apparatus, either by painting it with a white paint or by covering it with a white material.* Two screens, S and S', are required for shielding the window from direct illumination by L_T and L_S, and a third screen S_1 is needed between the sub-standard and the test lamp in order to prevent direct light from L_S from reaching any part of L_S † This screen generally takes the form of a small cap on the sub-standard lamp, the mean spherical candle-power of which is determined with the cap in position

It is desirable that the positions of the two lamps L_T and L_S should be as nearly as possible symmetrical with respect to the sphere window, but they should not be too close together, or the size of S_1 will be such as to produce an excessive distortion of the light distribution from the sub-standard. Further, as will be shown later, the lamps should not be too close to the surface of the sphere (see p 216), so that the best compromise is, generally, to place each about midway between the centre of the sphere and its surface.

Sphere Details.—From the above general description of the theory and practical application of the sphere it will be clear that in the design and use of any particular instrument of this nature very careful consideration must be given to (*a*) the nature and size of the window, (*b*) the size and position of the screen or screens, (*c*) the effect of objects within the sphere (including the source itself) and their position, and (*d*) the nature of the paint used for coating the interior surface. These details will, therefore, be con-

* Care must be taken to ensure that the distribution of the flux directly emitted by the lamp is not affected by this whitening, which should, therefore, be confined to surfaces which receive no direct light from the actual luminous source under normal working conditions

† This precaution is necessary, for otherwise the opaque parts of L_T would absorb *direct* light from L_S as well as light reflected by the walls of the sphere. It is this latter absorption only that takes place when L_T is alight

sidered one by one, and the application of the conclusions arrived at will then be shown by reference to spheres used in current photometric practice

The Sphere Window.—It was mentioned above that in most spheres the measurement of the illumination of the inner surface due to reflected flux is effected by measuring the brightness of the external surface of a translucent window provided in the wall of the sphere The alternative method, *viz*, that of measuring the illumination directly by means of a brightness photometer sighted, through an aperture in the sphere wall, towards a portion of the surface opposite, has not generally been used There seems to be no reason why this more direct method of measurement should not be adopted instead of that involving the interposition of a window, provided that a satisfactory photometer with a field not exceeding 2° to 3° in diameter is available for the work, and the inside surface of the sphere is sufficiently matt.

Meanwhile the window method is that most generally used, the normal brightness of the window being obtained, usually, in one of two ways In many cases it is convenient to regard the window as a luminous source and to measure its candle-power in the direction of the normal by means of a bench, photometer head, and comparison lamp, as described in the last chapter (pp. 165–168) Alternatively the brightness of the window may be measured by making it one of the comparison surfaces in a photometer head of special design, the other comparison surface being illuminated by a movable comparison lamp Such special photometer heads are shown diagrammatically in Figs. 124 (*a*) and (*b*). The ordinary Lummer-Brodhun head may be adapted for this work by inserting a thin sheet of silvered glass in front of the plaster surface, as shown in Fig 124 (*c*)

The former of the two alternative methods just described, *viz.*, that of candle-power measurement, can only be adopted when the brightness of the window is high. For, while the candle-power of the window is proportional to the product of its brightness and its area, the latter is severely limited by the fact that the screen between the window and the source, which must be at least as large as the window in the case of most light sources, has to be reduced as much as possible (see p. 213). Generally it may be assumed that the diameter of the window should not exceed one-tenth of the sphere diameter, and should, if possible, be less than this. Assuming this as an upper limit, the minimum candle-power which can be measured by this method in a sphere of given size can readily be calculated.

It has been shown that the illumination of the inner surface of the window due to reflected light is $I_0\rho/R^2(1-\rho)$, where I_0 is the mean spherical candle-power of the source and R is the radius of the sphere If τ be the transmission factor of the window its brightness is, therefore, $I_0\rho\tau/\pi R^2(1-\rho)$, so that if its radius be r its candle-power in the direction of its normal is $I_0\rho\tau r^2/R^2(1-\rho)$ If $\rho = 0{\cdot}8$, $\tau = 0{\cdot}3$, $R = 50$ cm, and $r = 5$ cm, this expression becomes $(0{\cdot}012)I_0$. Since the inverse square law cannot be assumed to hold to an accuracy better than 0 3 per cent. when the distance between the window and photometer is less than 20 r, it follows that, assuming an illumination of 10 metre-candles is required at the photometer

screen, the candle-power of the window cannot be found by the ordinary bench method of moving the photometer head and com-

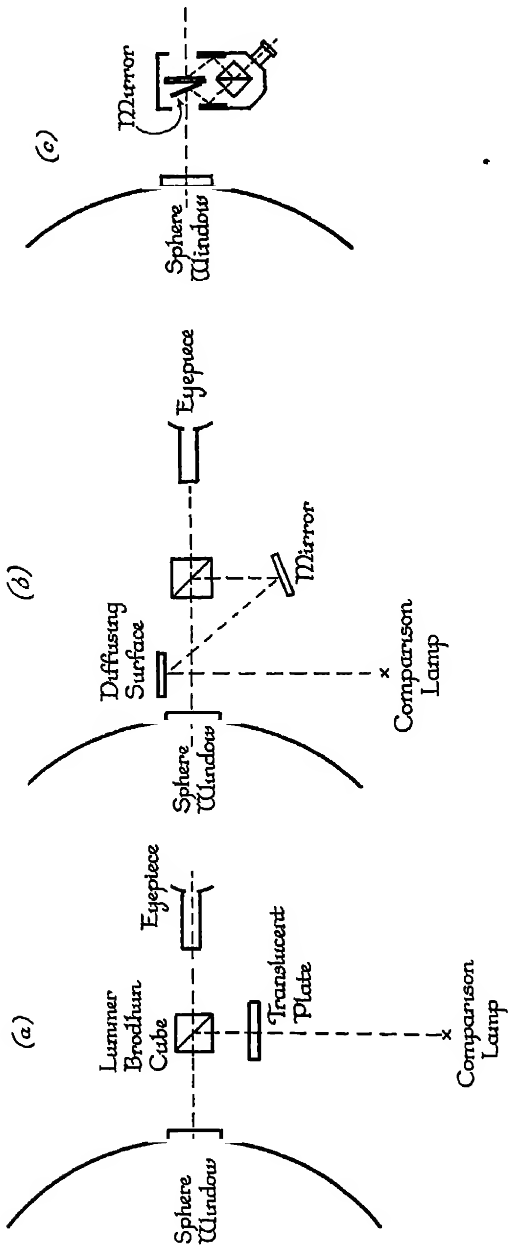

FIG 124 —Forms of Photometer Head for Use with the Integrating Sphere

parison lamp together unless I_0 exceeds 4,000 $R^2(1 - \rho)/\rho\tau$, i e., about 830 candles in the case considered above.

The above example shows that it is generally impossible to use a variable distance between the sphere window and the photometer

head For all sources, except those of very high candle-power, the photometer head is fixed at some convenient distance from the window, and the photometric balance is obtained by moving the comparison lamp A distance of from 10 to 30 cm is often convenient, although this naturally depends on the brightness and size of the window and on the illumination desired at the photometer screen Departures from the inverse square law due to the size of the window are quite immaterial so long as the distance from window to photometer head remains unaltered between the measurement of the sub-standard and the test lamp

In the case of lamps of low candle-power it is better to abandon altogether any attempt at measuring the candle-power of the window and, instead, to use the window surface itself as one of the comparison surfaces in the photometer (see Fig 124, (*a*) to (*c*))

Whichever method be employed for measuring the brightness of the sphere window, it is convenient to have an iris diaphragm close to the outer surface, so that the effective area may be reduced when sources of large candle-power are being measured When the candle-power of the window is measured directly, it is necessary to keep the aperture of the diaphragm constant between the test lamp measurement and the standardisation. If the photometer head be not fixed in position, it must be remembered (*a*) that the distance to be used for calculation by the inverse square law is that between the photometer head and the diaphragm (see p 108) ([42]), and (*b*) that with very short distances, departures from the inverse square law may be introduced not only on account of the size of the window, but also due to the possible effect of light returned by the photometer screen to the window, and thence reflected back to the photometer (see p 104). It is clear that a diaphragm cannot be employed in this way when the window forms part of the photometer head. In this case it is often found convenient to use *two* widely separated diffusers with an iris close to that one which forms part of the sphere surface (see Fig 125) The brightness of W_2 is then approximately proportional to the aperture of D A more satisfactory arrangement is that in which W_2 is a window in a small auxiliary white sphere with an opening in the plane of D The brightness of W_2 is then approximately proportional to the flux admitted into this sphere, *i e.*, to the area of the opening in D.

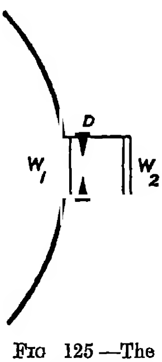

FIG 125 —The Double Window for an Integrating Sphere

In all cases it is desirable that the window should be readily removable from the sphere so that it can be cleaned effectively, and so that the sphere wall can be repainted without damage to the window surface Some form of shallow cylindrical framework is generally used to carry the window This fits into a circular aperture in the sphere wall, and a register is provided to ensure that, when the window is replaced, its surface is exactly flush with the inner surface of the sphere The inside of the cylinder should be painted a matt black or covered with black velvet to avoid reflection of light from the window towards the photometer

The choice of a suitable material for the window presents some

difficulty Since the function of the window is to enable the total flux received by its inner surface to be deduced from a measurement of the brightness of its outer face, it is clearly necessary that these two quantities shall bear a constant ratio to each other, no matter what the angle of incidence of the flux at the inner surface, *i.e.*, the window must give as close an approximation as possible to perfectly diffuse transmission.* This condition is very difficult to satisfy without reducing the transmission factor to an inconveniently small figure Opal glass depolished on the inner surface is often used, while two pieces of glass ground on their adjacent surfaces and on the surface forming the inner face of the window have also been employed ([43]) In any case the inner surface of the window should

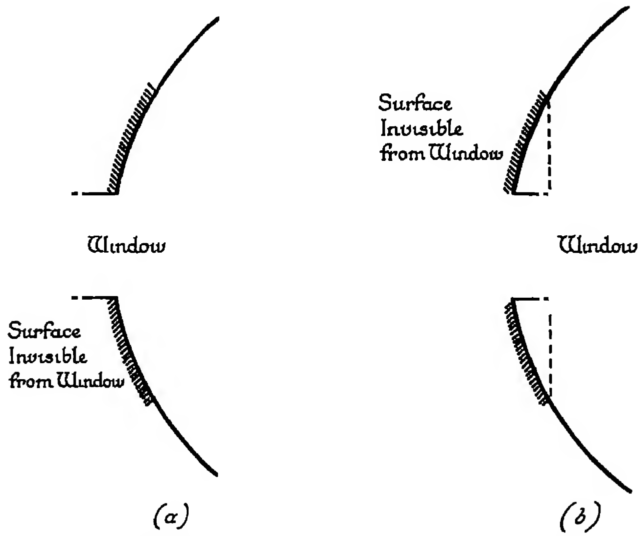

FIG 126 —The Effect of Displacement of the Sphere Window

be matt, and it should be absolutely flush with the inner surface of the sphere, otherwise part of the incident flux will fail to reach the window (see Fig 126, (*a*) and (*b*)). The edges of the glass or other translucent material forming the window should be painted white so that as little light as possible is lost by diffusion from them

The material used should be one that can readily be cleaned. Further, it must not alter the colour of the light transmitted through it, and it must therefore be as non-selective as possible Opal glass usually shows a selective absorption at the blue end of the spectrum. If this material be used, it follows that the thickness should be the minimum consistent with adequate diffusion of the transmitted light It has been found that a thickness of less than 1 mm is generally sufficient ([44]), or a flashed opal may be used. If ground glass be employed, the kind of glass selected should be as colourless as possible A slight tinge of green may be corrected by placing an

* Since the window is generally viewed in a constant direction it is more important that the brightness in a fixed direction should be independent of the direction of the incident flux than that it should be the same at all angles of view ([55]).

equal thickness of clear glass of the same kind on the comparison lamp side of the photometer In any case this artifice may be employed for the purpose of avoiding the introduction of a colour difference in the photometer head due to slightly selective transmission of the window

The Screen.—It has already been pointed out (see p 208) that one or more screens have to be placed in the sphere in order to shield the window (or area whose brightness is to be measured) from direct illumination by the source The size and position of the screens naturally affect the distribution of the flux within the sphere, and therefore exert a noticeable effect on the performance of the sphere as an integrator. It will be clear from Fig 127 that the first reflected light cannot reach the window from the zones *ac* and *bd*, in the latter case because no direct light reaches this part of the sphere, in the former because the screen shields this portion of the sphere wall from the window. The influence of this on the window illumination can readily be calculated as follows ([45]) —

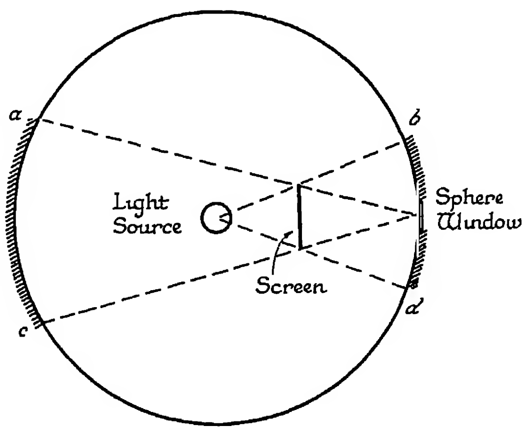

FIG 127 —The Effect of the Screen within the Sphere

The average illumination of the sphere wall by both direct and reflected light $E = E_d/(1 - \rho)$, where E_d is the average value of the direct component Hence $E = E_d + \rho E$, which shows that the reflected component is ρE. By multiplying both sides of this equation by ρ the value of this component is found to be $\rho E_d + \rho^2 E$, so that this expression represents the theoretical illumination of the window. If δ represent the fraction of the total flux from the source which falls upon the screened areas, the actual illumination of the window is $E' = (1 - \delta)\rho E_d + \rho^2 E$, if the absorption by the screens of the flux in the second and subsequent reflections may be neglected. It follows that the departure from the theoretical value is $E'/\rho E = (1 - \delta)(1 - \rho) + \rho$, or $E' = \rho E\{1 - \delta(1 - \rho)\}$ If E_s and E_s' be the theoretical and actual values of window illumination in the case of a sub-standard lamp for which the screened flux is δ_s, $E_s' = \rho E_s\{1 - \delta_s(1 - \rho)\}$ The error introduced into a measurement by comparison with a sub-standard is therefore $(E'/E_s')(E_s/E) - 1 = (\delta_s - \delta)(1 - \rho)$ approx. This is zero when $\delta = \delta_s$ If the substitution method be used the screened areas are equal for both lamps, a, say, and δ and δ_s are then respectively equal to $a\theta$ and $a\theta_s$, where θ represents the ratio of the average illumination over the screened area to the average illumination over the whole sphere It should be noticed that the error diminishes as ρ increases (as might be expected *a priori*), so that the paint used for the sphere should have a high reflection factor (see p 217)

So long as the test lamp and the sub-standard give approximately the same distribution of light $(\delta - \delta_s)$ is very small. For all other cases, however, it is clearly desirable to reduce both δ and δ_s as much as possible. This can only be done by reducing the size of the screen and adjusting its position in relation to the lamp and the window. The minimum size of screen which can be used is fixed by the sizes of the source and of the window The latter must be completely shielded from all direct light, not only from the filament, arc crater, *etc.*, but also from every part of the lamp structure which is to be regarded as contributing anything towards the flux emitted in the direction of the window For example, in the case of an incandescent lamp with a shallow translucent shade over it, not only the lamp filament, but also the whole of the shade, should be invisible from any point of the sphere window.

When the light source is at the centre of the sphere, it has been calculated [46] that the minimum value of the screened area is obtained if the screen be placed at a distance from the lamp of from 0 3 to 0·4 times the sphere radius [47] If the screen diameter does not exceed one-sixth of the sphere diameter $a = 0{\cdot}12$, and the error is therefore equal to $0\ 024\ (\theta - \theta_s)$, if $\rho = 0\ 8$ Hence if an accuracy of 1 per cent be aimed at, $(\theta - \theta_s)$ must not exceed 0 4, and as the value of θ_s is generally known, the limits between which θ must lie are readily ascertainable. An approximate idea of the value of θ may be obtained from the "spherical reduction factor" of the source under consideration, if this be known (see p 88) In other cases the candle-power may be measured in the two horizontal directions concerned, and for all sources but those of exceptional distribution the ratio of the mean of these two values to the m s c p as measured in the sphere will give a good approximation to the value of θ If the sub-standard be a vacuum electric incandescent lamp (squirrel-cage filament), $\theta = 0\ 78$ [48], so that the limits of θ are then 0 4 and 1·2.

When both test lamp and sub-standard are in the sphere together, it will be clear from Fig 123 that the areas screened by S and S' are different in area and differently placed, according as L_T or L_S is alight. For accurate compensation it is necessary that the fraction of the direct flux from L_T which reaches the areas shaded by S and S' when that lamp is alight should be the same as the fraction of the direct flux from L_S which reaches the areas shaded by these screens when L_S is alight Compensation of this kind is difficult to arrange, and all that can generally be done is to have the screens as small as is compatible with efficient shielding of the window, and to place them at distances from the lamps equal to between $0{\cdot}3d$ and $0\ 4d$, where d is the distance between the lamp and the window [49].

The use of translucent screens has been recommended in order to compensate for the error due to shading by the addition of a certain amount of transmitted light [50] Although very good compensation may be achieved, for a given arrangement of source and screen, by suitably adjusting the transmission factor of the latter, the method may lead to results which are in considerable error if the distance between source and screen, or the light distribution of the source, be materially altered from the standard arrangement [51]. A

reflecting screen with foliated black patches has also been devised with the object of producing compensation [52] There is but little advantage to be gained by the use of such devices, for the conditions which it has been shown to be necessary to fulfil when using an opaque screen are readily complied with in all cases likely to be met with in practice

Effect of Objects within the Sphere.—It is clear that any object placed within the sphere will not only disturb the distribution of the reflected flux, but it will also reduce the amount of this flux if it receive direct light from the source. It will be convenient to deal with these two effects separately

If A be the superficial area of the object [53] relative to that of the sphere it can be shown that, *if the object receive no direct light from the source*, the average illumination of the sphere wall due to reflected light is reduced in the ratio 1 $1 + \{\alpha' A/\alpha(1 - \alpha' A)\}$, where α and α' are the absorption factors of the sphere wall and of the object respectively. For the total flux F from the source is equal to $4\pi R^2 E_D$, where E_D is the average direct illumination of the sphere wall F is also equal to the total flux absorbed by the sphere and the object Hence, if E_R be the value of the illumination by reflected light when the sphere is empty, $F = 4\pi R^2 \alpha E_D + 4\pi R^2 \alpha E_R$ When the object is in the sphere, however, the flux absorbed by it is equal to the total reflected flux, $4\pi R^2 \rho(E_D + E_R')$, multiplied by $\alpha' A$, so that

$$F = 4\pi R^2 \alpha E_D + 4\pi R^2 \alpha E_R' + \alpha' A \,.\, 4\pi R^2 (1 - \alpha)(E_D + E_R').$$

From these equations it follows [54] that

$$E_R/E_R' = 1 + \{\alpha' A/\alpha(1 - \alpha' A)\} \quad . \; . \; . \qquad \text{(i)}$$

If $\alpha' = \alpha$, the percentage reduction becomes approximately $100A$ so long as α and A are both small. If, however, $\alpha' = 1$, the percentage reduction is approximately $100A/\alpha$

If, now, the object receives direct light from the source it is clear that the amount of flux lost due to absorption by the object is $\alpha' I \omega$, where ω is the solid angle subtended by the object at the source, and I is the candle-power of the source in the direction of the object There is thus a further reduction of the illumination of the sphere walls in the ratio $(4\pi I_0 - \alpha' \omega I) \cdot 4\pi I_0$, i e , in the ratio

$$\{1 - \alpha' \omega I/4\pi I_0\} \;\; 1. \quad . \quad . \quad . \; . \qquad \text{(ii)}$$

The above expressions (i) and (ii) give an indication of the probable magnitude of the effect produced by any given object in a sphere photometer With regard to expression (i.) it must be remembered that, since the distribution of the reflected light is disturbed by the presence of the object, it can no longer be assumed that the illumination of the sphere walls due to reflected light is everywhere the same The above expressions only indicate the *average* reduction in illumination The amount of the reduction at any particular region of the sphere due to a specific object must depend upon the proximity of that object to the region in question. For example, it has been found that with a disc of area A (i e , superficial area $2A$) relative to the area of the sphere the percentage reduction of the illumination of the sphere window is about $10A$

in the case of black discs, and about $3A$ in the case of white discs ($\alpha' = \alpha$), when the source is in the centre of the sphere and the disc is approximately half way between the source and the window ([55]) These figures serve to confirm the theoretical formulæ deduced above if it be assumed that $\alpha = 0.2$, and that $I \simeq I_0$ It follows from these results that objects within the sphere should be painted a dead white in order to reduce as much as possible their effect on the illumination of the sphere wall by reflected light

It must not be forgotten that the lamp itself, particularly if it include reflectors or other opaque parts, must be regarded as disturbing the *reflected* flux, and therefore altering the illumination of the window ([56]). Those parts which do not receive direct light from the actual luminous source (see footnote, p 208) should therefore be whitened, and, further, the correction calculated from the formula given above should be made except when a sub-standard of exactly the same type as the test lamp (*i e*, having similar shades, opaque parts, *etc*) is available, so that all necessity for correction is avoided owing to exact equality of the disturbing effect in the case of both test lamp and sub-standard Generally no such sub-standard is available, and the correction must be made. From the nature of the case, however, the amount of this correction is difficult to estimate with accuracy, and it has therefore been recommended that in cases where the correction is found to be large the simultaneous method of measurement should be used.* By this method sources may be measured for which the area of the opaque surfaces does not exceed one-fortieth of the area of the sphere.

Even a simple electric incandescent lamp without shade or reflector may cause appreciable absorption of light if the bulb be noticeably blackened ([57]) For example, if such a lamp have an absorption factor in the single glass thickness (excluding surface reflection losses) of 10 per cent, it follows from the formula given on p. 215 that the change produced in the illumination of the window is 1 per cent if the bulb be spherical and of radius one-tenth that of the sphere, the absorption factor of the sphere surface being taken as 0 2 If such a lamp be compared with a sub-standard in which the blackening of the bulb is negligible, an error of 1 per cent will be introduced into the measurements

As mentioned above, the uniformity of the (reflected) flux distribution over the surface of the sphere, postulated in the simple theory, is disturbed by the presence of an absorbing object, and this disturbance is greater the nearer the object is placed to the surface of the sphere. It follows that the source to be tested and its auxiliary apparatus should be placed as near the centre of the sphere as may be possible having regard to other considerations ([58])

Paint for Integrating Spheres.—It will be evident from the foregoing paragraphs that a satisfactory paint for integrating spheres should fulfil, as far as possible, the following conditions —

(*a*) It should have a matt surface when dry ([59])

(*b*) It should be quite non-selective, so that the repeated reflec

* If all the surfaces in question are white, or can be whitened, the correction to be made is approximately equal to the ratio of the area of these surfaces to the area of the sphere So long as this ratio does not exceed 1 per cent the substitution method may quite safely be employed.

tions within the sphere do not appreciably alter the colour of the light ([60])

(c) It should have a very high reflection factor in order to reduce the effect of the screens and other objects within the sphere (see p 213) A high reflection factor also assists in the diffusion of the flux within the sphere, and so helps to compensate for imperfect fulfilment of condition (a) ([61])

(d) It should be easy to apply, tenacious, and permanent, particularly as regards freedom from colour change with lapse of time

Of these four requirements, the second and third are the most important So far no paint which fulfils the other three conditions sufficiently for practical purposes has been found to be absolutely non-selective Many different paints have been used by various workers, e g , Keene's cement ([62]), magnesium oxide, baryta with "zapon" lacquer ([63]), and zinc white with various binding materials ([64])

Two formulæ have been proposed as the result of careful investigation of the matter The first of these was put forward by the Verband Deutscher Elektrotechniker ([65]) It consists of (a) a single coating of a "body" paint made of lead white or baryta ground into a varnish composed of equal parts by weight of copal and turpentine, and (b) three coats of cover paint, made up as follows 100 parts of zinc white are mixed with 8 parts of water, and 100 parts of the resulting water paint are then thoroughly mixed with 6 parts of a water glue consisting of 100 parts of fresh colourless cabinet-maker's glue dissolved in 500 parts of water

The paint used at the Bureau of Standards ([66]) is a zinc-oxide paint prepared by slowly adding 4 parts by weight of ZnO to a mixture of 1 part alcohol and 4 parts of a special cellulose lacquer. The mixture must be constantly stirred as the zinc oxide is added, and the stirring must be continued for an hour or more until a smooth thick paste is obtained This paste is thinned for use by slowly adding to it a mixture of about 2 parts alcohol and 1 or 2 parts of water-white turpentine The paint as used is thinner than an ordinary oil paint. It dries very quickly and must be brushed out as applied Contact with water must be avoided, as water coagulates the paint The drying should not be hastened by artificial means. About three hours should be allowed between the first two coats, and six or seven hours between coats after the second The special cellulose lacquer is made by dissolving 15 parts by weight of camphor in 100 parts of alcohol and then adding 10 parts of colourless celluloid in small pieces, stirring constantly all the time until the celluloid is completely dissolved This may take about ten or twelve hours

The above formulæ for the preparation of the paint have been given in some detail, as it is necessary to renew the surface of a sphere quite frequently The V D E. ([67]) recommends that repainting be done at least once a year, the surface paint being washed off with water, leaving the body paint untouched The period which is allowed to elapse between repainting must necessarily depend on the amount of use made of the sphere and the nature of the sources measured in it Flame sources and arcs, which produce a very marked upward current of heated air, soon cause a noticeable blacken-

ing of the top of the sphere. The fact that this blackening is more or less localised makes it far more important than a general uniform darkening of the whole surface of the sphere; for if one portion of the sphere surface have a lower reflection factor than the remainder, the flux reaching that portion will receive less than its proper weight in the estimation of the total flux. Hence, if the light distribution of the two sources compared be not exactly the same, an error will be introduced into the measurements ([68])

The extent to which any paint is selective as regards colour may be tested quite simply on the photometric bench ([69]) Two lamps, adjusted to colour match, are placed so that one of them illuminates one side of the photometer directly, while the other illuminates a flat surface coated with the paint under examination The light reflected from this surface illuminates the other side of the photometer, and the difference in colour introduced by the paint is seen as soon as adjustment to equality of brightness has been made.

If the spectral distribution of the light given by the test lamp be the same as that of the light from the sub-standard, selective reflection by the sphere surface is unimportant, but in all other cases an error is introduced of a magnitude which depends on the difference between the colours of the lights to be compared. The amount of the error committed in comparing two sources giving respectively lights which match a black body at two different temperatures may be determined by means of an electric incandescent lamp. The candle-power of the lamp is measured in any convenient *single direction* at two different efficiencies, *viz* the efficiencies at which it gives lights of the colours in question. The mean spherical candle-power is then measured at the same two efficiencies and, since it may be assumed that the ratio of the mean spherical to the fixed direction candle-power is unchanged by the change in efficiency, it follows that the difference between the ratio of the mean spherical candle-powers and the ratio of the single-direction candle-powers is the error due to the selective absorption of the sphere surface This is clearly equal to the error which would be made in comparing two lamps giving respectively lights matching in colour the light given by the electric lamp operating at the two efficiencies used in the experiment

If it can be assumed that, to the order of accuracy necessary in the case of a correction factor, the colour of the sphere surface is continuous throughout the spectrum, *i.e.*, that its only effect is to alter the colour temperature of the light it reflects, then all necessity for correction may be removed very simply by placing over the sphere window a colour filter (*e g*, a Wratten photometric filter) which is just sufficient to neutralise the colour of the sphere surface. In general it will be found that No 78 C (see p. 243) is suitable, since the coloration is generally in the direction of a lower colour temperature A lamp, previously colour-matched with the comparison lamp on the photometer bench in the ordinary way, is placed inside the sphere, and the colour filter is moved across the sphere window until a colour match is obtained in the photometer head The arrangement of window, filter and photometer head is then left undisturbed and photometric measurements are made as if the sphere surface were perfectly non-selective ([70]).

Laboratory Spheres.—It will be seen from the foregoing paragraphs that the requirements which should be met in a satisfactory sphere are very exacting, and, as is so often the case, they are to a certain extent incompatible with ease of construction and convenience in use. It follows that the exact form of construction adopted for a sphere must depend on its size and the nature of the work for which it is intended (71). Spheres more than 1 metre in diameter are frequently made to divide into two parts along a meridian, the halves being mounted on rollers so that they can be readily separated for repainting (72). The lamps may be introduced into the sphere through an opening at the top, or a door of segmental form may be provided at the side. The 88-inch sphere at the Bureau of Standards is not divided, but is built up on a steel network with reinforced concrete, finished off inside to a truly spherical surface (73). At the top is a large circular hole, 23 inches in diameter, covered with a flat wooden disc which can be lowered from above in annular sections, so that a lamp can be lowered into the sphere from above if desired. On one side of the sphere is a hinged segmental door of maximum dimensions 37 × 16.5 inches. Opposite the door is the window, of depolished opal glass, which is arranged to form one comparison surface in the photometer head (see Fig. 124), the other being a piece of similar glass illuminated by a comparison lamp which travels along a 1.5-metre bench. The lamp socket is carried on hinged rods so arranged that lamps can be readily brought to the door of the sphere for changing, and can then be returned automatically to their correct position within the sphere. At a point about 27 inches ($0{\cdot}6\,R$) in front of the window are two vertical rods which hold a runner for carrying the screens. These are of four sizes, *viz.*, 11, 21, 30 and 38 cm. in diameter.

For work in a laboratory where a large number of lamps have to be measured in rapid succession, it is essential to use a sphere designed in such a way that as little time as possible is lost in changing lamps. Since the size of the sphere used in such circumstances does not generally exceed about 1 metre in diameter, it may conveniently be constructed of zinc or of sheet steel (74). A convenient arrangement for changing the lamp is the double door device shown in plan in Fig. 128 (75). When this door is swung anti-clockwise around its central vertical axis, so that lamp A is in the sphere and ready for measurement, lamp B can be removed from its socket and a fresh lamp C can be put in its place. As soon as the measurement on A is completed, the double door is swung round clockwise so that C is now in the sphere. Another lamp D can then be substituted for A.*

The sphere window at W is of diffusing opal glass and is covered with a small hemisphere. A second window in this hemisphere acts as one of the comparison surfaces in the photometer head, which is of the Lummer-Brodhun type. For increasing the range of the instrument, diaphragms are placed over W inside the hemisphere. Some of these are provided with colour filters for use in the measure-

* A somewhat similar device is that shown in Fig. 129, which is self-explanatory. The cylinder C, carrying three identical sphere segments, can be rotated about its axis so that, while one lamp L_2 is being measured, the lamp just finished with, L_1, can be removed and a new lamp, L_3, can be inserted ready for measurement (76).

ment of gas-filled lamps, the comparison lamp L being a vacuum lamp L is enclosed in a travelling box in which the side facing the

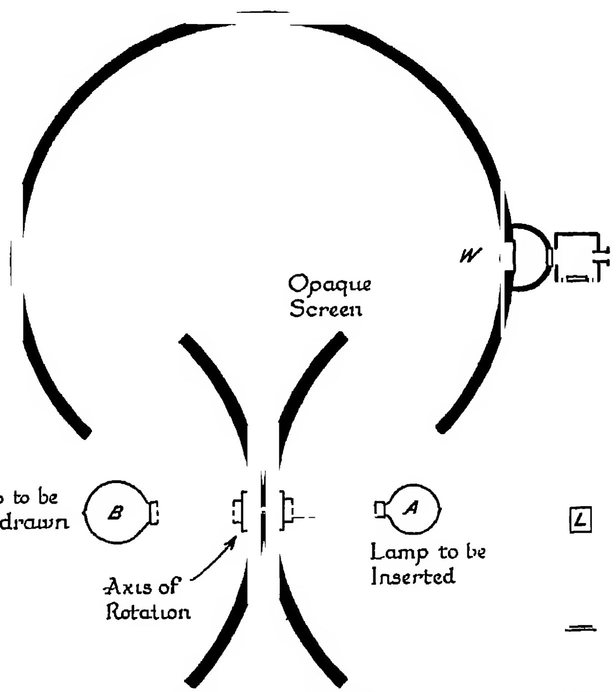

FIG 128.—Double Door Device for Rapidly Changing Lamps in the Sphere

photometer is covered with one or more sheets of translucent glass, which therefore become the real comparison source The box is also

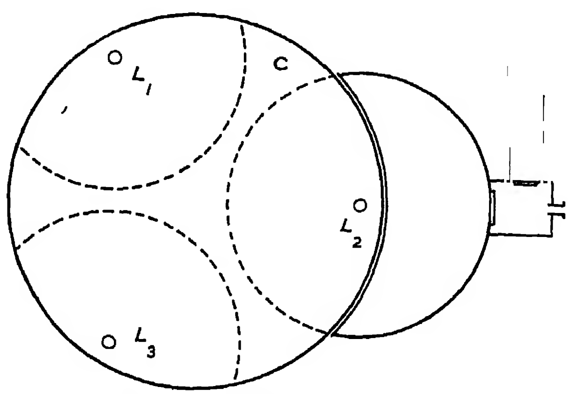

FIG 129 —Lamp Changing Device

provided on one side with a small window and vertical cross-wire. The shadow of this wire is formed on a vertical translucent glass scale placed at the side of the bench, and this device is used to give

FIG 130.—The Sphere Photometer Room.

[*To face p* 221

the position of L, just as in the Sharp-Millar illumination photometer (see p. 348) The bench is arranged as shown diagrammatically in the plan, so that the operator whose duty it is to change the lamps in the sphere can also take the readings on the bench A second operator stationed near W adjusts the balance in the photometer head by moving L.

Fig. 130 shows on the left a 1-metre sphere used at the National Physical Laboratory In this the lamp is carried at the end of a rod passing through a small segment of the top of the sphere which can be raised, within a framework attached to the sphere, by means of a cord and counter-weight attachment carried on pulleys at the top of the framework

Instead of using the inverse square method of varying the illumination on the comparison lamp side of the photometer, a second smaller sphere may be employed on this side, and the illumination may be varied by continuous alteration of the surface of the diffusing window of this second sphere ([77]) For work to commercial accuracy a lamp in a whitened cube provided with a window of adjustable area may be used as a comparison source ([78]) Such a cube is shown in front and side elevation in Fig 131. Over the front

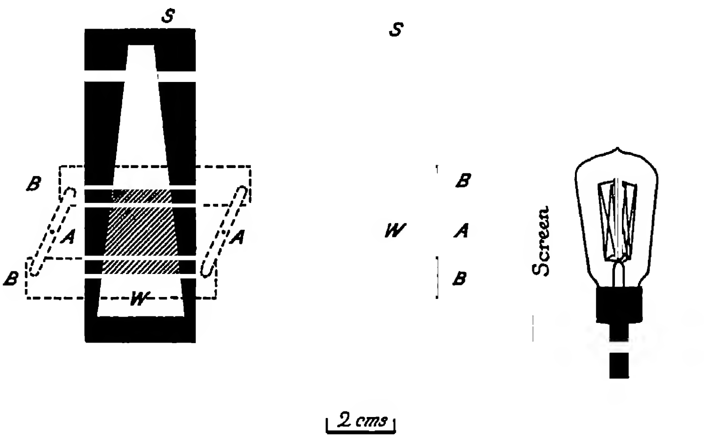

FIG 131 —A Comparison Source of Adjustable Candle-power for Use with a Photometric Integrator

of the diffusing window W moves a sliding shutter S with a wedge-shaped opening, so that the breadth of W is changed in proportion to the linear displacement of S. S bears a linear scale of candle-powers, and adjustment to a constant of unity is achieved by means of two horizontal bands B, B, which are carried on straps pivoted about their centre points A, A, and which move over the inner face of the window W The height of W can thus be adjusted, without altering the position of its centre, by simply altering the inclination of the straps bearing the bands B, B.

Measurement of Flame Sources.—When flame sources are measured in a sphere, it is necessary to arrange efficient ventilation, and this can only be done by the provision of at least two openings, one near the bottom and another at the top of the sphere. If very

small openings be used, efficient air supply can only be ensured by means of a forced draught, and this is liable to create undesirable air currents, with consequent disturbance of the flame Too large an opening leads to a serious departure from the theoretical conditions governing the distribution of the reflected light, since an opening must be regarded as an area of zero reflection factor It is clear that if this area relative to the total sphere area be A, and if the average value of the direct illumination on this area be E'_d, while that on the remainder of the sphere wall is E_d, then the illumination of the window is reduced from $\{(1-A)E_d + AE'_d\}\rho/(1-\rho)$ to $\{(1-A)E_d\rho\}/\{1-(1-A)\rho\}$. If $E'_d/E_d = \theta$, the magnitude of the error is therefore

$$A\{\theta + \rho(1-\theta)(1-A)\}/\{1-\rho(1-A)\}\{1-A(1-\theta)\}$$

If A be small ($<0{\cdot}01$, say) this is approximately equal to $A\{\theta + \rho(1-\theta)\}/(1-\rho)$, which, when $\rho = 0{\cdot}8$, becomes $A(\theta + 4)$, so that if the error due to the apertures is not to exceed 1 per cent, their total area A must not be more than 0·002 This, in the case of a sphere of 1 metre radius, allows two circular holes, each of about 12·5 cm diameter, assuming that θ is not much greater than 1, *i e*, that the directions of maximum candle-power of the source are not directed towards either aperture. It must be remembered that the actual error of a measurement made by either of the methods described on pp 207-8 above is much less than that just calculated so long as both test lamp and sub-standard are measured with the sphere in the same condition, *i e.*, so long as the apertures remain the same for the measurement of both B_S and B_T. The error may be diminished still further by placing a whitened screen over the aperture, but at a sufficient distance from it to allow adequate ventilation ([79]). This screen should be of such a size and so placed as to prevent the aperture from receiving any direct light from the source The whole of the lost flux is then reflected flux, and its amount being closely proportional to the illumination of the window, the error introduced into the measurements is very small

Sub-standards.—From the description (p 207) of the methods employed for the measurement of mean spherical candle-power by means of the integrating sphere, it will be clear that this instrument cannot be used to obtain absolute values of m s c p. without the use of a sub-standard measured by some absolute method, such as one of those described earlier in this chapter The type of source generally chosen for a sub-standard is naturally that most readily measured by absolute methods, *viz* a vacuum tungsten filament electric lamp. Such a lamp can be rotated without change of candle-power, and an accurate measurement of its mean spherical candle-power can therefore be obtained in comparatively few measurements by one of the methods described on p. 204 ([80]) For the measurement of gas-filled lamps a blue colour filter may be used on the comparison lamp side when the test lamp is in the sphere. The transmission factors of these filters are measured at the standardising laboratory, as described on p 263. Unless the sphere paint be quite non-selective there will be an error in the measurements made by this method, as explained on p 218 above The amount of this error may be determined by the method there described. The error may

be avoided by using gas-filled lamps as sub-standards, although these lamps cannot be relied upon as regards constancy of candle-power to the same extent as vacuum lamps Since the absolute measurement of the m s.c.p. of gas-filled lamps is a very tedious process, it is usual to measure them in the sphere by comparison with a sub-standard lamp of the vacuum type Since this comparison involves a colour difference, it should generally be carried out at the standardising laboratory

When using the method in which both sub-standard and test lamp are in the sphere together, the sub-standard and screen S_1 (Fig 123*b*) must be regarded as a single unit, and the mean spherical candle-power of the combination must be found by an absolute method. Similarly, in the determination of mean hemispherical or mean zonal candle-power by the substitution method (p 224, *infra*), the sub-standard and screen H (Fig 133) must be measured as one unit

Measurement of Mean Hemispherical Candle-power.—By a suitable modification of the methods described above for the measurement of mean spherical candle-power the sphere may also be used to measure the mean hemispherical or mean zonal candle-power of a source (see p 87) In the case of the former measurement it is generally the lower hemisphere that is in question The source is inserted in an opening at the top of the sphere as shown in Fig 132 ([81]) The radius of this opening must be at least twice that of the smallest sphere which will contain the light source, but must not exceed about 0 5 R, where R is the radius of the sphere ([82]) It should be either totally uncovered, or else covered with a surface blackened on the under-side. The screen S_1 placed over the sub-standard lamp must be sufficiently large to screen the whole of the opening or blackened surface from the direct light from this lamp. The measurement is then made in the usual way, the reading B_T obtained with the test lamp being doubled in calculating the mean hemispherical candle-power of L_T from the mean spherical candle-power of L_S, since m h s c p = (lumens)/2π

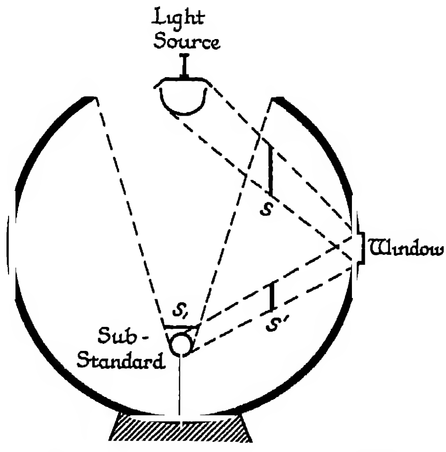

FIG 132 —The Measurement of Mean Hemispherical Candle-power

A less accurate method is that illustrated in Fig 133 ([83]) The source is surrounded by a box screen H, which, being blackened on the inside, absorbs all the light from the source except that emitted in the hemisphere or zone for which measurements are desired This screen must be whitened on the outside, but even so it disturbs the distribution of reflected flux in the sphere to a degree depending on its size, so that this method should only be used in the case of small sources.

A similar arrangement may be used in making measurements by the substitution method The screen H used with the test lamp in the sphere is placed over the sub-standard so that the light from it is absorbed exactly over the same zonal areas as in the case of the test lamp The mean spherical candle-power of the combination of sub-standard and screen is then measured as described on p. 204, and the value thus obtained is multiplied by the ratio $2B_T/B_S$, where B_T and B_S are respectively the brightness of the sphere window when the test lamp and the sub-standard are placed in the sphere, each being shaded by the screen H.

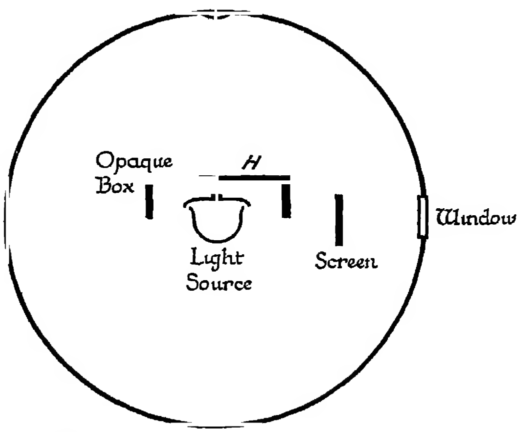

FIG 133 —The Measurement of Mean Hemispherical or Mean Zonal Candle-power

It is clear that, with a source of finite dimensions, a sharp cut-off at the edge of a given zone can only be obtained when the screen is large compared with the dimensions of the light source. In the same way, accurate delimitation of the hemisphere can only be obtained by the method illustrated in Fig 132 when the radius of the opening is large, and as this is limited by other considerations to a maximum of 0 5 R, it follows that the diameter of the sphere used for the measurement of mean hemispherical candle-power of a source must be at least eight times the diameter of the source Exact positioning of the source with reference to the edge of the opening is necessary, and an auxiliary photometer has been devised (84) for determining the photometric "centre of gravity" of a source of light This piece of apparatus is shown in section in Fig 134 The light from

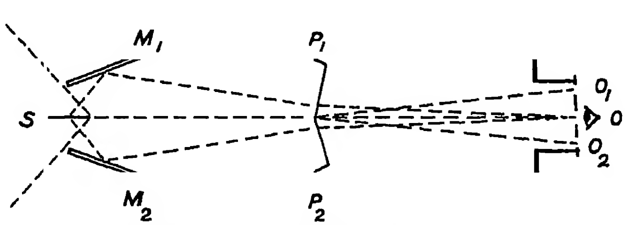

FIG. 134 —The Photometric Centroid Finder

the upper and lower parts of the lamp respectively illuminate the two sides of the Bunsen screen S, which is placed at a distance $r\sqrt{3}$ from the source, where r is the radius of the circular opening at the top of the sphere The two sides of the screen are seen in juxtaposition at O by means of the mirrors M_1, M_2, and the prisms P_1, P_2 When a balance has been obtained the eye is shifted to O_1 or O_2,

and then sees the source directly with the screen S as a horizontal black line dividing it into two parts The position of the line on the source is noted, and the latter is then placed in the sphere so that this line lies in the plane of the edges of the sphere opening.

Non-Spherical Integrators.—Certain disadvantages attendant on the spherical form, notably the difficulties of construction and the awkwardness of standing any object in a sphere, have led to suggestions for a modification of form. Of the regular polyhedra, the simplest to construct is the *cube*, and this form of integrator has, therefore, received considerable attention ([85])

It is clear that, if a light source be placed in the centre of a cube, and if a window be provided at the centre of one of the sides, the line joining the source and the window forms an axis of symmetry of the apparatus and, by means of four imaginary planes passing through this axis, the cube can be divided into eight portions, each of which is similarly situated with respect to the source and the window (see Fig. 135). It follows that the flux reflected to the

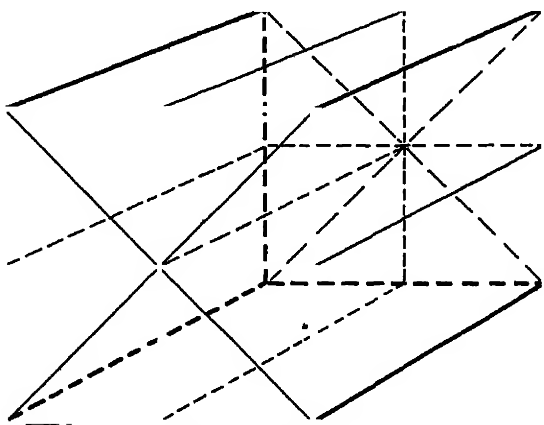

FIG. 135.—Symmetrical Division of a Cubical Integrator.

window from any given area in one of these eight portions will be exactly equal to the flux reflected to the window from an equal and equally bright area similarly situated in any of the other seven portions. Thus if a light source had a flux distribution such that its candle-power was uniform (but not necessarily the same) in each of the eight angular regions defined by the four imaginary planes above mentioned, the reflected flux received at the window would be proportional to the mean spherical candle-power of the source. It follows from this general consideration of the problem that the best position for a source in an integrating cube is at the centre of the cube, with its axis of symmetry perpendicular to the line joining the source and the window.

The amount of flux received at the window as a result of the incidence of a given amount of flux at each part of the cube surface has been determined empirically ([86]), and, using the results thus obtained, it is possible to calculate for any source of which the polar curve is approximately known the error which will be made in

comparing it with a source of some other known flux distribution. The results of this calculation applied to typical light sources in common use are given in the table below .—

TABLE SHOWING THE ERRORS OF MEASUREMENT DUE TO THE USE OF AN INTEGRATING CUBE FOR CERTAIN TYPICAL LIGHT SOURCES.

Light Source	Polar Curve Type (see Fig. 136)	Apparent M S C P in cube (Relative to Point Source)
Point source . . .	—	1 00
Electric incandescent lamp (vacuum, squirrel-cage filament) . .	*A*.	0 99
Electric lamp (gas-filled, ring filament)	*B*.	1·005
Gas lamp (upright mantle) .	*A*.	0 99
Gas lamp (inverted mantle) . .	*C*	1·05
Arc lamp	*D*	0 955

N B.—The above figures have been calculated on the assumption that (*a*) the source is placed at the centre of the cube, with its axis of symmetry vertical, and (*b*) the reflection factor of the cube surface is about 85 per cent.

As a compromise between the simple cube and the sphere, the proposal has frequently been made to use a cube in which the corners are blocked out so as to form a fourteen-sided polyhedron ([87]) Experiments have also been made with a rectangular box modified in this way ([88]), and on the regular icosahedron ([89]). Another form of integrator which has been used is a hemisphere, either open, or closed with a plane cover ([90]). The statement has been made that the form of the integrator is not of great importance, but this is incorrect, except when the light distribution of the test lamp is identical with that of the sub-standard. In all other cases no adequate theoretical treatment can be given, but from the general principles outlined in previous sections it may safely be concluded that the inaccuracy increases with (*a*) the difference in light distribution between test lamp and sub-standard, (*b*) the departure of

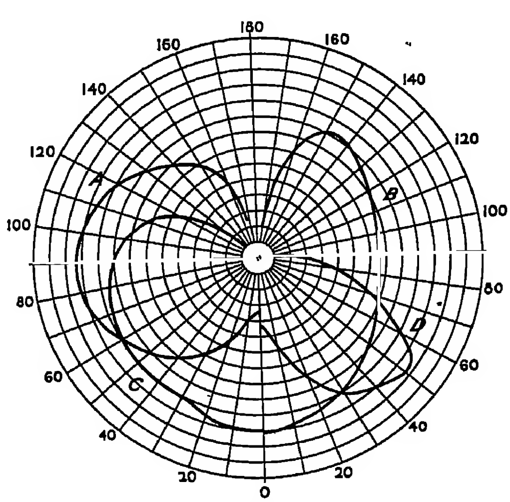

FIG 136 —Typical Polar Curves for the Sources referred to in the above Table

the integrator from the true spherical form, and (c) the absorption factor of the inner surface of the integrator. The departure from the theoretical flux distribution due to the presence of foreign objects is also greater in the case of a non-spherical integrator, and it is probable that lack of perfect diffusion from the inner surface would be more important than in the case of a sphere ([91]).

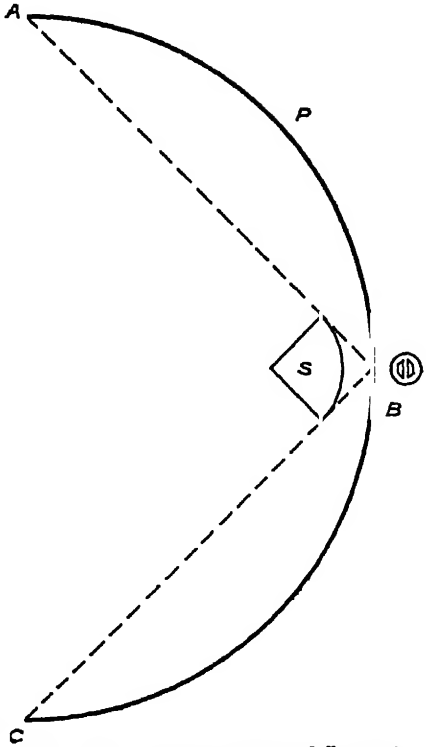

FIG. 137.—The Hemispherical Integrating Photometer.

A modified form of hemispherical integrator has been devised for the measurement of large sources ([92]) This consists of a whitened hemisphere ABC (Fig 137) with a translucent window at B. S is a special form of compensating screen, which is of such a size and so placed as to shield the window completely from the open face of the hemisphere. The side of S facing the window is a convex mirror with a central black foliated pattern, as shown in Fig. 138 The window receives light from any point P of the hemisphere, both directly and by reflection in S, and the shape and dimensions of the foliated pattern are determined empirically, so that a given amount of flux reaching the hemisphere produces the same effect on the window B, no matter to what part of the hemisphere this flux may be directed The source is placed with its axis in the plane AC, and two measurements are made, the source being turned about its axis through an angle of 180° for the second measurement. The "simultaneous method" of measurement is employed. It is clear that this form

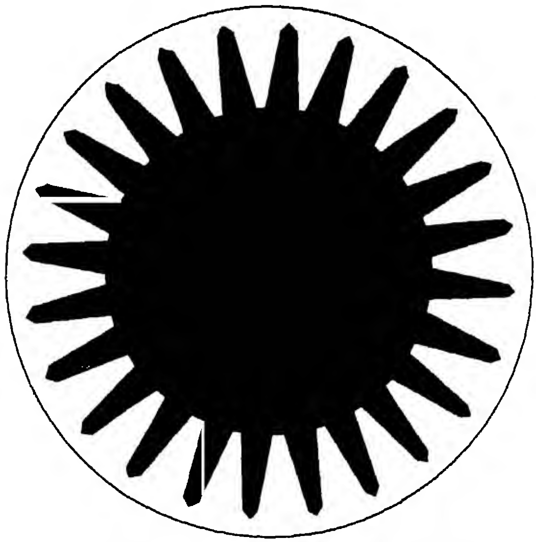

FIG. 138.—Compensating Screen for the Hemispherical Integrator

of integrator lends itself readily to a measurement of the total flux in the beam of light given by a projector (see Chapter XIV, p. 422)

General Conclusions regarding the Practical Use of a Sphere.—While it is difficult to lay down any definite rules regarding the employment of an integrator for the practical measurement of the mean spherical candle-power of a source, the following general principles may be regarded as of fairly wide application to such problems as are met with in ordinary photometric practice where an accuracy of 1 to 2 per cent. is aimed at.—

(1) The integrator should preferably be spherical in form, and the closer the approximation to the spherical form the more certainly is it possible to calculate the magnitude of the error likely to be made in measurements under any given set of conditions.

(2) The integrator must be coated internally with a paint which dries with a surface which has a high reflection factor and is as matt and as non-selective as possible. The surface must be renewed at frequent intervals, a year being the maximum period allowed to elapse between re-painting

(3) The true substitution method of measurement is preferable to the simultaneous method, but it demands a larger integrator When the true substitution method is used the ratio of the surface area of the integrator to that of the light source (including any surfaces, such as shades or reflectors, which affect the light distribution of the source under normal working conditions) should be not less than 100 times the ratio of the absorption factor of these surfaces to the absorption factor of the sphere surface. When the simultaneous method of measurement is used, the ratio of the surface area of the integrator used to that of the test source and its auxiliary apparatus may be as low as 40.

(4) Any auxiliary parts of the light source which it is necessary to have in the sphere (excluding the surfaces which affect the light distribution of the source under working conditions) should be painted white or covered with a white material.

(5) The window should behave as nearly as possible as a perfect diffuser for transmitted light, it should be non-selective, and its inner surface should be matt and flush with the inner surface of the sphere Its diameter should not exceed one-tenth of the diameter of the sphere, and may with advantage be smaller than this.

(6) Each screen should be just large enough to shield the window from the direct light coming from any part of a source (either test lamp or sub-standard) which contributes to the total flux from that source under ordinary working conditions, *e g*, light from a reflector, frosted globe, *etc.*, must be prevented from reaching the window directly by interposing a screen Screens should be painted with the same material as that used for the sphere surface, they should not be larger than is necessary to shade the window completely, and they should preferably be placed at a distance from the source equal to about one-third of the distance between the source and the window

(7) The sub-standard used should, if possible, have a light distribution which is similar to that of the test source. If this can be arranged, the true substitution method can be used and the errors due to non-compliance with recommendations (1), (2), (5) and

(6) are much reduced. Where this is impossible, as in the measurement of sources giving a concentration of light in certain directions, the test source should be so oriented that the regions of maximum illumination of the sphere are visible from the window, and the simultaneous method of measurement is generally preferable.

(8) When the true substitution method is employed, the source should be placed as nearly as possible at the centre of the sphere. When the simultaneous method is used, the two sources should be placed symmetrically with respect to the sphere window, and not too close either to each other or to the surface of the sphere.

(9) When the cube or fourteen-sided polyhedron is used instead of the sphere, the light source should be placed in the centre and should have its axis of symmetry perpendicular to the line joining the window of the integrator and the source The window should be in the centre of one side, and not at an edge or corner

Too much emphasis cannot be placed on the fact that the above rules are only generalisations, and any attempt to apply them universally without an intelligent appreciation of the theory of the sphere is certain to lead to large errors in particular cases The integrating photometer is an instrument of the utmost value when properly used, but its unintelligent application may easily be attended by unsuspected errors quite outside the limits of accuracy of ordinary photometric measurement

The sphere has been applied to several problems in photometry other than the measurement of candle-power. Some of these applications will be described in Chapter XIII, pp. 378 and 390 It can be used, in conjunction with a lamp, as a comparison source of adjustable candle-power, since the flux emitted from a window in the sphere is strictly proportional to the area of the window ([93])

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(1) See, *e g*, L Bell, Astrophys J, 45, *1917*, p 1, J Teichmuller, Licht u Lampe, 2, *1913*, pp. 802 *et seq*

(2) See Report of Frank. Inst Committee, J, 90, *1885* (supplement)

C Heim E T Z, 7, *1886*, p 384, and 8, *1887*, p 414, Z f I, 7, *1887*, p 35

H Krüss E T Z, 8, *1887*, p 356

Messrs. Queen & Co. El World, 18, *1891*, p 184

E Rousseau Comptes Rendus des Travaux du Comité Int des Essais Electriques de l'Exposition d'Anvers, *1885*; L'Electricité à l'Exposition Univ d'Anvers, *1885*, p 233

P H Ledeboer Lum El, 26, *1887*, p. 58.

(3) It has been found that the candle-power of a gas-filled electric incandescent lamp is higher when the lamp is mounted "tip-up" than when it is pendent. See G W. Middlekauff and J F Skogland, *loc cit*, note (16), *infra*

(4) F W Hartley Electrician, 11, *1883*, p. 76, Lum El, 10, *1883*, p 58; J of Gas Lighting, 41, *1883*, p. 1008, Lum. El, 31, *1889*, p 220

W J Dibdin Soc Chem Ind, J, 3, *1884*, p 277, and 4, *1885*, p 250; J. of Gas Lighting, 44, *1884*, p 59, Lum El, 31, *1889*, p 220

S Elster J G W, 30, *1887*, p 1094; Centralbl f Elektrot, 10, *1888*, p 33, Lum El, 27, *1888*, p 85

H Kruss. Centralbl f. Elektrot, 10, *1888*, p 117, E T Z, 8, *1887*, p 356, Z f I, 8, *1888*, p 70, Central-Ztg f. Opt u Mech, 9, *1888*, p 74, J G W., 30, *1887*, p. 1145, and 39, *1896*, p 265, Z f I, 9, *1889*, p 33

F Vogel E T Z, 8, *1887*, p. 31

E Liebenthal "Prakt Phot," p 287

D Coglievina. Der Gastechniker, 7, *1887*, p 193

(5) The surface of the mirror would, naturally, be perpendicular to the photometer screen shown in Fig 112.

E Voit and H Krüss Lum El, 7, *1882*, p. 402.

A. Palaz "Photométrie," p 170 (190).

E Liebenthal "Prakt. Phot," p 288.

(6) See p 113, and F Uppenborn, E.T Z, 11, *1890*, p 138, Z f I, 10, *1890*, p. 334 "Lehrbuch d. Phot," p 253.

On the determination of the reflection factor of a mirror, see chap XIII, p. 381.

(7) A. Blondel Ecl El., 2, *1895*, p 385

(8) A. P Trotter Inst El Eng, J, 21, *1892*, p 360

F. Laporte Ecl El, 20, *1899*, p 302; Soc Int des Elect, Bull, 16, *1899*, p 288

C P. Matthews El World, 35, *1900*, p 824, Am Gas Light J, 73, *1900*, p 371

L W Wild Illum Eng, 2, *1909*, p 193

G H Stickney and S L E Rose. Illum Eng Soc N Y, Trans, 6, *1911*, p 641, Am Gas Light J, 95, *1911*, p 210

(9) F von Hefner-Alteneck E T Z, 4, *1883*, p 445; Lum El, 10, *1883*, p 498

E P Hyde and F E Cady Bureau of Standards, Bull, 2, *1906*, p. 415, El. World, 48, *1906*, p 956, E T Z, 29, *1908*, p 83

(10) E Brodhun See E. Liebenthal, "Prakt Phot," p 227

M Schanzer [K Satori] E u M, 31, *1913*, p. 442, Z f I, 34, *1914*, p 62; Electrician, 71, *1913*, p 863, El World, 62, *1913*, p 653

H. Krüss J G W., 55, *1912*, p 1148, Z f Bel, 18, *1912*, p 421

A Vernon-Harcourt El. Rev, 23, *1888*, p 39, Lum El, 29, *1888*, p 286, Electrician, 21, *1888*, pp 412 and 474; El World, 12, *1888*, p 159, J of Gas Lighting, 52, *1888*, p 118

C C Colby and C M Doolittle Illum Eng Soc N Y, Trans, 18, *1923*, p 273.

(11) H Krüss J G W, 41, *1898*, p. 253, 50, *1907*, p 1017, J of Gas Lighting, 100, *1907*, p 901, Z f I, 37, *1917*, p 33

M Herschkowitsch. J.G W, 44, *1901*, p 650, Z f Bel, 7, *1901*, p 321, Z. f. I, 21, *1901*, p 365

Van R. Lansingh Am. Gas Light J, 85, *1906*, p 92

(12) H Krüss Central-Ztg f. Opt u Mech, 8, *1887*, p 85, Der Gastechniker, 7, *1887*, p 241

R W. Williamson and J. H. Klinck Frank Inst, J, 149, *1900*, p 66

S L E Rose Illum. Eng Soc N Y, Trans, 8, *1913*, p 379

See also Illum Eng, 3, *1910*, p 506

(13) Alternatively a simple linkage may be used to ensure that L remains upright as it moves round M.

(14) This formula assumes that distances are in mm, and that the values of d_0 have been based on a standard illumination of ten metre-candles at the photometer surface

(15) Since the light is incident on the glass at an angle of about 45°, the true correction for mirror thickness is $2\,t\,\sec\theta/n$ where $\sin\theta = (\sin 45°)/n$

This equals $2\,t\Big/\sqrt{n^2 - \frac{1}{2}}$ which, for $n = 1{\cdot}5$, reduces to $1{\cdot}51\,t$.

(16) It has been found that at constant voltage, both the current consumed and the candle-power are different when a gas-filled electric incandescent lamp is rotated, the current and candle-power changes being always in opposite directions The direction of the change depends on the speed of rotation and the type of lamp The candle-power change may be as much as 15 to 20 per cent See G. W Middlekauff and J F. Skogland, Bureau of Standards, Bull, 12, *1915*, p 587; El. World, 64, *1914*, p 1248; Electrician,

77, *1916*, p 252, E T Z, 39, *1918*, p 128, Z. f Bel, 22, *1916*, p 111, Lum El, 34, *1916*, p 89; Rev Gén de l'El, 1, *1917*, p 127

See also D H. Tuck, El World, 65, *1915*, p 78, and C H Sharp, Illum Eng Soc N Y, Trans, 9, *1914*, p 1021, El World, 64, *1914*, p 992

(17) Though the m s c p is not the mean of the values of candle-power thus measured (see p 91, and J Abady, J of Gas Lighting, 87, *1904*, p 108)

C Hering Illum Eng Soc N Y, Trans, 4, *1909*, p 354

(18) E Brodhun. Z f I, 15, *1895*, p 337.

J S Dow and V H Mackinney Optical Convention, Proc, *1912*, p 70; Electrician, 69, *1912*, p 691, J of Gas Lighting, 117, *1912*, p 277, E T Z, 34, *1913*, p 129

C H. Sharp Illum Eng, 5, *1912*, p 264

J. W Roper J of Gas Lighting, 129, *1915*, p. 143, Am Gas Light J, 102, *1915*, p 178

See also P Devigne, Z f Bel, 20, *1914*, p 241

(19) E Rousseau L'Ingenieur Conseil, 6, *1884*, p 251; Comptes rendus des travaux du Comité international des essais electriques de l'Exposition d'Anvers, *1885*, p 85, L'Eclairage à l'Exposition Universelle d'Anvers, p 233, Lum El, 26, *1885*, p 60

H Krüss E T Z, 8, *1887*, p 356, Centralbl f Elektrot, 9, *1887*, p 746.

F Vogel E T Z, 8, *1887*, p 31.

(20) J A Fleming Inst El Eng, J, 32, *1902*, p. 119, Z f Bel, 9, *1903*, pp 377, 389, 401 and 415, Electrician, 50, *1903*, p 481, Industria, 17, *1903*, pp 117, 151, 221, 235 and 250

See also F W Carter, El Rev, 47, *1900*, p 197, J of Gas Lighting, 76, *1900*, p 904, Brit Patent, No 8,704 (*1900*)

(21) The direction 45° below the horizontal has also been proposed (A J Rowland, Frank Inst, J., 150, *1900*, p 241, Electrician, 46, *1900*, p 362)

(22) E T Z, 31, *1910*, p 302, 32, *1911*, p 402; J G W, 53, *1910*, p 785, Z f Bel, 16, *1910*, p 299, British Standard Specification for Tungsten Filament Lamps (No 133)

(23) W Moller E T Z, 5, *1884*, pp 370 and 405

(24) A Crova Congrès des Electriciens, Paris, *1889*, C r, p 205, Lum El, 33, *1889*, p 475

C P Matthews Phys Rev, 6, *1898*, p 55, J de Phys, 7, *1898*, p 443

(25) Am I E E, Trans, 14, *1897*, p. 90 (carbon lamps), Nela Convention, Proc, *1897*, p 399

E P Hyde and F E Cady El World, 48, *1906*, p 956, and 49, *1907*, p 1260, Illum Eng (N Y), 1, *1907*, p 889, Bureau of Standards, Bull, 3, *1907*, p 357, Electrician, 60, *1907*, p 220, Ecl El, 50, *1907*, p 289

C Paulus. Z. f Bel, 14, *1908*, pp 195 and 206, J G W, 51, *1908*, p 933

(26) J. of Gas Lighting, 104, *1908*, p 33

(27) E Liebenthal "Prakt Phot," p 332 (See also Z. f. I., 19, *1899*, pp 193 and 225)

(28) F Uppenborn E T Z, 28, *1907*, pp 139 and 168, Electrician, 59, *1907*, p 87, Ecl El, 50, *1907*, pp 435 and 460, 51, *1907*, p 140

(29) See British Standard Specification for Electric Lamps, No 161, *1924*

H Armagnat Rev Gén. de l'El, 3, *1918*, p 603

J T Macgregor Morris and others Illum Eng, 10, *1917*, pp 7, 10, 59 and 61, El Rev, 80, *1917*, p 401, Gas J, 137, *1917*, p 160, Electrician, 78, *1917*, p 527

P Eitner J G W, 57, *1914*, p 656, J of Gas Lighting, 127, *1914*, p 99

G Zevi Elettrotecnica, 11, *1924*, p 612

(30) See, *e g*, Nela Convention, Proc., *1897*, p 399, Electrician, 39, *1897*, p 414

L Bloch E T Z, 26, *1905*, p 646

L Bell Am I E E, Trans, 19, *1902*, p 5

A J Wurts Am I E E, Trans, 20, *1902*, p 103

(31) For many types of electric lamps it has been found that a measurement of candle-power in one (sometimes two) definite directions is sufficient to give the mean spherical candle-power to a very close approximation See E Liebenthal, Z f I, 19, *1899*, pp 193 and 225, J G W, 43, *1900*, p 154, K Zickler, E u M, 33, *1915*, pp 469, and 481 E T Z, 37, *1916*, p 67, N A Halbertsma, E u M, 35, *1917*, p 65 For mirror apparatus designed to give the m s c p at a single reading by this method, see H Krüss, Z f I, 37, *1917*, p 33, E T Z, 38, *1917*, p 538, also El World, 67, *1916*, p 162

(32) J of Gas Lighting, 96, *1906*, p 98

(33) E P Hyde and F E Cady Bureau of Standards, Bull, 4, *1907*, p 91, El Rev. and West Elect, 61, *1912*, p 942, Nela Bull, 1, *1917*, p 192

(34) C H Sharp Phys Rev, 11, *1900*, p 181, Z f I, 20, *1900*, p 225

(35) A Blondel Ecl El, 3, *1895*, pp 406, 538 and 583, J de Phys, 5, *1896*, p 222, C R, 120, *1895*, pp 311 and 550, E T Z, 16, *1895*, p 608, Z f I, 15, *1895*, p 303, Illum Eng, 3, *1910*, p 311

J Rey Soc Int des Elect, Bull, 12, *1895*, p 211

H Krüss J G W, 38, *1895*, p 516

See also a different form of lumenmeter described by E J Houston and A E Kennelly, El. World, 27, *1896*, p. 509, and 38, *1901*, p 395, Electrician, 47, *1901*, p 898.

(36) A Blondel. Ecl El, 8, *1896*, p. 49, El World, 28, *1896*, p 175; Illum. Eng., 2, *1909*, p 785, and 3, *1910*, p 311.

(37) C P. Matthews Am. I E E, Trans, 18, *1901*, p 677, and 20, *1902*, p 59; Am Gas Light J, 74, *1901*, p 607, and 77, *1902*, p 708, El World, 38, *1901*, p 549, 40, *1902*, p 746, and 42, *1903*, p. 578, El Rev (Chicago), 41, *1902*, p 593, Z f Bel, 9, *1903*, pp 91 and 335, Ecl El, 34, *1903*, p 215.

W P Thompson Brit Patent No 14,688 (*1902*)

G B Dyke Phys Soc, Proc, 19, *1904*, p 399; Phil Mag, 9, *1905*, p 136, Chem. News, 90, *1904*, p 266

C Léonard Ecl El, 40, *1904*, p 128, and 45, *1905*, p 329

E P Hyde Bureau of Standards, Bull, 1, *1905*, p 255, El World, 44, *1904*, p 687

E P Hyde and F E Cady Bureau of Standards, Bull, 4, *1907*, p 91

C P Steinmetz Western El, 40, *1907*, p 542

H. Krüss J G W, 51, *1908*, p 597, E T Z, 30, *1909*, p 359, J of Gas Lighting, 103, *1908*, p 385, Illum Eng, 2, *1909*, p 710, El World, 52, *1908*, p 307, and 53, *1909*, p 1112, Am Gas Light J, 89, *1908*, p 404

J. Sahulka E T Z, 39, *1918*, p 253, Electrician, 82, *1919*, p 255; Illum Eng, 11, *1918*, p 195; Elettrot., 6, *1919*, p 300, E T Z, 44, *1923*, p 665, E u M, 41, *1923*, p 530; Rev Gén de l'El, 15, *1924*, p 879, Illum Eng, 16, *1923*, p 212.

R Hiecke E u. M (Lichttechnik), 42, *1924*, p 307

(38) W E Sumpner Phys Soc, Proc, 12, *1892*, p 10, Phil Mag, 35, *1893*, p 81, J of Gas Lighting, 61, *1893*, p. 15, Electrician, 30, *1893*, p 381, Z. f. Bel, 2, *1896*, pp 75 and 85; Chem News, 66, *1892*, p 301

(39) This may also be shown thus —

The total flux emitted by the source must be equal to that absorbed by the walls of the sphere, *i e*, to the total flux reaching the walls, multiplied by the absorption factor Hence $F = (1 - \rho)(F + 4\pi r^2\Phi)$ where Φ is the flux reaching unit area of the sphere by reflection Hence $\Phi = \rho F/4\pi r^2(1 - \rho)$. (R Ulbricht, Das Kugelphotometer, p 15) See also E Mascart, Lum El, 28, *1888*, p 180, Soc Int des Elect, 5, *1888*, p 103, Rev. Int de l'El, 6, *1888*, p 345

(40) R Ulbricht. E T Z, 21, *1900*, p 595

(41) See, *e g*, L Bloch, E T Z, 26, *1905*, pp 1047 and 1074, and 27, *1906*, p 63, Ecl El, 45, *1905*, pp 436 and 501, Electrician, 56, *1906*, p 1057, Illum Eng (N Y), 1, *1906*, p 421, El Rev, 58, *1906*, p 445, Illum Eng, 1, *1908*, p 274

Various Authors Z. f I, 26, *1906*, p 316

H Krüss Hamburg Naturwiss Verein, Verh, 15, *1907*, p lxxviii

A. A Perrine El World, 54, *1909*, p 727, Illum Eng, 3, *1910*, p 268.

E Dyhr E T Z, 31, *1910*, p. 1295

T. Hirobe and H Matsumoto Dept of Communications, Tokyo, Electrotechn Lab. Report, No 23, May, *1915*.

E B Rosa and A H Taylor Bureau of Standards, Bull, 18, *1922*, p 281.

Also papers quoted in the notes which follow

(42) N A. Halbertsma Z f Bel, 22, *1916*, p 43.

(43) See, *e g*, A. Crova, Ann Chim Phys, 6, *1885*, p 342, J de Phys 5, *1886*, p 193

(44) E B Rosa and A. H Taylor Bureau of Standards, Bull, 18, *1922*, p 281

(45) N K Chaney and E L Clark Illum Eng Soc N Y, Trans, 10, *1915*, p 1

(46) R Ulbricht Das Kugelphotometer, p 93, E T Z, 26, *1905*, p 512

N K. Chaney and E L Clark [R B. Chillas] Illum Eng Soc N Y, Trans, 10, *1915*, p. 1

(47) Further, in the case of a small source the two screened areas (*ac* and *bd* in Fig 127) are equal if the distance between source and screen is one-third of the sphere radius Equality of these areas clearly tends towards equality of δ and δ_s in the general case where the lamps may have any distribution whatever

(48) See p 107, and C. C Paterson and B P Dudding, Phys Soc, Proc, 27, *1915*, p 230, Phil Mag, 30, *1915*, p. 34, N P L, Coll Res, 12, *1915*, p 51

(49) Ulbricht has proposed (E T Z, 28, *1907*, p 777) to correct for the presence of the screens by adding the percentage $(20s - 25s')$ to the measured value of L_T, s and s' being respectively the (single side) areas of the screens S and S'

(50) R Ulbricht E T Z, 21, *1900*, p 595

C H Sharp and P S Millar Illum Eng Soc N Y, Trans, 3, *1908*, p 502, Illum. Eng, 1, *1908*, p 1031

(51) N K Chaney and E L Clark Illum Eng Soc N Y, Trans, 10, *1915*, p 1.

(52) F A Benford, Jun. Illum. Eng Soc N Y, Trans, 11, *1916*, p 997; Z f Bel, 22, *1917*, p 50, E T Z, 38, *1917*, p 578

(53) The surface is here assumed to be either convex or flat everywhere. The area of a concavity is to be reckoned as that of the plane surface defined by its edges In other words, A is the area of a piece of paper which just covers the object when stretched tightly over it.

(54) R. Ulbricht "Das Kugelphotometer," p 25 The expression given in E T Z, 26, *1905*, p 512, is in error

(55) E B Rosa and A. H Taylor. Illum. Eng Soc N. Y., Trans, 11, *1916*, p 453,

(56) See, *e g*, B Monasch, E T Z, 27, *1906*, pp 669 and 695, El. World, 48, *1906*, p. 441, Illum Eng (N Y), 1, *1906*, pp. 586 and 700, Ecl El, 48, *1906*, pp 316, 356 and 391

(57) J W. T Walsh Illum Eng, 18, *1925*, pp. 12 and 42.

(58) R Ulbricht "Das Kugelphotometer," p 33

E B Rosa and A. H Taylor Bureau of Standards, Bull, 18, *1922*, p 281.

(59) R Ulbricht Illum Eng., 3, *1910*, p. 387

(60) F E Cady Frank Inst, J, 189, *1920*, p. 787, El World, 77, *1921*, p 368

(61) The difficulty of maintaining a surface of high reflection factor has, however, led to the suggestion that a grey surface should be adopted (E Presser, Elektrot Anzeiger, 23, *1906*, pp 885 and 912, El World, 49, *1907*, p 806) This is not advisable for the reason stated on p. 213

(62) E B Rosa and A H. Taylor Illum Eng Soc. N Y, Trans, 11, *1916*, p 453

(63) B Monasch E T Z, 27, *1906*, p 669; Illum. Eng (N Y), 1, *1906*, p 586, El World, 48, *1906*, p 441, Ecl El, 48, *1906*, p. 316

A H Taylor. Illum Eng Soc N Y, Trans, 16, *1921*, p 587.

(64) E Winkler-Buscher. E u M, 28, *1910*, p. 659.

D McFarlan Moore Illum. Eng Soc N Y, Trans, 16, *1921*, p 599

C C Paterson, J W T. Walsh, A K Taylor and W Barnett Inst El Eng, J, 58, *1920*, p 83

See also E Dyhr, E T Z, 31, *1910*, p 1295

(65) A. Utzinger E T Z, 36, *1915*, p 137, Z f I, 36, *1916*, p 23

R Ulbricht "Das Kugelphotometer," p 66

(66) A H. Taylor Illum Eng Soc N Y, Trans, 16, *1921*, p 587.

(67) A Utzinger *Loc. cit supra*

(68) J W. T. Walsh. Illum. Eng, 18, *1925*, pp 12 and 42 See also L Weber, Central-Ztg. f Opt u Mech., 5, *1884*, p 53

(69) M Corsepius Gesell deut Naturforscher u Aerzte, Verh, 80, *1908* (2, i), p 66

(70) This arrangement cannot, however, be used when the sphere window itself forms one of the comparison surfaces in the photometer head

R C Fox Opt Convention, Proc, *1926*

(71) Laboratory spheres are made by the Foote, Pierson Co, of New York, by Franz Schmidt and Haensch, of Berlin, and other manufacturers of scientific apparatus

(72) M Corsepius E T Z, 27, *1906*, p 468, Illum Eng (N Y), 1, *1906*, p 482, Ecl El, 48, *1906*, p 78, El World, 47, *1906*, p 1194, and 48, *1906*, p 23

E W Marchant Illum Eng, 3, *1911*, p 37

(73) E B Rosa and A H Taylor Illum. Eng Soc. N Y, Trans, 11, *1916*, p 453

(74) See also El World, 73, *1919*, p 1219, K Schmidt, Helios, 25, *1919*, p 313, and A L Powell and J A. Summers, El World, 77, *1921*, p. 216, also Rev. Gén de l'El, 15, *1924*, p 1196

(75) This device is used in the Sharp-Millar spheres made by the Foote, Pierson Co, of New York.

(76) Used at Philips' Glow Lamp Works, Eindhoven, Holland

(77) R von Voss E T Z, 38, *1917*, pp. 188 and 605 (see also N A Halbertsma, *ibid*, p 263), Z f I, 38, *1918*, p 200, Electrician, 80, *1918*, p. 630, and 81, *1918*, p 418, Rev Gén de l'El, 2, *1917*, p 807, E u M, 35, *1917*, p 292, and 36, *1918*, p 131

E Zopf Licht u Lampe, *1919*, p 94

(78) A. H Taylor Frank Inst, J., 194, *1922*, p. 543

(79) E B Rosa and A H Taylor Bureau of Standards, Bull, 18, *1922*, p 281

See also C O Bond Am Gas Inst, Proc, 7, *1912*, p. 291, Am Gas Light J, 97, *1912*, pp 418 and 426

(80) E Winkler Schweizerische E T Z, 4, *1907*, pp 85, 97 and 110

(81) R Ulbricht. E T Z, 26, *1905*, p 512, and 27, *1906*, p 50, Ecl El, 46, *1906*, p 432

(82) R Ulbricht "Das Kugelphotometer," p 79

(83) M Corsepius Illum Eng, 1, *1908*, pp. 801 and 895, Gesell deut Naturforscher u Aerzte, Verh, 80, *1908* (2, i), p 66

(84) R Ulbricht E T Z, 28, *1907*, p 777, El World, 50, *1907*, p 418, E T Z, 30, *1909*, pp 322 and 507, Z f I, 30, *1909*, p 322, Illum Eng, 1, *1908*, p 553

(85) W E Sumpner Illum Eng, 3, *1910*, p 323, Electrician, 65, *1910*, p 72, Z f Bel, 16, *1910*, p 437, El World, 55, *1910*, p 1220, Z d Vereines v Gas- u Wasserfachm in Ost-Ungarn, 51, *1911*, p 456

L W Wild. Illum Eng., 3, *1910*, p. 549

C C Paterson, J. W. T Walsh, A K Taylor and W Barnett. Inst El. Eng, J, 58, *1920*, p 83, Electrician, 83, *1919*, pp 625, 723 and 730, Illum Eng, 12, *1919*, p 335, Elettrot., 7, *1920*, p 645

J T Macgregor Morris Illum Eng, 15, *1922*, p 283

(86) H Buckley Inst. El Eng, J, 59, *1921*, p 143, Gas J, 154, *1921*, p 436, El World, 77, *1921*, p 839, E T Z, 42, *1921*, p 1072

(87) G. W. O Howe. Illum. Eng, 3, *1910*, p. 391 This form of integrator is made

by Messrs. Everett, Edgcumbe and Co., Hendon (El Rev , 93, *1923*, p 808 , J Sci Inst., 2, *1925*, p 201 , Illum Eng , 18, *1925*, p. 49).

See also J. T. Macgregor-Morris and A H Mumford, J Sci Insts , 2, *1925*, pp 353 and 385

(88) L O. Grondahl. Illum Eng Soc N Y , Trans , 11, *1916*, p 152

(89) K S Weaver and B. E Shackleford. Illum Eng Soc. N Y , Trans , 18, *1923*, p. 290 ; E T Z , 45, *1924*, p 1193 , Licht u. Lampe, *1923*, p 382

(90) B Monasch. E T Z , 27, *1906*, pp. 669, 695 and 803 , Illum Eng (N Y), 1, *1906*, pp 586 and 700 , El World, 48, *1906*, p 441 ; Ecl El , 48, *1906*, pp 316, 356, and 391 See also R Ulbricht, E T Z , 27, *1906*, p 803

(91) L Bloch Illum Eng 3, *1910*, p 388. On the use of the cone see R Ulbricht, Z f Bel., 28, *1922*, p 43, and E T Z , 43, *1922*, p 1262

(92) F A Benford, Jun. Illum Eng. Soc N Y , Trans , 11, *1916*, p 997 , 13, *1918*, p 323, 15, *1920*, p 19 , Optical Soc Am , J , 6, *1922*, p 1040

(93) See, *e g* , J of Gas Lighting, 104, *1908*, p. 33 , L Bloch, E T Z , 40, *1919*, p 602 , J G W , 62, *1919*, p 355 , Gas J , 147, *1919*, p 571 ; Licht u Lampe, *1919*, p 151

E. Karrer and A Poritsky Opt Soc Am., J , 8, *1924*, p 355 , Z. f. I., 44, *1924*, p 423

See also p 221, above.

CHAPTER VIII

HETEROCHROMATIC PHOTOMETRY

In the descriptions of photometric apparatus and methods given in previous chapters it has been tacitly assumed that the lights being compared are either alike, or nearly alike, in colour, so that while the eye of the observer is endeavouring to make a judgment of equality of brightness of the comparison surfaces it is not embarrassed (1) by a difference of hue In other words, it is assumed that measurement of *quantity* is not interfered with by a difference in *quality* of the things compared

Unfortunately this condition, which until now it has been necessary to assume fulfilled in order to simplify the treatment of the subject, is completely satisfied only in a very small proportion of the problems met with in practical photometry It has already been said (p 126) that the standards of candle-power at present available give a light which is very much yellower than that given by modern light sources under working conditions It follows that the measurement of such sources by comparison with the standards must, fundamentally, involve a considerable colour difference It is true that steps are taken to ensure that this colour difference is very much reduced, if not eliminated, in every-day photometry, but the difficulty is thereby only transferred to another link in the chain of measurements by which the source under test is compared with the primary standards, and nothing can remove the necessity for heterochromatic photometry at one stage or another of the series of comparisons.

An observer faced with the problem of making a photometric measurement by the comparison of two surfaces differing markedly in colour is tempted at once to condemn the operation as senseless, and the result obtained as almost without meaning (2), and he is to a certain extent justified by the physical principle that things which differ in kind cannot be compared in degree except by some quality which is common to both. Thus, in the case of two lights of different colours, while there is no theoretical difficulty in comparing their relative energies expressed in watts, this quantity being common to all forms of radiant energy, there is very considerable difficulty in comparing their relative effects on the retina, since these effects are different in kind as well as in degree This argument, however, if pushed to its logical conclusion, would almost deny the possibility of photometry at all. The position has been well expressed by C Fabry (3), as follows :—

"Confining oneself to the region of pure theory, one would therefore be tempted simply to condemn the problem as, by its very nature, contrary to reason. But it is not only from the theoretical point of view that the problem of heterochromatic photometry must be presented, its interest is pre-eminently practical and even

commercial; the problem *demands* solution, even if it be partly by means of a convention."

As a matter of common experience, although it may be difficult, if not impossible, to say with certainty when two contiguous red and green luminous surfaces have the same brightness, yet if their brightness be varied there is certainly a point on one side of equality at which the red is definitely brighter than the green, while there is similarly a point on the other side of equality at which the green is certainly the brighter, always assuming that the observer is normal as regards colour vision (4). It is the object of heterochromatic photometry to reduce, as far as possible, the region within which definite inequality just appears on each side The same degree of precision as in the case of homochromatic photometry is not attainable (5), and, unfortunately, the physiological phenomenon of simultaneous contrast (see p 70) causes the colours of two adjacent bright fields to appear even more widely separated in the spectrum than they really are (6) The various methods which have been used by different workers at different times will be described in this chapter, and the particular advantages and difficulties inherent in each of them will then be appreciated It is essential that the principal phenomena of colour vision, described in outline in Chapter III., should be borne in mind when any problem of heterochromatic photometry is under consideration (7)

One source of error which must be avoided in all photometry involving colour difference is the reduction of the brightness of the comparison surfaces to below the limit at which the Purkyně effect begins to be noticeable (see p 65). Thus, unless colour difference has been completely eliminated, the illumination of the photometer field should not fall much below 10 metre-candles, for otherwise the variation of sensation with brightness change will be different on the two sides, *e.g*, a balance made at 10 metre-candles will no longer be a balance to the eye if the illumination of both comparison surfaces be reduced to 1 metre-candle (8). This consideration at once rules out one of the early methods of heterochromatic photometry, *viz*, that in which acuteness of vision was used as a measure of brightness (9). A têst chart, such as a number of lines of small type printed in black on white paper, or some geometrical pattern, was illuminated in turn by the lights to be compared, and the illumination was gradually altered in each case until a given line of type or a pattern of a certain fineness became just distinguishable (10). The illuminations under these conditions were assumed to be equal, and the candle-powers of the sources were calculated accordingly (11).

The same objection applies also to the elimination of the colour difference by reducing the brightness of the comparison surfaces to within the photochromatic interval of the eye (12) (see p. 67) The sensitivity of both these methods is, moreover, very low.

The Compensation or Mixture Method.—In this method of heterochromatic photometry an ordinary direct-comparison photometer head is used, but the colour difference is reduced by illuminating one or both of the comparison surfaces with light from both the sources which are being compared While the ease with which a setting of the photometer head can be made is much improved by

the reduction of the colour difference thus achieved, the sensitiveness is correspondingly reduced by reason of the very fact that the photometer surfaces do not each receive light from one source alone (see p 154) In J Wybauw's form ([13]) a Bunsen photometer is used, with an auxiliary mirror to reflect light from one of the sources on to the side of the disc remote from the source (see Fig 139) In Grosse's form, which is much more complicated ([14]), one side of the photometer receives n per cent. of its light from lamp L_1, and $(100 - n)$ per cent from L_2, while the other side receives n per cent from L_2 and $(100 - n)$ per cent from L_1 n can be varied at the option of the observer according to the degree of colour difference present, but as the colour difference is diminished by increasing n, so also the sensitivity is reduced, until when $n = 50$ both colour difference and sensitivity vanish together ([15])

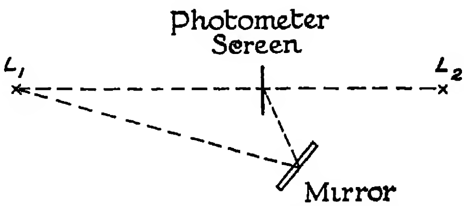


FIG 139 —Wybauw's Compensation Photometer.

An instrument depending on the formation of two sets of shadow patterns, one by each source, has been described by W B. von Czudnochowski ([16])

Direct Comparison with Small Colour Differences.—The direct comparison of sources giving lights of markedly different colours is, as stated at the beginning of this chapter, inaccurate and unsatisfying to the observer, but if the colour difference be comparatively slight a measurement may be made, with a suitable form of photometer head ([17]), to an accuracy of about the same order as that obtainable by either of the methods already described in Chapter VI, so long as the brightness of the comparison surfaces is considerably in excess of the value at which the Purkyně effect begins to operate If this precaution be not observed very large errors may be introduced (see p 66) If the two fields of a Lummer-Brodhun contrast photometer, for example, be illuminated by lights differing slightly in colour, a position of the photometer head can be found at which the contrast on both sides of the field appears to be equal This may be taken as the position of balance When the colour difference is about the same as that between the light from two tungsten filament vacuum lamps operating respectively at 7 and 5 3 lumens per watt, it is found that an observer, following the procedure described on pp 165 to 167, will repeat his measurements from day to day to an accuracy of about 1 per cent, while two normal-sighted observers will generally agree with each other to about the same accuracy When, however, the colour difference is increased beyond this limit, not only do different observers disagree markedly, but the same observer becomes inconsistent from day to day. It is a somewhat curious psychological effect that, with a considerable colour difference, a single observer may obtain very consistent readings during the course of a single set of observations, but on another occasion his readings, although again consistent among themselves, are quite different from his previous set ([18]). There is apparently an

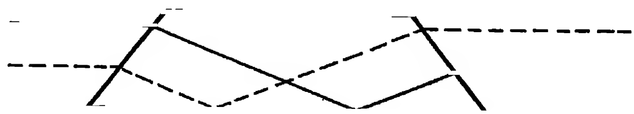
unconscious adoption of a certain criterion of equality which, although remembered throughout a single set of measurements, is forgotten if a considerable period elapse between one set and the next.

The Cascade Method.—One method of overcoming the uncertainty of a measurement involving a large colour difference, such as, for example, the comparison of lamps operating at a normal efficiency of 8 lumens per watt with those giving light of the same colour as that from a pentane lamp (1·3 l p w), is to divide this colour difference into a number of steps, using sets of lamps operating at intermediate efficiencies Thus at the National Physical Laboratory, between the international standards of candle-power and the sub-standards used for the measurement of lamps of normal efficiencies, five sets of sub-standards are interposed, operating respectively at 2 1, 3, 4, 5·5 and 7 lumens per watt The lamps in each set are carefully compared with the set of lower efficiency by a number of observers, and thus, from the results of five comparisons, the value of the highest efficiency set in terms of the international standards, is obtained ([19])

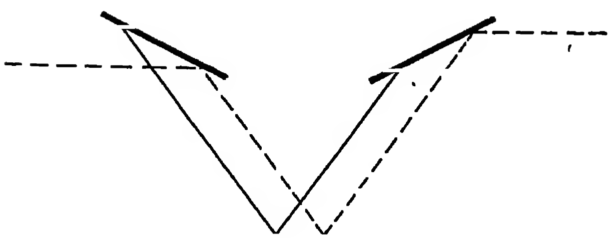
Although, when many readings are taken, the results obtained by the cascade method for any one observer are not more consistent with the average of a number of observers than when a comparison entailing the whole colour difference is made, it appears that the day-to-day consistency of a single observer is better by the cascade method ([20]) The method also has the advantage that when once the intermediate sub-standards have been measured, they are available for the measurement of lamps of any efficiency lying within the range they cover In the work above quoted, the cascade method and the ordinary comparison method were found to give results in agreement to 0 3 per cent. The chief disadvantage of the cascade method is the unavoidable one that an observer who tends to weight either side of a comparison field, for example the blue, will gradually increase his error as the series is built up It is for this reason that the method can only be satisfactorily used when the mean results of a large number of observers can be obtained. The mean values so assigned to the lamps of any set in the cascade can then be used for the measurement of lamps which they happen to match in colour, so that only a few observers are then needed for this measurement, just as in any other case of homochromatic photometry

Some part, at any rate, of the consistent differences which are found to exist between observers when making a heterochromatic comparison may well be due to the fact that the luminosity curve of the retina near the fovea is not everywhere exactly the same (see p 69). The error due to this cause may be reduced by making two or more sets of measurements with the comparison fields differently arranged in the observer's field of view This may be achieved by inserting in the eyepiece of the photometer head a prism of the form shown in Fig. 140 (*a*), or an arrangement of mirrors as shown in Fig. 140 (*b*) If either of these systems be rotated through an angle of 90°, the image seen through them is rotated through 180°. Thus if the photometer field be symmetrically divided into two parts, as in the case of the Lummer-Brodhun contrast photometer,

by making measurements with the prism in each of four positions 45° apart, any irregularities in sensitivity of the central part of the observer's retina should be compensated. A simple left-to-right reversal of field may be obtained by rotating the photometer through



(a) Prism



(b) Mirrors

FIG 140 —Inversion Systems.

180° about its horizontal axis, and then turning it through 180° about its vertical axis, the observer being now on the opposite side of the bench.

Use of Characteristic Equations.—For many purposes it has been found convenient to use the characteristic equation connecting the candle-power of a vacuum electric lamp with its current or potential so as to enable a large range of efficiencies to be covered by a single standard ([21]) For this purpose the characteristic equation of a lamp may be taken as $\log I = A(\log x)^2 + B \log x + C$ ([22]), and measurements of I at three or more values of x suffice to determine the constants A, B and C The method may clearly be extended by treating a combination of a lamp with a blue glass as a single unit and finding the values of the constants for the combination It will be seen that, as in the cascade method, colour difference is not avoided, but with a comparatively small number of standardisations by several observers a standard is available for homochromatic comparisons over a wide range of efficiencies.

Homochromatic Methods.—The most fundamental method of making a photometric comparison between two lights which differ in colour is to resolve each into components which are visually monochromatic and to compare the intensities of these components pair by pair If I_ν and $x_\nu I_\nu$ be the respective intensities of any one pair of similar components whose wave-number interval is $\delta\nu$, and whose mean wave-number is ν, then the ratio of the two original

intensities is $\Sigma x_\nu I_\nu \delta\nu / \Sigma I_\nu \delta\nu$, where Σ denotes summation throughout the visible portion of the spectrum. In this way a single heterochromatic comparison is converted into a large number of homochromatic comparisons These comparisons are carried out in a special form of instrument, known as a spectrophotometer, in which a composite light is resolved into its spectral components and photometric measurements are then made on as many of these components as may be necessary. This method is, clearly, of universal application, and it will be described more fully in the next chapter. It is, however, exceedingly tedious, and is liable to large errors unless numerous precautions are taken. For many purposes, therefore, some fundamentally less accurate method is preferable for ordinary work. The methods now to be described are of this kind, and,

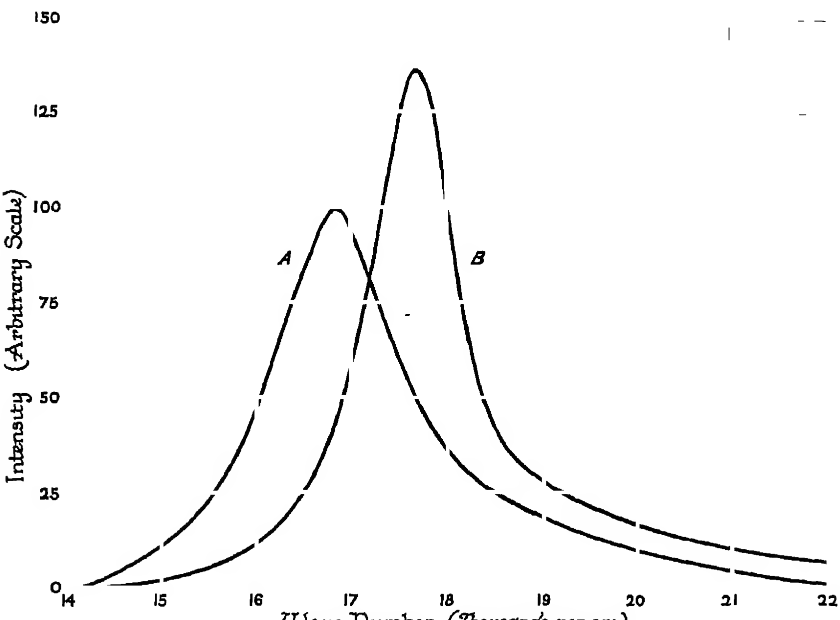


FIG. 141.—The Principle of the Crova Method.

although based ultimately on spectrophotometric determinations, they enable such determinations to be relegated to the standardising laboratory, where they can be performed most conveniently and accurately.

The Crova Wave-number (Wave-length) Method.—One of the earliest of these methods is that of A. Crova ([28]), which may best be explained by reference to Fig. 141. Curves *A* and *B* represent, respectively, the relative luminous intensities throughout the visible spectrum of the light given by two sources These curves are obtained with a spectrophotometer The ordinate scale is so arranged, for each curve separately, that the areas of the two curves are equal, *i.e.*, the total candle-power is the same for each source If these curves intersect at wave-number ν it is clear that equality at this wave-number is a criterion for equality of the integral light. It follows,

therefore, that for sources having respectively the spectral distributions exhibited in curves *A* and *B*, comparison at wave-number ν gives the same result as a comparison of the integral lights. Hence, by placing in front of the eyepiece of the photometer a medium which only transmits a narrow portion of the spectrum on either side of ν, practically homochromatic observations may be made, and the results obtained will be valid for the integral light given by the two sources [24] The curves of Fig. 142 show the Crova wave-numbers

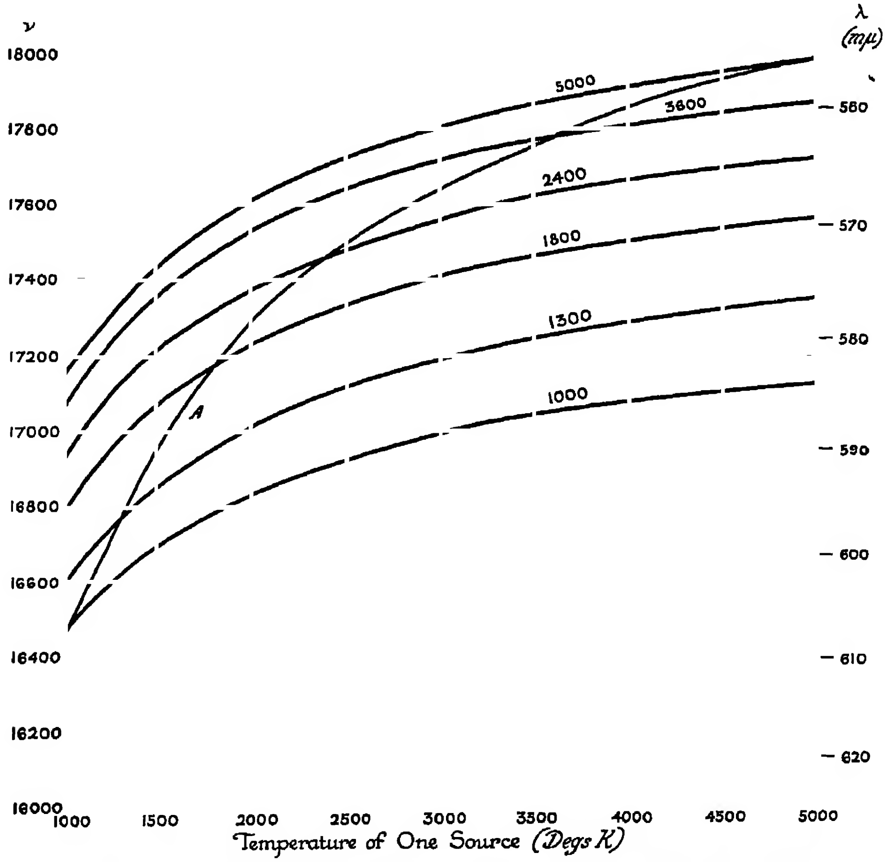


FIG. 142—The Crova Wave-number for pairs of Grey Body Sources at different Colour Temperatures

for the comparison of two "black-body" sources at the temperatures shown respectively (*a*) on the curve, and (*b*) on the axis of abscissæ. For comparison between a carbon lamp operating at 2 3 lumens per watt (colour temperature 1,980° K) and a tungsten lamp at 10 lumens per watt (2,430° K.), the Crova wave-number is approximately 17,370 [25]. A suitable solution having, at a thickness of 25 mm , a well-defined transmission band in this region consists of $CuCl_2$, 86 gm.; $K_2Cr_2O_7$, 60 gm ; HNO_3 (sp gr. 1·05), 40 c.c with water to 1 litre at 20° C. [26]. Curve *A* of Fig 142 is the line of limiting Crova wave-numbers, *i.e* , the line which gives the Crova wave-number for two black-body sources at the temperatures

$T \pm \delta T$, when δT is vanishingly small To a fair approximation, the Crova wave-number for two sources may be found by taking the arithmetic mean of the limiting Crova wave-numbers for the respective temperatures of those sources.

A method somewhat similar to that of Crova was developed by Macé de Lépinay and Nicati (27), who assumed that for bodies giving approximately black-body radiation $I/I_R = f(I_G/I_R)$, where I is the integral candle-power for all frequencies, and I_R and I_G are the candle-powers in regions of the spectrum confined to the red and the green respectively Further, they found that $f(I_G/I_R) = [1 + 0\ 208(1 - I_G/I_G)]^{-1}$ represented the results for a number of ordinary sources, so that the ratio of the integral candle-powers of two sources could be obtained from the results of two comparisons made (*a*) with a red transmitting medium, and (*b*) with a green medium in front of the eyepiece. Both of these methods of heterochromatic photometry depend on the energy distribution in the spectrum of the sources to be compared. They have been used with some success for sources giving a continuous spectrum with approximately the same energy distribution as a black body at some temperature. For sources with a discontinuous spectrum, however, they are quite useless, except that Crova's method may be used for approximate work when once the Crova wave-number has been determined by spectrophotometry. These methods make no claim to be fundamental in any sense of the word, and both suffer from the disadvantage that, since the transmission factors of the coloured media used are necessarily small, the brightness of the photometric field is reduced to an undesirable extent if any ordinary form of photometer head be used with sources of normal candle-power

Colour Filter Methods.—It is clear that if, when two sources giving lights of different colours are being compared, there be placed between the photometer and one of the sources a coloured transparent medium of such a tint as to cause the light from one source to match that from the other, at least as far as visual sensation is concerned, the difficulty of colour difference disappears, and the whole problem is reduced to one of homochromatic photometry, except that it now becomes necessary to determine the transmission factor of the transparent medium for light of the colour given by the source with which it is used. This is again a problem of spectrophotometry (28), and therefore is not susceptible of the same accuracy as that obtainable in homochromatic photometry (see p. 287) An alternative method of measuring the transmission factor of the medium is to make a direct measurement of the candle-power of a source giving light of the same spectral distribution as that with which the filter is intended to be used. This measurement is made first without the medium, and then with the medium placed between the source and the photometer, any of the more accurate methods of heterochromatic photometry being used for the second comparison (29)

It will be seen that one of these measurements involves a colour difference as great as that which the medium is designed to eliminate, so that no gain would seem to result, but in the first place the colour difference may be divided into two steps by using a comparison lamp which gives light of a colour about midway between that of the source with and without the medium, and there is, further, an

important practical gain, since the transmission factor may be determined once for all by a large number of observers working on several different occasions, and the mean value thus obtained may then be used in conjunction with the results of one or two observers working with the medium Since these observers have now no colour difference to contend with, the mean of a comparatively few results of theirs will be equal in accuracy to the mean of the results which would be obtained by the larger number of observers working on every occasion on which the medium is employed The gain of time in this procedure, when many similar sources have to be compared with a standard of a different colour, may be very great (30).

The medium may be either a glass, a stained gelatine film, or a cell with parallel glass walls filled with a chemical solution of some kind. The glass is to be preferred from the point of view of permanence (31) and convenience in use, and cobalt glasses have been used at the Bureau of Standards for obtaining light of the colour given by a tungsten lamp operating at 6·6 lumens per watt from a lamp at 2 5 lumens per watt (32) Some colours are, however, difficult to reproduce with certainty in glass (33), and gelatine films mounted between glass form a very good substitute, the range of colours obtainable being far greater with this medium Such filters are, however, less permanent. Their use has been carefully investigated by Mees, who has developed a special set of filters for photometric work (34) These filters are termed "photometric" filters, and, as shown by the table below, they are designed to enable a colour match to be made between any two electric glow lamps which are giving light anywhere in the range from 1 2 l p w. to daylight.

WRATTEN PHOTOMETRIC FILTERS (35)

Filter No.	When used with a 6 7 l p w lamp, matches a lamp giving	Matches a 6 7 l p w lamp when used with a lamp giving	Approximate transmission factor when used with a lamp giving 6 7 l p w.
Blue .			per cent.
78	Daylight	—	13
78A	(30) l p.w	1·2 l p w.	35
78B	15	3·5	50
78C	11	5	70
Yellow .			
86C	5	10	80
86B	3·5	15	76
86A .	1 5	(25)	74
86 .	—	Daylight	70

The figures given in the above table can only be regarded as approximate, since both the colour and the transmission factor of any given type of filter are liable to vary from specimen to specimen Further, the colour match is by no means perfect when a large

colour difference is bridged by means of these filters. The values of transmission factor are for the complete filter regarded as a unit, surface reflection losses being included A filter (No 78AA) intermediate in colour between Nos 78 and 78A is supplied for use with a gas-filled lamp to give light of daylight colour, while No. 79 gives daylight colour when used in front of an acetylene flame.

A variable tint filter has been devised for enabling an approximate colour match to be obtained over a range of efficiencies ([36]). The comparison lamp is placed in an enclosure with a translucent glass window, over which moves a metal slide containing two smaller windows, Y and B (see Fig 143) These are covered one with a yellow and the other with a blue filter (*e g.*, Wratten 86 and 78). The heights of the windows are inversely as the transmission factors of the two filters for the light from the comparison lamp It follows that as the slide is moved across the front of the window the candle-power is not altered, while the colour of the light changes continuously from yellow to blue The method has been extended by H. E. Ives ([37]), who employs three colour filters—red, green and yellow—so that the accuracy of the colour match can be improved and the range of the instrument extended to cover most practical light sources

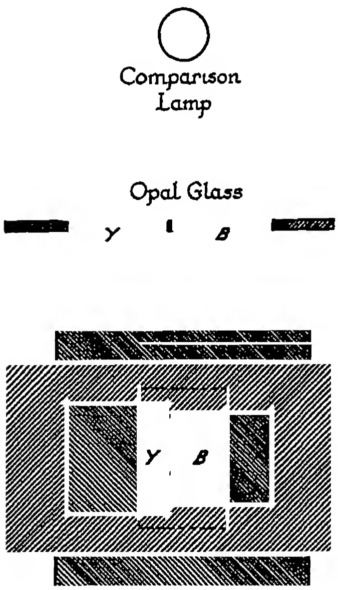


FIG 143—A Variable Tint Filter with Constant Transmission.

The Leucoscope. — A different form of variable tint filter may be obtained by making use of the rotatory dispersion of quartz When plane polarised light passes through a plate of quartz cut with its surfaces perpendicular to the optic axis, the plane of polarisation is rotated by an amount which varies with the frequency of the light ([38]). The result is that, if the incident polarised light is composite, the plane of polarisation of the transmitted light will be different for each monochromatic component, *i e*, the plane of polarisation of the component of frequency ν is rotated through an angle α_ν, where α_ν is a function of ν, $f(\nu)$ If, therefore, a Nicol prism or similar analyser be placed in the path of the light transmitted by the quartz, the intensity of the component ν in the light transmitted by this Nicol will be proportional to $\sin^2(\phi - \alpha_\nu)$, where ϕ is the angle of rotation of the second Nicol measured from the position of extinction with the quartz plate removed.

Thus it follows that the spectral transmission curve of a system consisting of a quartz plate between two Nicol prisms may be altered in a manner which is calculable theoretically, since the value of α_ν is known for all values of ν, and is proportional to the

thickness of the quartz plate. By using plates of different thicknesses the flexibility of the combination may be much increased A still greater range of transmission may be obtained by using two quartz plates sandwiched between three Nicol prisms, two of which are capable of independent rotation.

The device as applied to the problem of heterochromatic photometry ([39]) is shown in Fig 144 L_1 is the sub-standard or test

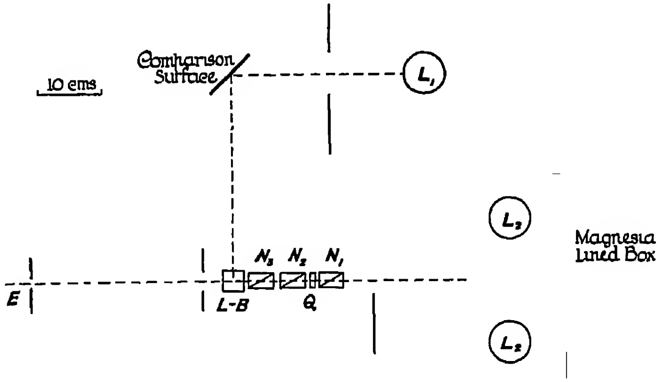


FIG 144 —The Leucoscope Photometer

lamp, and L_2, L_2 two lamps in a whitened box, which forms the comparison source. N_1 is the fixed Nicol, and Q the quartz plate N_2 and N_3 are two Nicols capable of united rotation with respect to N_1 and Q This rotation gives the colour match N_3 can then be rotated separately with respect to N_2 in order to obtain the intensity match. *L-B* is a Lummer-Brodhun cube, to enable this match to be made by the eye at *E*.

The single quartz plate system can be used in combination with a gas-filled lamp having a colour temperature of 2,830° K to obtain a colour match with the light from a black-body radiator at any temperature between 3,100° and 4,000° K. For the range 4,000° to 7,000° K. the two-plate system must be used ([40])

Chemical Solutions.—Following a suggestion by C. Fabry ([41]), certain chemical solutions have been developed for use with sources having spectral distributions of the black-body type These solutions have the advantages of ready reproducibility from specification and easy adjustment by alteration of concentration and thickness They are used in cells with plane polished walls of special colourless glass having the form shown in Fig 145 ([42]) The central solid glass frame is accurately ground to 1 cm. thickness The two faces are not cemented on, but, after cleaning with nitric acid and distilled water, are laid in close contact with the glass frame, and are held in position by rubber bands while a seal of paraffin is run round the edge with a hot metal point

The yellow solution is composed of .—

Cobalt ammonium sulphate	100 gm
Potassium bichromate	0 733 gm
Nitric acid (1·05 sp. gr.) .	10 c c.
Water . . .	to 1 litre of solution.

This solution may be used with a 2 7 l p.w (4 w p c) carbon lamp to bring its light to a colour match with that given by a lamp of lower efficiency. The concentration of the above specified solution

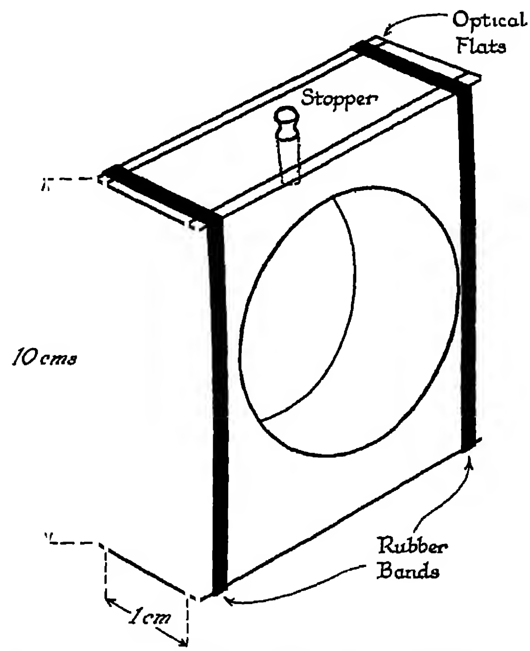


FIG 145 —A Form of Glass Cell for Colour Filter Solutions

is then altered to produce the best possible colour match, and the transmission is found from the relation $\log_{10} \tau = -0{\cdot}245\ C^{0{\cdot}9}$, where τ is the transmission factor for a thickness of 1 cm. of the solution as compared with clear water (43), and C is the concentration, that is, the number of c.c. of solution as above specified in 1 c c of the solution used

The same solution may also be used to bring to a colour match with the carbon lamp the light from any higher efficiency source giving visible radiation of the "black-body" type by using the solution on the test lamp side of the photometer In this case the transmission factor for 1 cm thickness, as compared with clear water (43), is found from the relation

$$\log_{10}\tau = -0{\cdot}366\ C^{1{\cdot}05}.$$

The blue solution, for use in front of a 2·7 l p w carbon lamp, is composed of

Nickel ammonium sulphate .	50 gm.
Ammonium sulphate . .	10 gm
Ammonia (0·90 sp. gr) . .	55 c c
Water . . .	to 1 litre of solution

For this solution the transmission factor per cm. thickness, as compared with clear water (43), is given by the relation

$$\log_{10}\tau = -0{\cdot}539\ C^{1{\cdot}03},$$

the concentration being altered in this case by diluting with water containing 10 gm. ammonium sulphate per litre ([44])

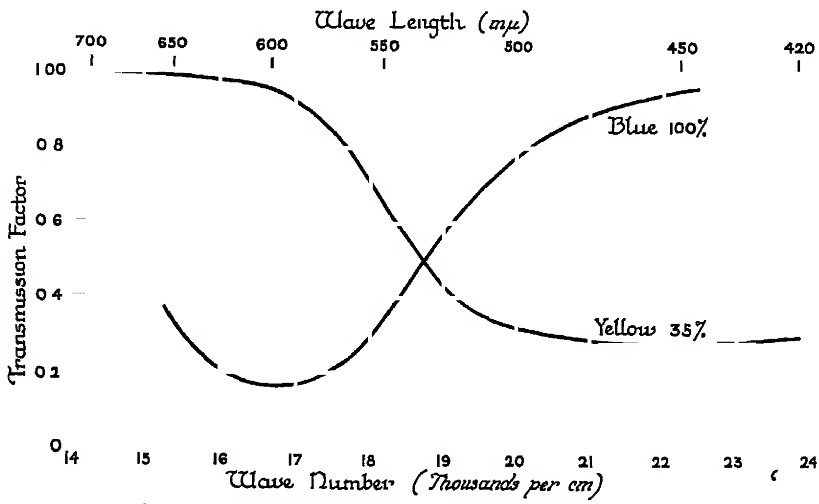


FIG 146 —Transmission Curves of Colour Filter Solutions

The spectral transmission curves of the yellow solution at 35 per cent. concentration, and of the blue solution at 100 per cent. con-

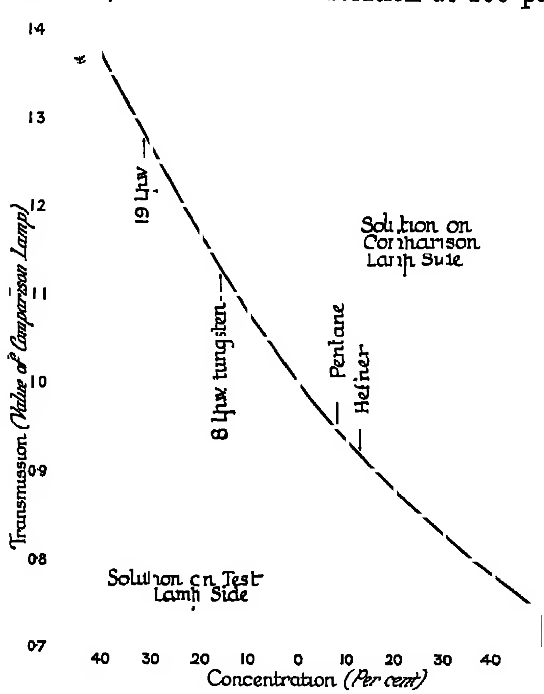


FIG. 147 —Transmission of the Yellow Solution with a 2 7 l p w Carbon Lamp.

centration, are shown in the two curves of Fig 146. It is found that (a) the colour match obtained by using the solution is only a

sensation match, and will therefore not hold accurately for observers with abnormal colour vision (see p 263, *infra*), and (*b*) Beer's law as to the equivalence of concentration and thickness [45] does not hold, as will be seen from the expressions for τ given on p 246. Figs 147 and 148 show respectively for the yellow and blue solutions the approximate concentrations and corresponding transmissions in a thickness of 1 cm. for a colour match with various sources, using as comparison source a 2 7 l p w. carbon lamp

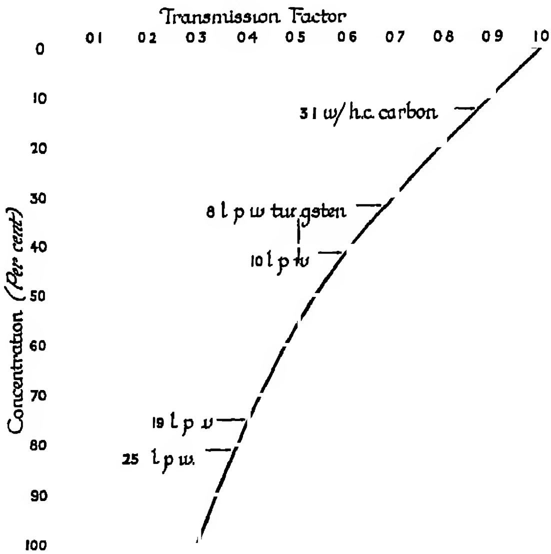


FIG 148 —Transmission of Blue Solution with a 2 7 l p w Lamp

An undesirable feature of the yellow solution is the existence of a pronounced temperature coefficient, which makes it necessary either to work at the temperature used in calibrating (20° C.) or to apply corrections obtained from the curve of Fig. 149, which shows the effect of temperature change on the transmission factors of solutions of two concentrations The blue solution has practically no temperature coefficient, but, on the other hand, it is less stable and slowly dissolves the glass of the containing vessel if allowed to stand in it.

Whatever the medium used for altering the colour of the transmitted light, it is essential that there should be complete absence of any curvature or unevenness in the surfaces, otherwise the lens effect introduced causes a change in transmission factor as the distance between the medium and either the source or the photometer surface is altered (see p. 183) [46] Any trace of scatter within the medium is also to be avoided (see p 181)

It may be mentioned again that a transmission factor determined on the bench in the manner outlined above can only be assumed to hold for light of the same colour as that with which it has been measured [47] In the case of a spectrophotometric measurement the transmission factor is determined for light of every frequency,

so that the integral transmission factor for light of a known spectral energy distribution can be found at once by calculation, since it is equal to $\int \tau_\nu K_\nu E_\nu d\nu / \int K_\nu E_\nu d\nu$, where τ_ν, K_ν and E_ν represent respectively the transmission factor, luminosity of radiation and energy per unit wave-number interval of the light considered, all at wave-number ν

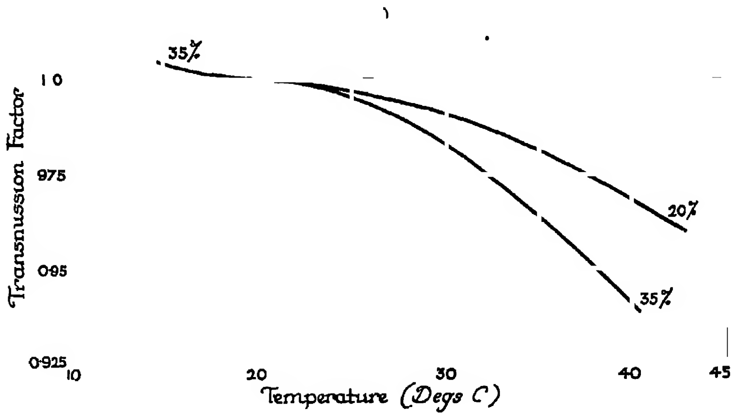


FIG 149 —Change of Transmission of the Yellow Solution with Temperature.

(see also p 289). As an example of the amount of change produced in transmission factor by alteration in the colour of the transmitted light, it may be mentioned that a piece of Wratten filter No. 78 having a transmission factor of 13 per cent for light from a 6 7 l p w. tungsten lamp has a transmission factor of 17 5 per cent when used with a lamp operating at 10 l p w The difference is naturally less in the case of the other filters, but it is still considerable. For media giving less even transmission curves than the photometric filters the effect may be very large.

Methods depending upon Flicker . (1) *Critical Frequency.*—An altogether different method which has been used for the measurement of lights of different colours is the production of an intermittent brightness alternating with complete darkness, and the determination of the lowest alternation frequency required for disappearance of flicker This frequency, termed the critical frequency for the light under investigation, gives a rough absolute measure of the brightness of the field, and thus, like the visual acuity method described above (see p. 236), does not depend on any comparison of juxtaposed surfaces. Like this method, however, its accuracy is exceedingly low ([48]), and it has therefore never been developed. As, however, the phenomena of critical frequency throw some light on the results obtained with the flicker photometer, it will be of value to describe briefly some of the results obtained by Ives ([49]), which are illustrated in Fig. 150 The lines of this diagram show that when the brightness exceeds a certain value (of the order of 1 photon), the critical frequency is connected with the logarithm of the brightness by a simple relationship of the form (Frequency) $= A \log$ (Brightness) $+ B$,

where A and B are constants depending on (i.) the colour of the light, (ii.) the ratio of the lengths of the light and dark periods, and (iii.) the steady brightness (if any) upon which the flickering brightness is superposed ([50]). In the case of red light, with equal periods of light and darkness and no steady brightness, the values of A and

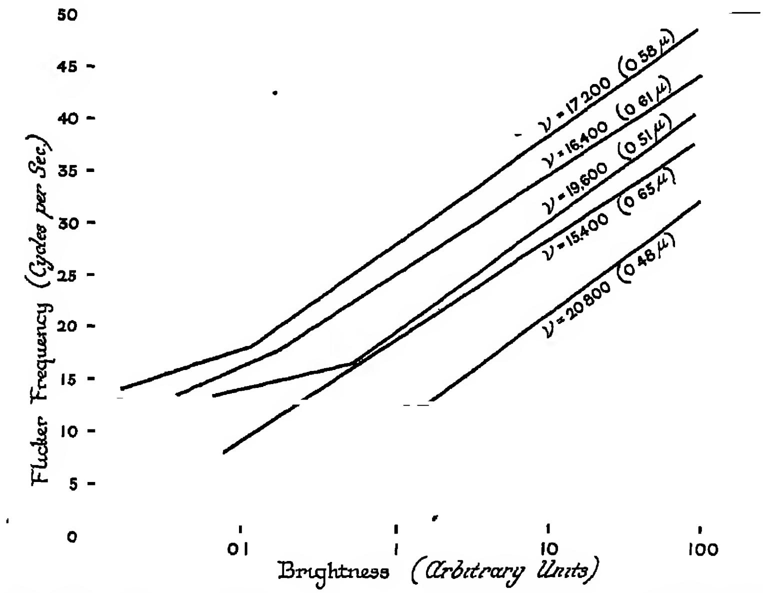


FIG. 150.—Variation of Critical Frequency with Field Brightness for Light of Various Frequencies. (The brightness unit is of the order of one photon.)

B are the same for any brightness, while for other colours there is a more or less abrupt change in the values of these constants when the brightness falls below a certain limiting value. In this region A is zero for blue light, so that the critical frequency is constant at all low values of brightness.

It will be noticed that the critical frequency exhibits a reversed Purkyně effect (see p. 65), for while at high values of brightness blue is weighted with respect to red (as compared with steady comparison), the opposite is the case at low values of brightness.

(2) *The Flicker Photometer.*—In this instrument the criterion of the equality of brightness of two surfaces is the disappearance of the flicker produced by presenting them alternately to the eye at a certain minimum frequency. This minimum is defined as the lowest frequency which will just cause flicker to disappear over the smallest possible range of variation of either brightness alone. The extent of this range over which no flicker is discernible determines the sensitivity of the photometer under the conditions prevailing when the experiment is made. The actual method of working of this form of photometer, and the precautions to be observed when using it, must be described in some detail, since when the colour difference between the lights to be compared is large it has been found that the errors of measurement, especially in the case of inexperienced

photometric observers, are considerably less with a flicker photometer than with a steady comparison instrument. This question will be referred to again later.

The earliest forms of flicker photometer were those of F. P. Whitman ([51]) and O N. Rood ([52]), which depended on the intermittent presentation to the eye of the two surfaces of a Ritchie wedge (see p 3), and that of Simmance and Abady (first form) ([53]), which was an adaptation of the Gas Referees' modification of the Bouguer-Foucault photometer (see p. 2).

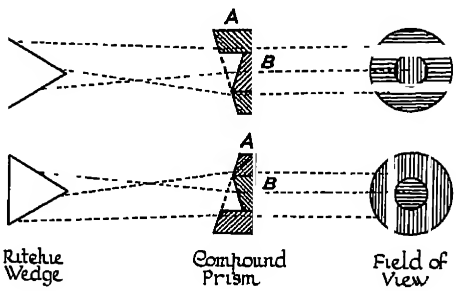


FIG 151 —The Principle of the Bechstein Flicker Photometer

A more accurate form is the Bechstein photometer ([54]), in which a two-part prism is caused to rotate in front of a Ritchie wedge The outer annulus (A) of this prism and its central part (B) have their refracting surfaces inclined in opposite directions, as shown in section in Fig 151. The result is that, as the prism rotates, each side of the wedge is seen alternately in the inner circle and the outer ring, and when the two sides are equally bright the flicker between annulus and centre disappears

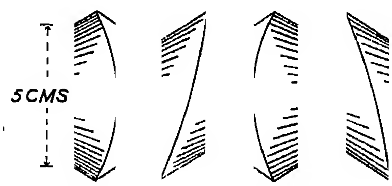


FIG 152 —The Simmance-Abady Flicker Disc

Other widely used forms of the flicker photometer are those designed by Simmance and Abady (second pattern), and by L Wild The former is, again, a Ritchie wedge device ([55]), the photometer "disc" consisting of a double truncated cone of plaster of Paris having the form shown in Fig 152 This figure gives the appearance of the disc in four positions at intervals of 90° The construction of the simple disc will be made clear by reference to Fig. 153 The two cones are cut along the lines AC and EG, the portions ABC and EGH are removed, and the remaining parts are then fitted together In use it is placed with its axis along the line joining the light sources to be compared, and is then rapidly rotated so that first one and then the other comparison surface is presented to the eye, the dividing edge moving to and fro across the field of view Since each conical surface owes its brightness entirely to the lamp towards which it is inclined, it follows that, if the angles which the generators of the two conical surfaces of the disc make with the line joining the sources are exactly equal, then, when the brightnesses of the surfaces are equal, the illuminations due to the two sources will also be equal and the usual

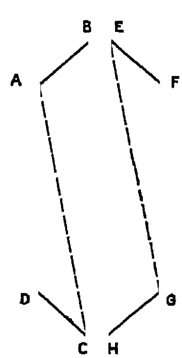


FIG 153. — The Construction of the Simmance-Abady Disc

photometric procedure can be followed. H Kruss has devised similar discs giving four and six instead of two reversals of field per revolution ([56]) Dow's cosine photometer (see note 87, p. 192) may also be used as a flicker instrument ([57]) All the above instruments suffer from the liability to angle error inseparable from the use of the Ritchie wedge (see p 152).

Wild's photometer is an adaptation of the Bunsen principle ([58]). A circular disc, of which one half is translucent and the other

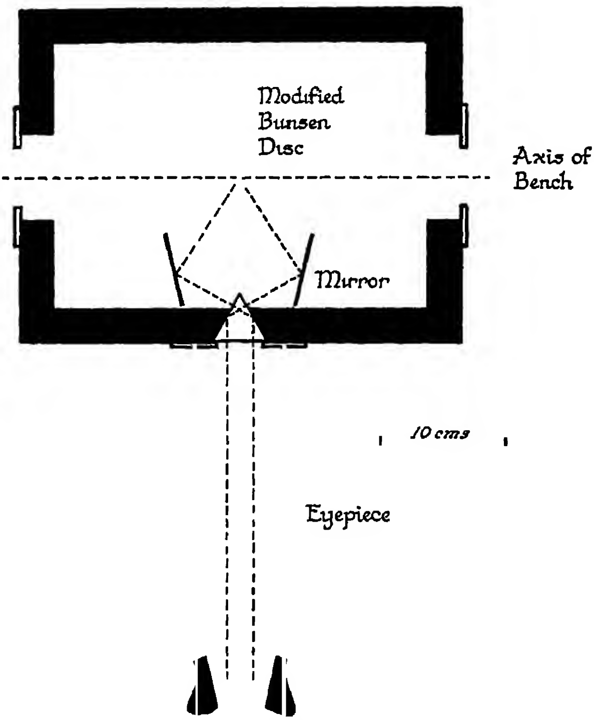


FIG. 154 —The Wild Flicker Photometer

opaque, is set in rotation, and a small portion of its surface is observed on both sides simultaneously by means of the mirror and prism arrangement shown in Fig. 154. What is seen by the eye is a circular field with a thin black line, due to the vertical edge of the prism. On either side of this line there is a flicker due to the fact that, as already noted (p. 153), the brightnesses of the translucent and opaque parts of the disc are not equal on both sides of the disc at the same position of the photometer head. The position of balance is therefore not found by absence of flicker, but by equality of flicker in the two halves of the field This is found to result in a much improved sensitivity, especially with a considerable colour difference, since it is not necessary to increase the speed to the point at which colour flicker entirely disappears in order to obtain a reading. With this photometer the liability to error owing to the oblique incidence of the light on the comparison surfaces is avoided.

It has been found (Wild, *loc cit.*) that an accuracy of 0·5 per cent. with homochromatic lights, and 0·9 per cent. with red and green lights, can be obtained with this instrument

H Kruss has designed a flicker form of the Lummer-Brodhun head, but without the contrast field (59) A simple back-and-forth movement of the dividing line between two adjacent co-planar fields has been used by A. H. Pfund in an adaptation of the graded contrast photometer (see p. 159) (60), and a flicker form of the Joly block photometer (see p 160) has been devised (61). Modern flicker instruments will be described later in this Chapter, but it will be desirable first to consider briefly the results of experimental work leading to the formulation of certain conditions which a satisfactory instrument of this type must fulfil

The Validity of the Flicker Method.—It is self-evidently essential that the results obtained by any method of photometry shall be in accordance with the ordinary laws of physical quantities, *i e*, that two brightnesses which are found to be each equal to a third shall also be equal to each other, and that the brightness which results from the superposition of two illuminations shall be equal to the sum of the brightnesses due to each illumination separately (62). With lights of the same colour these laws are rigorously obeyed whatever the method of photometry adopted. For lights of different colours the laws have been found to apply to the results of measurements by the steady comparison method, at any rate approximately (63), but the same is not universally true of the flicker method.

The most exhaustive work on the subject is that of H. E. Ives (64). The use of the flicker photometer depends upon the physiological phenomenon that colour difference between the two halves of an alternating field disappears at a lower speed of alternation than brightness difference. For lights of identical colour flicker disappearance takes place when the two parts of the field are equally bright as judged by the steady comparison (often termed the "equality of brightness") method. When there is a colour difference, flicker may disappear when the comparison fields are slightly unequal as judged by the steady comparison method, the difference depending on the conditions under which the instrument is used For example, the Purkyně effect (see p. 65), which causes a weighting of the bluer light at low intensities in the steady comparison photometer, is reversed in the flicker photometer (65) This is shown very clearly by the curves of Fig. 155, which exhibit Ives' results on the variation with field brightness of the luminosities of coloured lights as measured by the flicker method (66). The scale of abscissæ is a logarithmic scale of field brightness in photons (see p 52) Each curve refers to light of a certain colour and shows the variation, with field brightness, of the luminosity of light of that colour in terms of the luminosity of white light. The horizontal line represents the relative luminosities at high values of brightness, and at every point of the curve the respective brightnesses of the two fields being compared are the same fraction of those brightnesses which were found to be equal at high illuminations. The reversal of the Purkyně effect is seen from the fact that the lower curves (red) rise above the high brightness line, while the upper curves (blue) fall below it, when the field brightness is less than about 60 to 70 photons (67).

In the steady comparison method, when the field brightness is low, the effect of decreasing the size of the field of view is to decrease the Purkyně effect, *i.e.*, the red is weighted more and more as the size of the retinal image is decreased down to about 2° (yellow spot

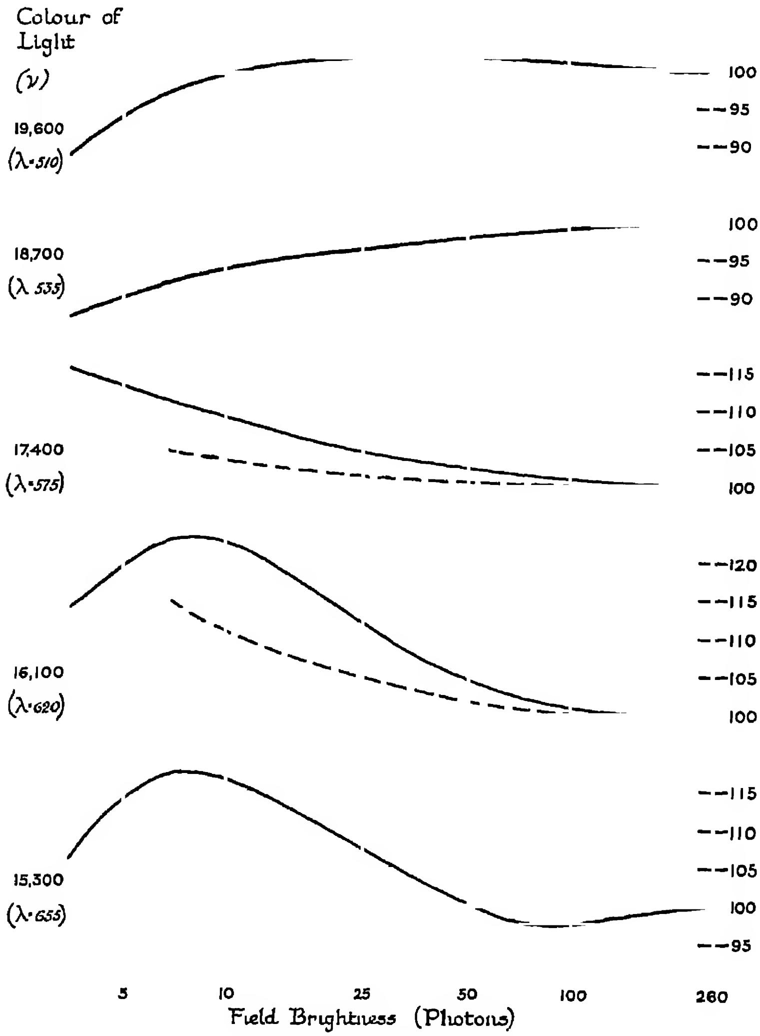


FIG. 155.—The Reversed Purkyně Effect in the Flicker Photometer

effect; see p. 66). In the case of the flicker photometer this effect also is reversed, and the effect of decreasing the size of the field is to weight slightly the blue end of the spectrum This is shown by the broken lines of Fig. 155, which refer to small fields This reduction of the reversed Purkyně effect with small fields is only what would be expected if this effect in the flicker photometer were, like the true Purkyně effect of the steady comparison method, absent at the rod-free part of the macula ([68]). The results of some

experiments by J. S Dow on (*a*) the effect of change of field size, and (*b*) the effect of change of brightness on the relative values obtained by the steady comparison and the flicker methods, are shown in Figs. 156 (yellow spot effect) and 157 (Purkyně effect) (69).

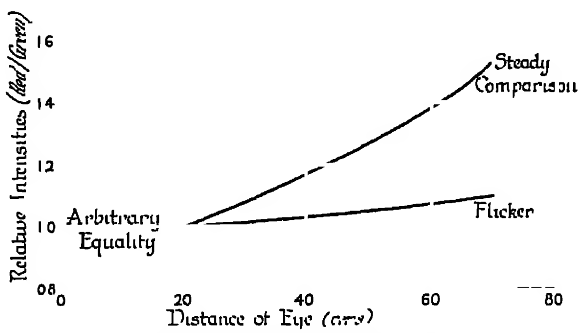


FIG 156 —Comparison of the Yellow Spot Effect in the Steady Comparison and Flicker Photometer

Dow varied the field size by altering the distance of the observer's eye from a fixed aperture through which the photometric field was viewed.

Errors and Reproducibility of Judgment.—Ives found that for a normal value of brightness of the photometer field (about 80

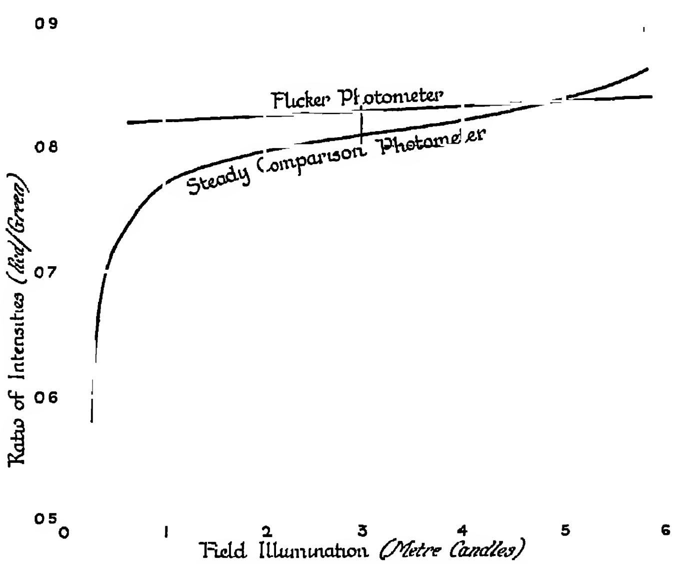


FIG 157 —Comparison of the Purkyně Effect in the Steady Comparison and Flicker Photometers

photons) the error of measurement in comparing different parts of the spectrum with white light was, on the average, four to five times as great with the steady comparison as with the flicker method, although in the case of experienced observers it was only about twice as great (70). The error by both methods was, naturally,

larger at the ends of the spectrum than in the middle. In the case of a lower level of brightness (3 photons) the difference between the errors obtained by the two methods was not nearly as great, being, in fact, negligible in the case of experienced observers. It should be noticed that the brightness of the photometer disc as ordinarily used is equivalent to about 3 candles per square metre seen through a normal pupil, or through an artificial pupil of 5 mm diameter (Lummer-Brodhun eyepiece) The brightness is therefore of the order of 60 to 80 photons. The reproducibility of the measurements by a single observer after an interval of two months was found to be rather higher in the case of the flicker photometer Apparently the criterion of equality is more liable to alter from day to day in the steady comparison than in the flicker method ([71]) The reproducibility for both methods diminishes as the brightness is decreased below about 6 photons.

Sensitivity.—The eye is more sensitive to flicker at high values of brightness than at low, so that higher speeds are necessary to cause disappearance of flicker when the illumination is increased. This is seen from the curves of Fig. 158, which show, for light of

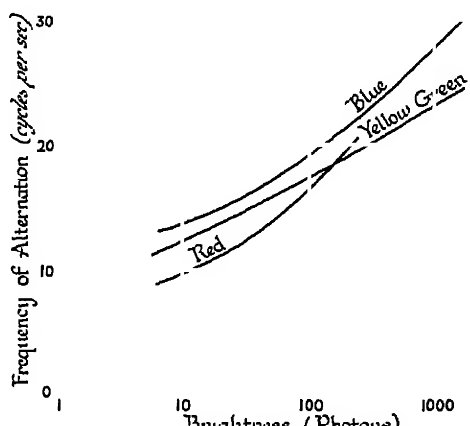


FIG. 158 —The Effect of Field Brightness on Flicker Frequency.

three colours compared with white light, the frequencies necessary to cause disappearance of flicker at various degrees of brightness ([72]). Ives found (*loc. cit*, *note* ([64])) that the sensitivity was noticeably increased by enlarging the field of view from 1 86° diameter to 8 6° × 5·16° By averted vision, when the image of the centre of the smaller field was 8° from the fovea, the sensitivity was again decreased Although this result appears to be contrary to the known fact that the peripheral part of the retina is more sensitive to flicker than the fovea (see p. 62), the explanation of the discrepancy is found to be that this extra sensitivity by averted vision is quite transitory, and is very quickly reduced to a negative value when this part of the retina is used continuously ([73]). Binocular vision has been found to give a slight increase of sensitivity in some observers ([74]).

Flicker Photometer Speed.—Some flicker photometers are so

designed that it is difficult, if not impossible, to control the speed of alternation. This seriously impairs the efficiency of the instrument, for the range of absence of flicker increases with the speed [75], so that it is clearly desirable to work at the minimum speed at which flicker just disappears. This minimum depends on the degree of colour difference worked with as well as on the field brightness, being roughly equal to bB^a alternations per second, where B is the field brightness in photons, and a varies from 0 26 to 0 13, while b varies somewhat irregularly between 5 and 10 as the colour of the light compared with white changes from red to violet [76].

Fig 159 shows the frequency required for the elimination of flicker when a white light (tungsten lamp) is compared with lights

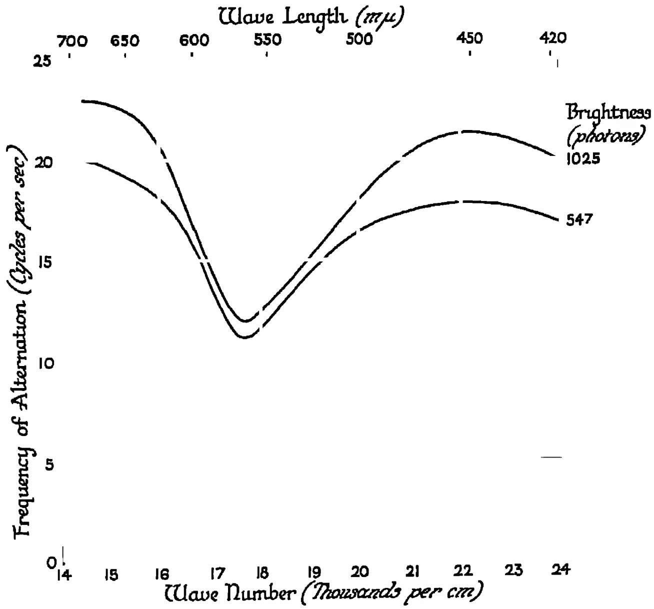


FIG 159 —The Effect of Colour Difference on Flicker Speed.

at different parts of the spectrum, while Fig 160 shows the variation of sensitivity with speed at three different values of field brightness, the curves in this case showing the range over which flicker is absent at various speeds when a green and a white light are compared.

In general it may be said that the minimum speed increases with (i) field brightness, (ii.) separation on the colour triangle (see p. 303), and (iii) irregularity of the periods of exposure of the two comparison surfaces [77]

Theory of the Flicker Photometer.—Ives and Kingsbury have proposed [78] a theory of the behaviour of the eye under a transient or fluctuating illumination of the retina which, whatever its relation to the actual phenomena, at any rate gives a rational foundation for the outstanding characteristics of the eye's behaviour when used with a flicker photometer They suppose that the retina possesses a certain " diffusivity " for light energy, *i e.*, that the incidence or extinction of light on the retinal surface is not instantaneously

accompanied by the full corresponding visual sensation, but that there is a certain rate of growth or decay of sensation, which depends on (*a*) the magnitude of the sudden change in retinal illumination which the eye is called upon to appreciate, and (*b*) the colour of the light causing that change of illumination The action may be illustrated by the behaviour of a lamp filament supplied with a

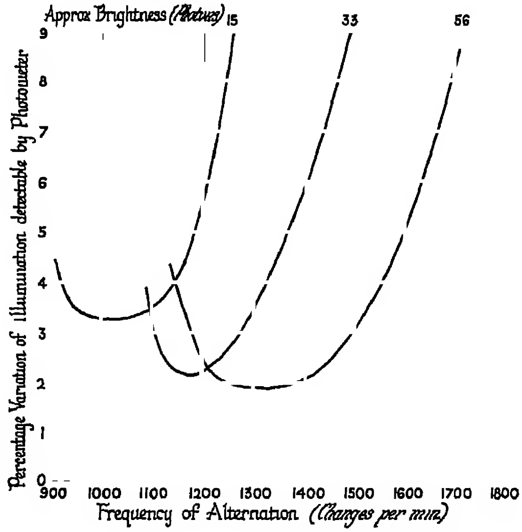


FIG. 160.—The Variation of Sensibility with Flicker Speed

regularly fluctuating current. For any given size of filament, change of temperature will lag behind the change of current which produces it, owing to the heat capacity of the filament, and, if the fluctuations be sufficiently rapid, the amplitude of the temperature oscillations will be much less than the amplitude of the I^2R oscillations. For the same frequency of oscillation a thick filament will produce more lag, and therefore a greater reduction of amplitude in the temperature oscillations, than a thin filament. If, then, the retina behave towards light of different colours in a manner analogous to the behaviour of filaments of different thicknesses under current oscillation, it will follow that—

(*a*) If two alternating lights of different colours, but of the same average *intensity*, fall on the retina, as shown by the imaginary curves *r* and *b* of Fig 161, the result will be an outstanding flicker, as shown by the resultant curve *a* This represents the result of taking two coloured lights on a *vertical* line in Fig 150.

(*b*) If two alternating lights of different colours fall on the retina, and their respective intensities are such that each has the same critical frequency, then, as shown in the imaginary curves *r′* and *b′* and *a′* of Fig 162, the result will still be a flicker, for, considering Fechner's law, it is reasonable to assume that equality of critical frequency implies an equality of the amplitude of oscillation of intensity when *expressed as a fraction of the mean intensity* This, then, represents the result of taking two coloured lights on a *horizontal* line in Fig. 150.

It follows that neither equality of mean intensity nor equality of critical frequency, but some intermediate condition, will result in an absence of flicker at minimum speed This theory has been developed ([79]), and has been found to be in satisfactory qualitative agreement with the experimental facts described above, so that it may be regarded as capable of providing something more than a useful mental picture of the behaviour of the eye towards a flickering light.

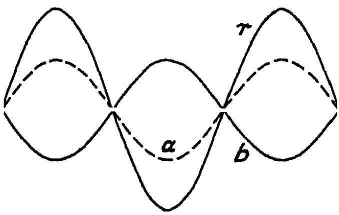


FIG 161 —The Theory of the Flicker Photometer (I)

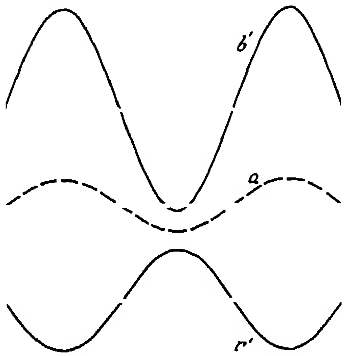


FIG. 162 —The Theory of the Flicker Photometer (II.)

Best Conditions for the Flicker Photometer.—Ives concludes ([80]) that for the comparison of lights of different colours the flicker photometer is the most sensitive, and gives the most reproducible results of any of the ordinary photometric methods. At high illuminations it agrees with the steady comparison method when the latter is freed from the psychological uncertainties inherent in its use ([81]). Further, by a series of careful experiments, he has proved that brightnesses which measure equal to the same by this method also measure equal to one another, and the sum of the parts is equal to the whole ([82]) He has suggested ([83]) that the following are satisfactory conditions for the use of the flicker photometer —

(i) An illumination of the comparison surfaces of at least 25 metre-candles (of the order of 120 photons with a natural pupil), and

(ii) A photometric field of 2° diameter, surrounded by a bright field of about 25° diameter, maintained at approximately the same brightness as the photometric field

In addition to the above conditions, the following have been laid down by A H Taylor ([84]) as essential for accurate work in flicker photometry .—

(iii) There should be no dark ring between the photometric field and the surrounding bright field referred to in (ii)

(iv.) When the moving parts of the flicker head are rotated slowly no shadows or unequally illuminated spots should be apparent in the field

(v.) The two halves of the flicker field should be visible for equal lengths of time during a complete cycle ([85]).

(vi.) The photometer head should remain stationary, and intensities should be balanced by moving one of the lamps. This lamp should move with little effort on the part of the observer

(vii.) In order to give smooth and steady running, it is recommended that a direct-current series-wound motor be run at or near its rated speed, a fly-wheel being used to give additional steadiness and a reduction of eight or ten to one being used between the motor and the moving parts in the head.

(viii.) The flicker photometer is not suitable for continuous work on account of its fatiguing effect. It should be used only for periods not exceeding about an hour.

Modern Forms of Flicker Photometer.—The conditions above laid down are fulfilled in at least two modern forms of flicker photometer. In the first of these, due to Ives and Brady ([86]), the Bechstein prism is used (see p 251 and Fig. 151). The test lamp or sub-standard illuminates the diffusing surface M (Fig. 163), while a

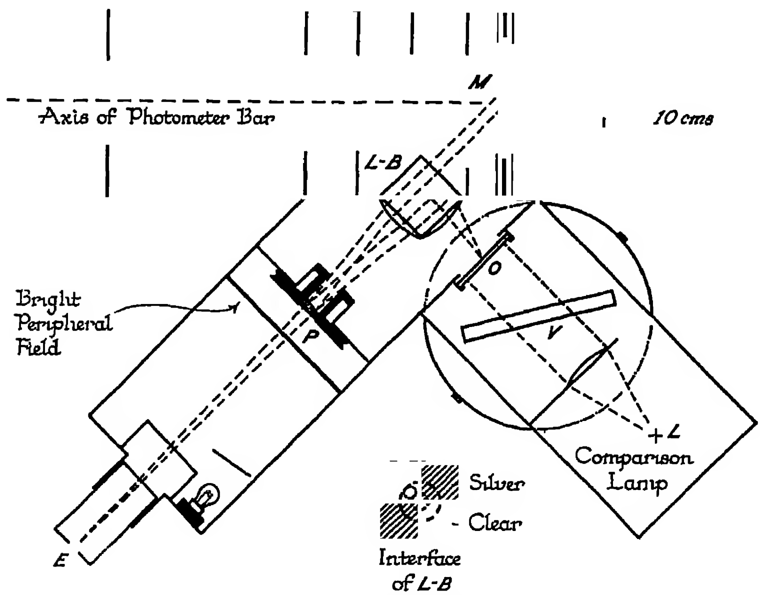


FIG 163 —The Ives-Brady Flicker Photometer

comparison lamp L illuminates an opal glass screen O. The brightnesses of these two surfaces are compared by means of a prism L-B, which is constructed on the Lummer-Brodhun principle, with a field of the form shown in detail at the bottom of the diagram. The field of view visible from E is made to travel over the interface of the prism by rotation of a small 10° prism P, as in the case of the Bechstein instrument. V is a variable neutral filter of the form described on p. 180.

Ives has also described a polarisation instrument, two forms of which are shown in Fig 164 ([87]) Two images of each half of the photometric field are formed by the double-image prism W, and the dimensions of the apparatus are so chosen that the horizontally

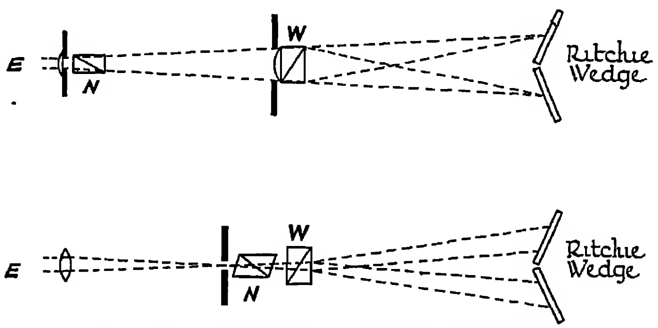


FIG 164.—Ives Polarisation Flicker Photometers.

polarised image of one half is superposed on the vertically polarised image of the other half The Nicol prism N is rotated, with the result that the total brightness of the field seen at E is equal to $B_1 \sin^2 \theta + B_2 \cos^2 \theta$, where B_1 and B_2 are the brightnesses of the two individual fields This expression is equal to

$$\frac{1}{2}\left\{(B_1 + B_2) - (B_1 - B_2)\cos 2\theta\right\}$$

so that the transition from one field to the other is not sudden, but follows a sine curve.

An altogether different type of instrument is that shown in plan in Fig 165 ([88]) S is a white surface of magnesium oxide illuminated, by way of the total reflection prism P, by the light from one of the sources to be compared W is a Whitman disc, which can be rotated at any desired speed about a horizontal axis Its surface is covered with magnesium oxide and is illuminated by the other source of light so that, when viewed by the eye at E, the field of view seen through the small aperture A is alternately occupied by W and by S. A is of such a size as to subtend an angle of 2° at E. It is cut with a sharp (back-bevelled) edge in a concave surface F, which is also covered with magnesium oxide and is evenly illuminated by the small lamp L, the light from which is transmitted through a piece of opal glass and is diffusely reflected from the white internal surface F. If P, S and W be so arranged that the effective position of the front surface of S coincides with the white surface of W, this latter plane may be taken as the position of an infinitely thin photometer screen, so that no correction for thickness is necessary L is adjusted to give F a brightness of about 8 candles per square metre, and the distances of the sources to be compared are then arranged so that the brightness of the field at A has approximately the same value. The test lamp and photometer are then fixed and measurements are made by moving the comparison lamp, the speed of rotation of W being reduced until flicker can be brought almost to disappear. Settings of the comparison lamp are then made to the point of

minimum flicker. It is to be noticed that the surfaces of *S* and *W* are made of the same material, and that they are so arranged that a slight angular twist of the photometer head causes the incident light to change inclination (relative to the line of view) in the same

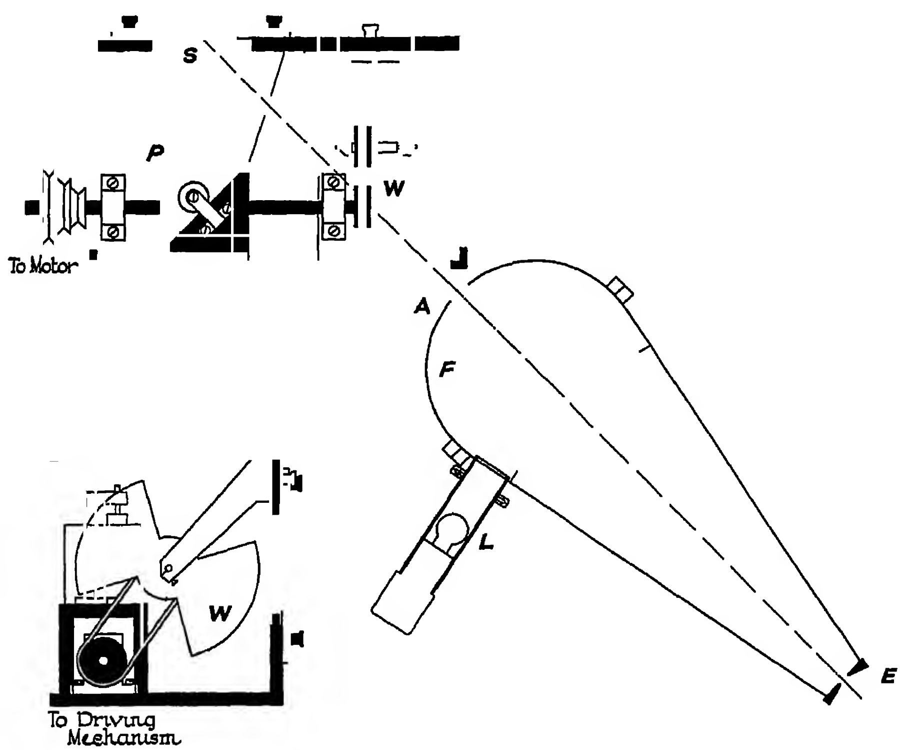


FIG 165 —Guild's Flicker Photometer.

direction in both cases. Hence the angle error noticed on p 164 with regard to the Lummer-Brodhun head is of much less importance in this photometer. *S* and *W* are easily removable, so that the white surfaces can be readily renewed by holding them over burning magnesium ribbon.

It will be clear that, in general, no form of sector disc can be used in combination with a flicker photometer owing to the introduction of stroboscopic effects.

General Conclusions.—There seems to be no doubt that, while the steady comparison method is the best for comparing lights which do not differ greatly in colour, for a direct comparison with a considerable colour difference the flicker photometer is more satisfactory, especially with unpractised observers, when the above conditions as to illumination and field size are observed [89]. One fact which cannot be too frequently insisted upon is that in all heterochromatic photometry, whether by the steady comparison method or by the flicker method, the brightness of the photometer field should always be well above the limit of the Purkyně or reversed Purkyně effect. In addition it is most desirable that the size of field employed

should be specified Wherever accurate photometric measurement is necessary the introduction of a colour difference should, as far as possible, be avoided by the use of a properly standardised colour filter, or some similar device. This relegates the heterochromatic problem to the standardising laboratory, where a large number of observers are available to make measurements on several occasions under the most favourable conditions as regards accuracy ([90])

The Choice of Observers.—It has been assumed throughout this chapter that the observers making measurements in heterochromatic photometry are normal as regards colour vision, *i e*, that the curves they would individually obtain for the luminosity of the spectrum would not differ appreciably from that shown in Fig 186 ([91]).

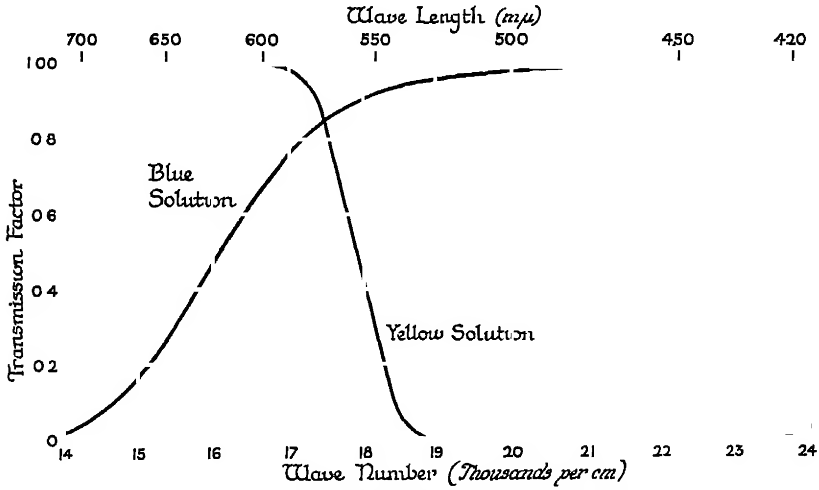


FIG 166 —Transmission Curves of Solutions for Testing the Colour Vision of Observers.

Undoubtedly there are many people who, while not noticeably colour-blind in the ordinary sense of the word, have peculiarities of colour vision, such as diminished sensitivity in some region of the spectrum, which render them unfit to make a comparison between lights of different colours ([92]).

It is clearly impracticable to have each observer make a complete determination for his own eye of the luminosity curve, and a more rapid method of testing for departures from normal colour vision consists in the determination by the observer of the relative transmission factors of specified yellow and blue solutions ([93]). For observers with normal colour vision these solutions have the same transmission factor when used in a thickness of 1 cm before a 2 7 l.p.w. (4 w p.c.) carbon filament lamp The solutions are, respectively,

(1) 72 grams potassium bichromate to 1 litre of water, and
(2) 57 grams copper sulphate to 1 litre of water

Only observers, or groups of observers, who find for these solutions approximately equal transmission factors when used with a 2·7 l p.w. lamp, should make observations in heterochromatic photometry.

although Ives and Kingsbury ([94]) have developed a method involving the use of absorbing media by means of which any observer can be corrected to normal.

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(84) Illum Eng Soc N Y, Trans, 16, *1921*, p 585.

(85) See also H E. Ives and E F Kingsbury, Phys Rev, 7, *1916*, p. 149 J S Dow found that unequal periods of exposure for the two surfaces in the photometer did not materially affect the accuracy of the measurement (Electrician, 58, *1907*, pp 609 and 647, Ecl El, 50, *1907*, p 397)

(86) Phys Rev, 4, *1914*, p 222, El World, 64, *1914*, p 919

See also E F Kingsbury, Frank Inst, J, 180, *1915*, p 215, J of Gas Lighting, 131, *1915*, p 393, Mechanical Engineer, 36, *1915*, p 215, Illum Eng, 8, *1915*, p 462

(87) H E Ives Frank Inst, J, 182, *1916*, p 541, J of Gas Lighting, 136, *1916*, p. 220, Phil Mag, 33, *1917*, p 360, Opt Soc Am, J, 1, *1917*, p 98

See also S Maisel, Russian Phys -Chem Soc, J, 39, *1907*, p 193, J de Phys, 7, *1908*, p 739

In G Yvon's photometer (Rev d'Opt 1, *1922*, p 499) the two beams to be compared are polarised in mutually perpendicular planes and are viewed through a rotating Nicol so that the resulting flicker field is free from a dividing line of any kind

(88) J Guild J Sci Instruments, 1, *1924*, p 182, El World, 83, *1924*, p 790

(89) E C Crittenden and F K Richtmyer Bureau of Standards, Bull, 14, *1918*, p 87, Illum. Eng Soc N Y, Trans, 10, *1915*, p 331, El World, 67, *1916*, p 499, Lum El 33, *1916*, p 112

See also L W Wild and others, Electrician, 60, *1907*, pp 122, 176 and 216 63, *1909*, pp 540, 638 and 639

L W Wild Illum. Eng, 3, *1910*, p 303, Electrician, 65, *1910*, p 74

P S Millar Illum Eng Soc N Y, Trans, 4, *1909*, p 769, Illum Eng, 3, *1910*, p 29

(90) H E Ives and E F. Kingsbury Phys Rev, 5, *1915*, p 230; Z f I, 36, *1916*, p 133.

G W. Middlekauff and J F Skogland Illum Eng Soc N. Y, Trans, 11, *1916*, p 164, Bureau of Standards, Bull, 13, *1916*, p 287; E T Z, 39, *1918*, p 69

A H Taylor Illum Eng Soc N. Y, Trans, 16, *1921*, p 574

(91) See, *e g*, F L. Tufts, Phys Rev, 25, *1907*, p 433.

W. Watson Roy Soc, Proc, 88, *1913*, p 404; J Parsons, "Colour Vision," p 235, W de W Abney and W. Watson, Roy Soc, Proc, 89, *1913*, p 232

(92) See, *e g*, C von Maltzew, Z f Psychol u Physiol d Sinnesorgane (Abth II), 43, *1909*, p. 76, F L Tufts, Am J Sci, 22, *1906*, p 531, O. N Rood, *ibid*, 8, *1899*, p 258

E Brodhun Gas World, 48 *1909*, p 182.

J S. Dow. Gas World, 48, *1908*, p 462

(93) H. E Ives Illum Eng. Soc N Y, Trans, 10, *1915*, p 203, Frank Inst, J, 188, *1919*, p. 217.

E. C Crittenden and F K Richtmyer Bureau of Standards, Bull 14, *1918*, p 87, Illum Eng Soc N Y., Trans, 10, *1915*, p 331; El. World, 67, *1916*, p 499; Lum El, 33, *1916*, p 112.

K. S Gibson Opt Soc Am, J, 9, *1924*, p 113

(94) Illum. Eng. Soc, N Y, Trans, 10, *1915*, p 259.

CHAPTER IX

SPECTROPHOTOMETRY

In ordinary photometry the light emitted by a source is measured as a whole, and quite irrespective of its spectral distribution, *i.e.*, of the relative intensities of the components of different frequencies. For many purposes, however, including heterochromatic photometry by certain methods (see p. 240), it is necessary to know in what manner the integral light is made up of its spectral components. In order to obtain this information it is necessary first to disperse the composite light by some device, such as a prism or grating (see p. 24), and then to measure the intensity of each component, or, in practice, the small group of components lying within a certain narrowly-restricted region of the spectrum.

This measurement was at first made by comparing the light emitted by a source within given limits of frequency with the *total* light given by the same or any other convenient source (1), or by using some "absolute" criterion for estimating its intensity (2), such as visual acuity (3) or the reduction necessary for extinction. In all modern spectrophotometry, however, the light emitted by the source under examination is compared with that given by a standard source in the same spectral region. If the spectral distribution of the light given by the standard source be known, a comparison of this kind at every part of the spectrum gives at once the spectral distribution of the light from the test source (4).

The Standard of Spectral Distribution.—The most fundamental form of source giving light of a known spectral distribution is the "black body," or "complete radiator," described in Chapters II. and V (pp 33, 132), which, when operating at the temperature of melting palladium (1,829° K), gives radiation having the spectral distribution shown in Fig 167 Unfortunately, this temperature is so low compared with that of most modern light sources that the comparison at the blue end of the spectrum becomes very unsatisfactory, and it is therefore desirable to use a black body at a higher temperature as a standard of reference (see p 133). The use of this form of black body is attended with considerable experimental difficulties, and its employment as a standard of spectral distribution is therefore generally confined to the standardising laboratory, some more convenient, though less fundamental, radiator being used in ordinary work The acetylene flame in various forms has been employed extensively for the purpose and its spectral distribution has been very carefully determined (5) Tungsten, although it does not radiate in exactly the same manner as a black body, does so sufficiently closely within the visible spectrum for the spectral distribution curve of a tungsten filament glow lamp to be used as a standard of reference within the limits of accuracy generally required (6).

True Temperature, Colour Temperature, and Black-Body Temperature.—It is convenient to refer here to the terms commonly used for describing the radiation characteristics of a body. The true temperature is defined as the temperature of the body as measured on the Kelvin thermodynamic scale ([7]), or, what is equivalent to it for most practical purposes, the gas thermometer scale given by

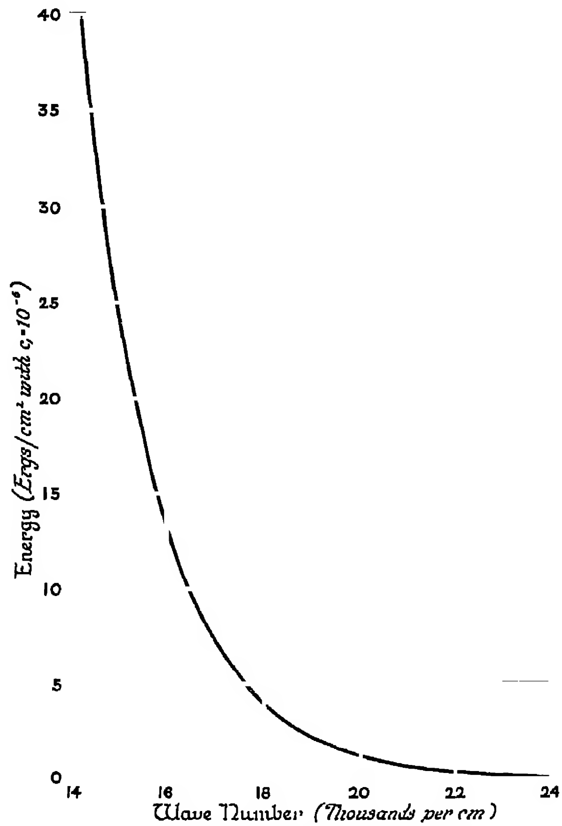

FIG 167 —The Energy Distribution from a Black Body at 1,829° K

the relation $P \propto T$, where P is the pressure of a given mass of an ideal gas, such as hydrogen, maintained at constant volume.

The "black-body" temperature (sometimes called the "brightness" temperature) of a body is the temperature of the black body whose brightness is the same as that of the body in question at a given part of the spectrum, generally a fairly narrow region in the neighbourhood of $\nu = 15{,}400$ ($\lambda = 0{\cdot}650\,\mu$). For a body which radiates at all frequencies exactly as a black body, the "black-body temperature" is necessarily identical with the true temperature For all other bodies giving purely thermal radiation ([8]), *i.e.*, most solid bodies radiating under open conditions, the "black-body temperature" cannot be greater than the true temperature, since the emissivity of such a body cannot exceed that of a black body at any part of the spectrum (see p. 34), and therefore at the particular frequency at which measurements are made.

For some bodies of practical importance in photometry (*e g.*, platinum and tungsten) the emissivity is a nearly constant fraction of that of a black body for all frequencies in the *visible* spectrum, but not for every frequency. In these cases the colour of the light emitted is nearly the same as that from a black body at the same true temperature If the emissivity be slightly higher at the red end of the spectrum, it will be possible to obtain a sensation colour match (*i e* , one which will appear correct to the eye) when comparing the body with a black body at a slightly lower temperature, while if the emissivity of the radiating body increase slightly towards the blue, the light from the body will give a sensation colour match with that from a black body at a temperature which is slightly higher than the true temperature of the body. In either case, the temperature of the black body which gives light of the same apparent colour as that given by the radiating body, is known as the "colour temperature" of that body [9], and this, clearly, may be either higher or lower than the true temperature of the body, owing to the inability of the eye to distinguish between the hues of lights of slightly different spectral composition [10].

Fig. 168 will illustrate the difference between the true and the "black-body" temperatures Curve *A* shows the radiation from

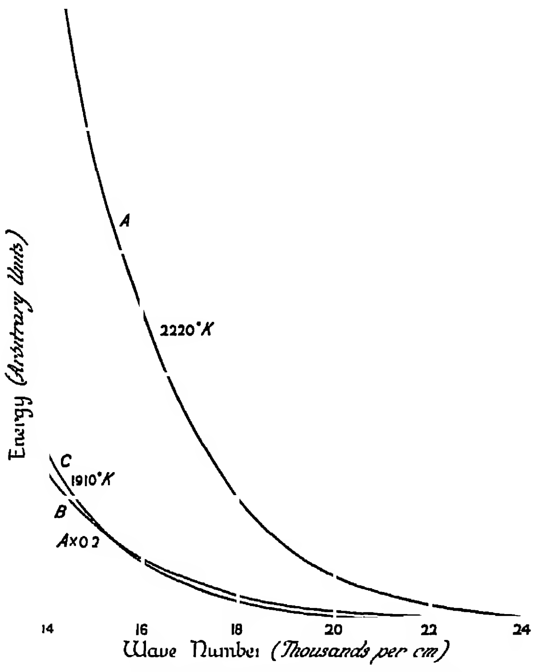

FIG 168 —The Relation between True Temperature and Black Body (Brightness) Temperature

a black body at a temperature of 2,220° K. Curve *B* shows the radiation from a grey body with emissivity 0·20, radiating at 2,220° K. Curve *C* shows the radiation from a black body at a temperature of 1,910° K It will be seen that *B* and *C* have the same ordinate

at $\nu = 15{,}400$ ($\lambda = 0{\cdot}650\ \mu$), so that the "black-body" temperature of the grey body, measured at this frequency, is 1,910° K. Since the ordinates of curve B are proportional to those of A, it follows that the light from a grey body will match in colour that from a black body at the same temperature, as indeed is obvious *a priori* In Fig. 169 curve A shows, as before, the radiation from a black body

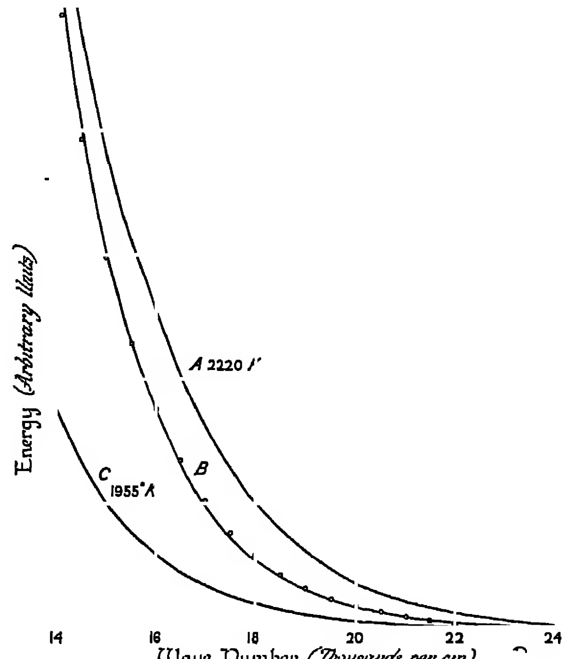

FIG. 169 —The Relation between True Temperature and Colour Temperature

at 2,220° K., while curve B now represents that given by a body at the same true temperature, but having an emissivity varying from 0·8 at $\nu = 15{,}000$ to 0 5 at $\nu = 20{,}000$ Curve C shows the radiation curve of a black body at temperature 1,955° K. It will be seen that the ordinates of curve B are everywhere so closely the same as those of curve C multiplied by the constant factor 2 93 (small circles) that the light given by the radiating body would match that from the black body at 1,955° K. very closely indeed. The "colour temperature" of the body would therefore be 1,955° K , *i e* , 265° lower than its true temperature.

Since, at any given temperature, a black body has the greatest possible emissivity at all frequencies (see p. 34), it follows that the brightness temperature of a body can never exceed its true temperature The colour temperature may, however, be either greater or less than the true temperature, as stated above.

The Tungsten Lamp as Radiation Standard.—A slightly selective radiator, such as a tungsten filament electric lamp, may, then, be used as a standard of spectral distribution by regarding it as the equivalent, for practical purposes, of a black body operating at the

"colour temperature" of the lamp. This colour temperature is given by the relation (curve *A*, Fig 170)

$$\log_{10} L/W = 2\,854 \log_{10} (T - 1{,}205) - 7{\cdot}840,$$

where L/W is the efficiency of the lamp in lumens per watt, corrected for the cooling effect of the leading-in wires and filament supports (11). From this relation it will be found that a tungsten lamp operating at an efficiency of 5 5 l.p w (1 8 w p m h c) may be assumed to emit

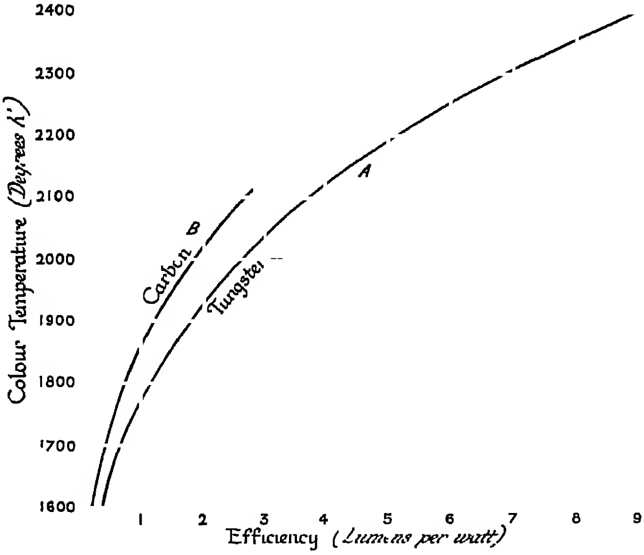

FIG 170 —The Relation between Colour Temperature and Efficiency for Electric Lamps

light of the same spectral distribution as a black body at 2,220° K., i e., its spectral distribution curve has the form shown in Fig. 168.

A similar relation for a carbon lamp is (curve *B*, Fig. 170)

$$\log_{10} L/W = 2\,789 \log_{10} (T - 1{,}270) - 7{\cdot}724.$$

Carbon behaves as a black body more closely than tungsten (12), but it cannot be used at any temperature above about 2,000° K

When electric glow lamps are used as standards of spectral distribution it has to be remembered that the effect of blackening of the bulb, which takes place gradually as the lamps are used, is to lower the apparent temperature of the filament (13)

The accuracy with which it is possible to make a colour match is, in the case of experienced observers, about one-quarter of 1 per cent. in temperature (14).

A colour temperature scale may, clearly, be set up by means of the leucoscope (see p 244), using a convenient source of known colour temperature (15)

The Spectrometer.—In the foregoing sections a description has been given of the manner in which a standard of spectral distribution may be obtained The remainder of this chapter will be devoted to a description of the methods which may be employed for comparing the spectral distribution of the unknown light with that from the

that the frequencies in the spectrum formed by a grating were so distributed that $d\lambda/dx$ was constant throughout the spectrum (to the accuracy with which $\tan\theta = \sin\theta$) In the case of a prismatic spectrum, however, no such simple relation between frequency and separation can be found, for the refractive index of glass is not a linear function of the frequency of the light, so that the spacing in the spectrum is uneven and is, moreover, different for different kinds of glass. It follows that, owing to the overlapping of the images of the slit, the total light reaching any given point of the spectrum is greater at the parts where the crowding of the images is denser, *i e*, where the dispersion is less, and *vice versâ* ([21]).

Although this effect has not generally to be allowed for in visual spectrophotometry where, as will be seen later, both the spectra compared are equally affected, it does enter into the slit-width correction to be described at the end of this chapter Moreover, in the determination of spectral energy distribution by any absolute method as, for example, by means of a thermopile (see p 319), the correction for dispersion must always be made, for what is required is the energy within a given wave-number (or wave-length) interval, and the interval included by the spectrometer slit is different for different parts of the spectrum. In fact, if the calibration curve of the spectrometer be plotted, its differential gives the multiplying factor to be applied at each frequency to convert prismatic intensity to the intensity in an even frequency spectrum For if $\delta\nu$ be the

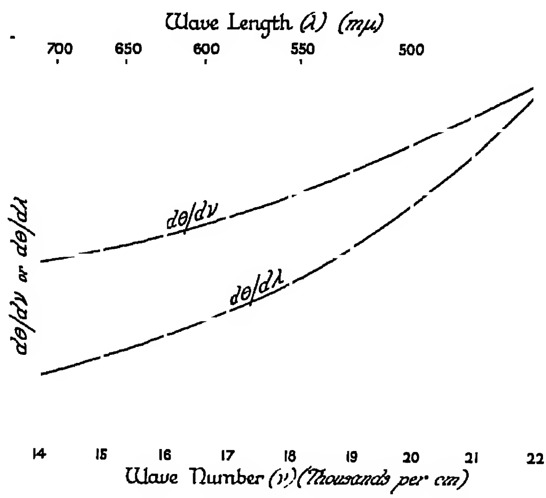

FIG 172 —Typical Dispersion Curve for a Glass Prism

effective slit width at wave-number ν, while I_p and I_ν are respectively the prismatic and the even-frequency intensities, then $I_\nu/\delta\nu = I_p$ A typical dispersion curve is shown in Fig 172.

The Spectrophotometer.—From what has been said above it will be clear that every spectrophotometer consists of the following two principal parts —

(i.) Apparatus for analysing the light given by the two sources to be compared, and for presenting to the eye, in a manner suitable

for photometric comparison, a limited region of the spectrum from each source

(ii.) Means for readily altering the brightness of one or both parts of the spectral field in a continuous and known manner.

The first part consists of an optical device similar in action to a spectrometer, the light from the two sources compared being admitted either into two separate collimators or, by two slits (or different parts of a single slit), into a single collimator. When two slits are used the light from the sources may conveniently be reflected on to them by means of total reflection prisms, as shown in Fig. 173 ([22]).

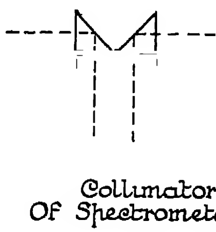

FIG. 173.—Double Prism Attachment for a Spectrometer.

There are two principal types of spectrophotometer field. In one of these the lights to be compared are presented to the eye in the form of two spectral bands placed one above the other, shutters being provided for isolating the particular portion of the spectrum being studied. In the other form of field the Maxwellian view is employed (see p. 109), so that the field appears homochromatic, although the light entering the eye is not strictly homogeneous. These two types of field will be referred to as the juxtaposed spectra type and the homochromatic field type respectively. The latter type generally gives a more satisfactory form of field as regards accuracy of equality match. The precautions to be observed as regards size of image when the Maxwellian view is used have already been mentioned (see p. 110). The purity of the field is naturally governed by the breadth of the ocular aperture or of the natural pupil, whichever is the smaller. The juxtaposed spectra type of field possesses the advantages that (i.) stray light is comparatively unimportant, and (ii.) any rapid changes in intensity of either spectrum can be at once seen, and the degree of uncertainty arising from this cause (see p. 287, *infra*) can be estimated.

The necessity for a fine and sharp line of demarcation between the comparison surfaces as seen by the eye is just as important in spectrophotometry as in ordinary photometry ([23]), and many of the older forms of spectrophotometer fail in this respect.

The means used for brightness control in ordinary photometry may also be employed in spectrophotometry, but the inverse square method is not generally suitable, since it is important to have very bright comparison surfaces (especially when working at the ends of the spectrum, where the luminosity is low), and the distances from the source to the spectrometer slit are, therefore, generally of the order of a few centimetres ([24]).

Stray light, due to reflections from the sides of the collimator or telescope if the diaphragm system be not perfect, and to scattering from the glass surfaces due to dust or scratches, is sometimes a cause of inaccuracy in spectrophotometry, particularly at the ends of the spectrum, where the luminosity is low. This trouble may be avoided by the use of suitable filters over the eyepiece. A filter having a high transmission in the red end of the spectrum and a low transmission elsewhere is Jena 4512 (or Corning G 24),

while for use in the blue end Jena 3654 (or Corning G 585) is suitable.

Early Spectrophotometers.—The first instrument (after those mentioned on p. 269) was that of Vierordt ([25]), in which the diaphragm method of brightness control was employed. This instrument consisted simply of a spectrometer in which the collimator was provided with a divided slit. Each half of the slit was illuminated by one of the sources to be compared, and the widths of the two halves were separately adjusted until the portions of the two spectra seen in the eyepiece were equally bright

The method of equality adjustment by alteration of slit width, even when symmetrically opening slits are used ([26]), is unsatisfactory, both from a practical and a theoretical point of view It is necessary, in order to obtain the requisite range at different parts of the spectrum, to use a slit width varying from at least 1 mm. to a few hundredths of a millimetre In the former case the spectrum is very impure (see p. 275), and in the latter case an error of 1 per cent. is produced by an inaccuracy of less than 0 001 mm , and diffraction effects introduce considerable uncertainty into the measurements. The results are further complicated by the fact that the correction to be applied on account of slit width varies according to the width employed. It is therefore desirable in spectrophotometry always to work with a fixed slit, so that the correction for slit width may be applied with certainty (see p 287)

The Lummer-Brodhun Spectrophotometer.—In this instrument ([27]) two separate collimators, C_1 and C_2 (Fig. 174), are used for the two sources being compared. The parallel beams from C_1 and C_2 traverse a Lummer-Brodhun prism L and the dispersion prism P, and then enter the telescope T, which is focussed on the interface of L. Since the division between the two parts of the field must be sharp, and must disappear at the point of balance, the dividing lines in L are necessarily horizontal ([28]), and when the contrast principle is used the field has the form shown in Fig 175 For the

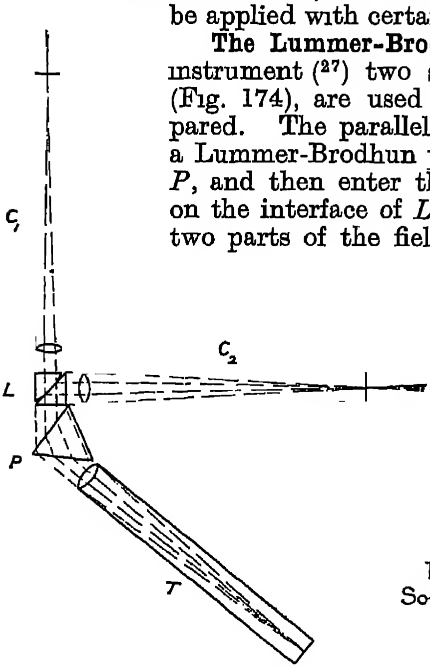

FIG. 174 —The Lummer-Brodhun Spectrophotometer

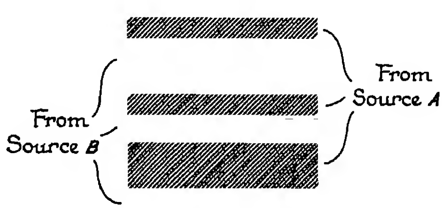

FIG 175 —The Contrast Field of the Lummer -Brodhun Spectrophotometer

adjustment of the two parts of the field to equality either a variable sector disc, a variable slit in one collimator, or alteration of the distance of one of the sources from its slit may be used.

A convenient form of disc for this purpose is that of Brodhun, described in Chapter VI (p. 178), or a special form of disc with openings as shown in Fig. 176 [29]. The curves forming the edges of the sectors are such as to give an approximately uniform percentage rate of change of transmission for a given alteration of d, the radial distance of the slit from the disc axis. The disc is mounted on a carriage, which can be moved along a "vee"-shaped track in a casting to which the base of the spectrometer is rigidly attached. The distance d is varied by moving the disc carriage along this groove by means of a screw, and is measured on a millimetre scale attached to the base at the side of the groove, a movement of 0·1 mm. at any position giving an alteration of about 0·8 per cent in the value of the transmission factor at that position.

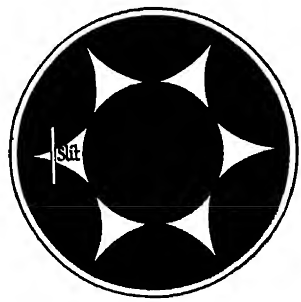

Fig. 176.—Hyde's Sector Disc for Spectrophotometry

Other instruments, somewhat similar in principle to the Lummer-Brodhun, are (i.) the Brace spectrophotometer [30], in which the

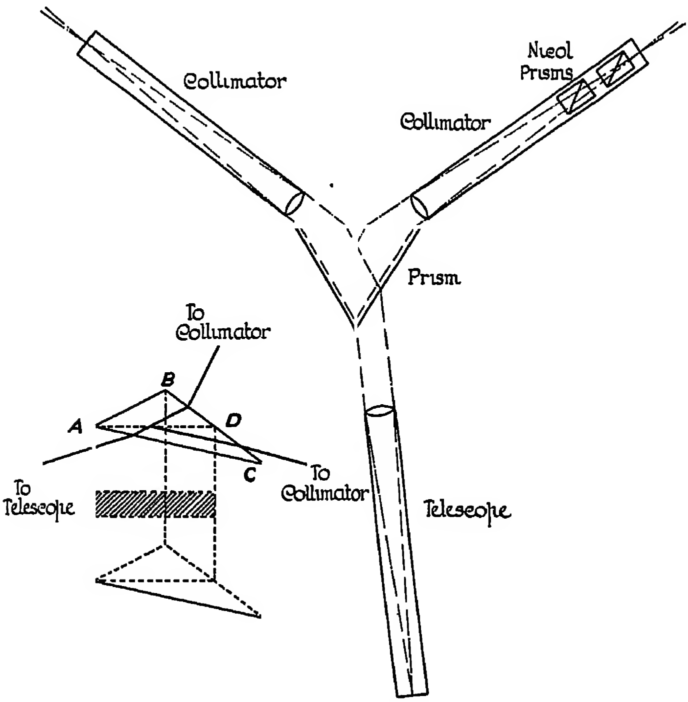

Fig. 177.—The Brace-Lemon Spectrophotometer.

photometer prism and the dispersion prism are combined as shown in Fig. 177, a central strip of the interface between the two halves *ABD* and *ACD* being silvered before these are cemented together, and (ii.) the differential spectrophotometer [31], in which the equality

of field is produced by variation of the collimator slits, the movement being so arranged that the sum of the two slits always remains constant. The diaphragm method of equality adjustment has also been used, the beams to be compared passing through stopped lenses before they reach the spectrometer ([32])

Another instrument, similar to the Lummer-Brodhun, but based on the Hilger constant deviation spectrometer, is that shown diagrammatically in Fig 178 ([33]) A second collimator is mounted above and parallel to that of the spectrometer by means of two supports *A* and *B*, one of which is provided with adjustments for securing exact parallelism of the collimators An image of the luminous disc of a 500 c.p. "pointolite" lamp is focussed on the slit of each collimator by means of lenses *C* and *D* and achromatic prisms *E* and *F* The photometric prism *G* is built of two parts, a parallelopiped and an isosceles

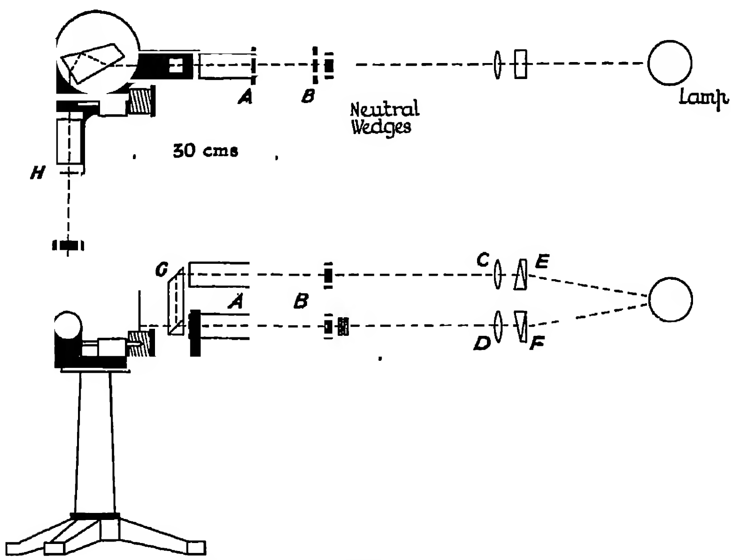

Fig 178 —Guild's Spectrophotometer

prism A central strip of the hypotenuse face of the latter is ground away, and the remainder put in optical contact with one of the short faces of the parallelopiped to form a composite prism of the shape indicated in the figure Light from the lower collimator passes straight through this prism except at the part of the interface where there is no optical contact. Light from the upper collimator is totally reflected down the prism by the first inclined face. At the part of the interface where there is no optical contact it is totally reflected in a direction parallel to the beam from the lower collimator, whereas at the regions of optical contact it passes straight down and is lost After passing through the dispersing system of the spectroscope the two beams are focussed as superposed continuous spectra in the focal plane of the telescope *H* The eyepiece of the latter is replaced by a slit, and an eye placed there sees the interface of the prism *G* as a photometric field of three horizontal strips, the central

one of which is illuminated by light from the upper collimator, and the outer ones by light from the lower collimator The brightness match is obtained by means of a series of sector discs of the type described on p 178, the interval between successive discs being bridged by means of a double neutral wedge placed in front of the lower slit

In all these instruments the homochromatic type of field is used.

Polarisation Spectrophotometers.—A number of different spectrophotometers depend on the polarisation method of intensity variation ([34]). In many of these a single collimator is used, and an image of one part of the slit formed by light polarised in one plane is compared with an image of the other half, either unpolarised, or polarised in the perpendicular plane A Nicol prism placed in the path of the light is used for producing the photometric balance, as in the Martens polarisation photometer (see p. 173) Of the spectrophotometers of this class the most commonly used is probably that designed by König and modified by Martens ([35]) Fig. 179 shows

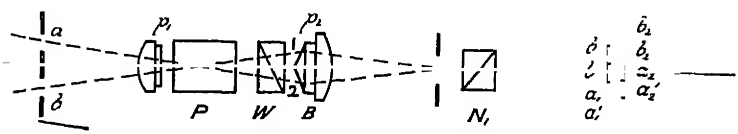

FIG 179 —The König-Martens Spectrophotometer

diagrammatically a section through the instrument perpendicular to the plane of the light rays, the bend of the rays at P being ignored for the sake of clearness The light from each of the two halves of the slit, a and b, after passing through the dispersion prism P, enters the Wollaston prism W and, after separation into two parts polarised in mutually perpendicular planes, passes through the biprism B and forms, therefore, four series of images of both a and b Of these four series of images of a, two, *viz* , a_1 and a_2, are polarised in one plane and are formed respectively by the two parts of the biprism, while the other two, a'_1 and a'_2, are polarised in the perpendicular plane One series of images of a, *viz.*, a_2, is caused to coincide with one series of images of b, *viz* , b'_1, and the remaining images are stopped off so that an eye placed at any point along the line formed by the two series of images sees the part 2 of the biprism bright by reason of light from a which is polarised in one plane, and the part 1 of the biprism bright by reason of light from b which is polarised in the perpendicular plane A Nicol prism placed in the path of the light at N_1, and capable of rotation about an axis lying along the direction of the light rays, gives a means of determining the ratio of the intensities of the two images just as in the ordinary Martens photometer. The position of the eye along the line of images (perpendicular to the paper) determines the frequency of the light for which the comparison is made. In practice the eye is not moved in order to make comparisons at different frequencies, but the prism P is rotated about an axis perpendicular to the plane containing the collimator and telescope axes until the image formed by light of the desired frequency is formed at the position of the eyepiece. The plane of the light rays through the spectrometer is

generally vertical in the König-Martens instrument, and horizontal in most other forms. Two prisms of small angle, p_1 and p_2, are introduced for the purpose of diverting light reflected from the surfaces of the various optical elements

Other spectrophotometers employing the polarisation method of brightness adjustment are those of Glan [36], whose instrument is very similar in principle to that of Konig-Martens, except that there is no biprism, so that two images of each slit are produced instead of four. In Glazebrook's instrument [37] two widely-separated beams are polarised in mutually perpendicular directions by two Nicol prisms, and equality is produced by the rotation of a third Nicol. In the Crova form [38] there are two Nicols in one of the beams to be compared, so that the brightness of this beam is proportional to $\sin^2 \theta$, where θ is the angle between the two Nicols The instrument suggested by Zenker is similar, but has three Nicols, the centre one being moved so that the brightness varies as $\sin^4 \theta$ [39]. All these instruments have the juxtaposed spectra type of field. H. Wild has devised a modification of his polarisation photometer [40] (see note (33), p 11), in which the criterion of equality is absence of polarisation as indicated by a Savart polariscope [41]. Konigsberger [42] has designed a spectrophotometric addition to his microphotometer (see note (71), p 408) in which the same criterion of equality is used.

The Brace instrument (see p. 279) has been improved by the insertion of a polarising device in one beam, instead of the variable slit originally used to obtain the photometric balance [43]. This modified form, known as the Brace-Lemon spectrophotometer, is shown in Fig. 177.

Instruments for use with Spectrometers.—Several instruments have been devised for use in combination with any form of accurate spectrometer [44]. Of these the first was that of G Hufner [45], who employed a rhomb of the form shown in Fig 91 (p 160) in front of the slit so as to produce an effective division of the slit with a very sharp line of demarcation. The light entering one half of the rhomb was polarised by means of a Nicol prism, and photometric balance was obtained by means of a movable Nicol placed in the ocular of the spectrometer. A similar instrument has been designed in which the diaphragm method of equality adjustment is used instead of the polarisation method [46] In Féry's instrument the double neutral wedge method of intensity adjustment is employed [47].

Other instruments designed to be used with an ordinary spectrometer are those of Ives (homochromatic) and Nutting (juxtaposed spectra) [48] The last named, as subsequently modified by Adam Hilger & Co , of London, has the form shown in Fig 180 The two parallel beams of light to be compared enter the apertures A and B of the photometer The beam entering the photometer at A is totally reflected at the face a of the prism P_1, and again at the silvered upper and lower parts of the face b. This face, however, is in optical contact with the prism P_2 over a central horizontal unsilvered strip about 2 5 mm broad, as shown in detail in the figure. It follows that the light from A which reaches this band travels on without reflection and is absorbed in the blackened wall

of the photometer box The light entering at B passes through a fixed Nicol prism N_1 and is transmitted directly through the central parts of the prisms P_1 and P_2 where the surface b is unsilvered, while the upper and lower parts of this beam are reflected to the wall of the box and there absorbed. Thus the beam leaving b consists of three parts, the upper and lower, unpolarised, from A, and the centre strip, plane polarised, from B. This tripartite beam then passes through a second Nicol prism N_2, which is capable of rotation at will about its axis The centre part of the beam is therefore reduced in intensity by the factor $\cos^2 \theta$, where θ is the angle between the optic axis of N_2 and the (fixed) optic axis of N_1 The light

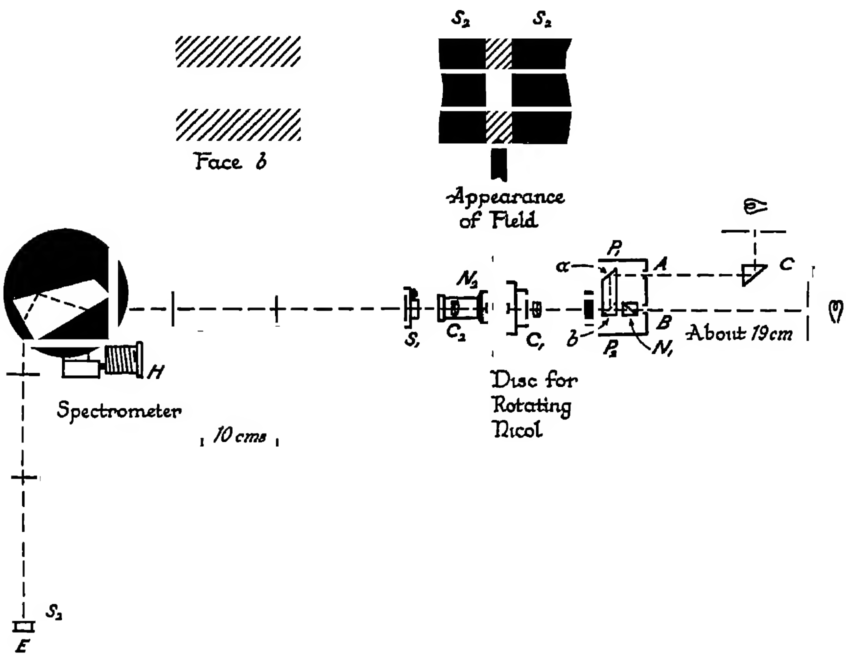

FIG 180 —The Nutting-Hilger Spectrophotometer.

passing through N_2 is made approximately parallel by adjusting the position of the lens C_1 according to the distance of the light source from the apertures A, B (49). C_2 is then moved so that the edges of the silvered parts of b are focussed sharply on the spectrometer slit. The appearance of the field seen at E is, therefore, as shown in detail at the top of the figure, the brightness and purity of the spectrum being governed by the width of the slit S_1 The spectral range seen is adjustable by means of sliding shutters S_2. These isolate a field of any desired breadth on either side of the pointer which indicates the exact region corresponding to the wavelength shown on the rotating cylinder head H of the prism The centre part of the field is brought to equality with the upper and lower parts by rotating N_2. If θ be the angle at which equality is found, the ratio between the intensities of the beams entering A and B respectively, viz , I_B/I_A, is $\cos^2 \theta/\cos^2 \alpha$, where α is an instrumental

constant found by a measurement in which the same sour illuminates both apertures, so that $I_B = I_A$.

In addition to the angular scale giving θ directly in degree there is a second scale giving values of $-2\log_{10}\cos\theta$. It follows th if the reading on this scale be a when the two sources are bein compared, and b when the same source is illuminating both ape tures, then $a = -\log_{10}\cos^2\theta$, and $b = -\log_{10}\cos^2\alpha$, so th $(b - a) = \log_{10}(\cos^2\theta/\cos^2\alpha) = \log_{10}(I_B/I_A)$

When the instrument is being used, not for the comparison two different sources, but for the measurement of the transmissio factor of a filter for light of various frequencies, the total reflectio prism C is removed, and the double prism arrangement shown i Fig. 181 is placed at a fixed distance from a single source Th prism arrangement forms two bean (with parallel axes), which enter th apertures A and B, and the filter placed in the path of one of then The measurement is then made a before, and in this case the tran mission factor of the filter is give by $\tau_\nu = \cos^2\theta/\cos^2\alpha$, where τ_ν is th factor for light of wave-number ν, th wave-number indicated by H whe the comparison is made On th other scale this becomes $(b - a) =$ $\log_{10}\tau_\nu$, so that $\tau_\nu = 10^{b-a}$.

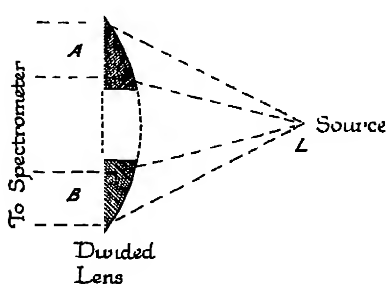

FIG 181 —The Double Prism for Use with a Spectrophotometer for the Measurement of Transmission Factors.

If the instrument be carefull adjusted so that $\alpha = 0$, the $b = 0$ and $\log_{10}(100\tau_\nu) = 2 + \log_{10}\cos^2\theta = 2 - a$, so that th percentage transmission $= 10^{2-a}$

This instrument possesses the double advantage that it can b used in combination with any good spectrometer, and the wid separation of the apertures A and B permits the measurement of th transmission factor of a filter of ordinary size at any part of it surface, and not only at the edge

In a later model two auxiliary Nicol prisms are used, one behin each of the apertures A and B By this means the two inciden

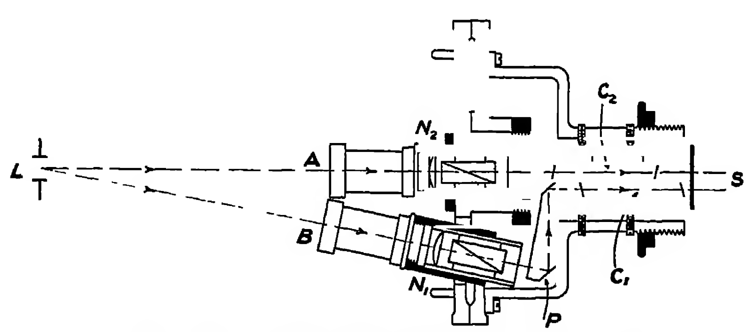

FIG 182 —The Bellingham-Stanley Spectrophotometer.

beams are polarised in planes mutually at right angles, and therefor $I_B/I_A = \tan^2\theta/\tan^2\alpha$, as in the Martens photometer (see p 175) ([50])

Another instrument designed for use in combination with a spectrometer is that shown in elevation in Fig. 182 ([51]) The two comparison beams from the light source L pass into the instrument at A and B. The beam B passes through a fixed Nicol prism N_1 and then, by way of a total reflecting prism P of special form, through the lower calcite plate C_1 The beam A passes through the movable Nicol N_2 and the upper calcite plate C_2 The calcite plates are of such dimensions and so cut with respect to the optic axis of the crystal that only the extraordinary ray is transmitted ([52]) The two beams illuminate S, the collimator slit of the spectrometer.

This instrument has been designed for use with the form of spectrometer shown in plan in Fig 183 ([51]) The light entering

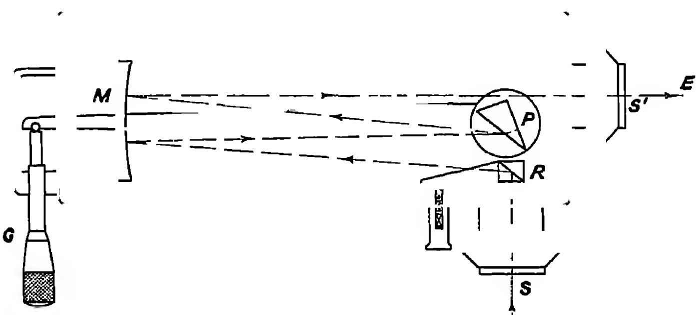

FIG 183 —The Bellingham-Stanley Spectrometer

S is reflected by the prism R to the concave mirror M, and thence to the dispersing prism P This prism is silvered on the back surface and so reflects the light back to M, from which it is again reflected to the eyepiece. The focal length of M is such that an image of S is formed at S'. The frequency of the light reaching S' depends upon the position of the prism P, which can be rotated by means of the arm and micrometer screw G.

The field of this spectrophotometer is, again, of the juxtaposed spectra type The photometric balance is obtained by rotating N_2 until equality is obtained It will be seen that the planes of polarisation of the beams reaching S are strictly parallel.

A source of error which has to be carefully considered in the design of polarisation spectrophotometers is the partial polarisation introduced by passage through glass surfaces, such as those of the dispersion prism. According to Fresnel's equation ([53]) (see p 112), if unpolarised light fall on a reflecting surface (such as a glass surface) at an angle of incidence θ, and if θ' be the angle of refraction, the reflection factor for light which is polarised in the plane of incidence is $\rho_I = \sin^2(\theta' - \theta)/\sin^2(\theta' + \theta)$, while for that polarised in the perpendicular plane it is $\rho_P = \tan^2(\theta' - \theta)/\tan^2(\theta' + \theta)$. The corresponding transmission factors will be $\tau_I = 1 - \rho_I$ and $\tau_P = 1 - \rho_P$. Since in a prism at the angle of minimum deviation (the usual arrangement in a spectrometer) the angle of incidence on entering the prism is equal to the angle of refraction on leaving it ([54]), the transmission factor of the whole prism for light polarised in the plane of incidence is $(1 - \rho_I)^2$, while for light polarised in the

perpendicular plane it is $(1 - \rho_P)^2$ (neglecting absorption in both cases). The values of these factors for a 60° prism for which $n = 1{\cdot}658$ are respectively 0·651 and 0 997.

When, therefore, a spectrum produced by such a prism is observed through a polarising prism, and the latter is rotated, the beam varies in intensity from $(1 - \rho_P)^2$ to $(1 - \rho_I)^2$. The matter has been treated fully by Hufner and also by Twyman ([55]), who compensated for the effect in the Hufner instrument by using a rhomboidal prism of such an angle that the amount of polarisation produced in it was equal and perpendicular to that produced in the dispersion prism.

The difficulty may be entirely avoided in either of two ways as follows (i.) the light, *after* traversing the dispersing prism, may pass through a *fixed* polariser before reaching the movable Nicol (as in the König-Martens instrument), or (ii) the polarising device for changing the relative intensities of the beams may be arranged so that both beams are always polarised in the same plane when passing through the dispersing prism (as in the Nutting and Bellingham-Stanley instruments)

It will be clear that all polarisation instruments must be liable to error if either of the incident beams contain any polarised light ([56]). This difficulty can only be overcome by adopting the procedure described in connection with the Martens photometer (see p 175). It is far preferable, however, to use some other type of instrument for the measurements in this case.

Flicker Instruments.—The use of the flicker principle is unnecessary in ordinary spectrophotometry, since colour difference is entirely eliminated. Instruments of the flicker type have, however, been designed ([57]) in connection with the determination of the sensitivity curve of the eye (see p 294), since for this purpose it is necessary to make a photometric comparison between the brightnesses of two fields which are respectively illuminated by light from two different parts of the spectrum

Photometric Procedure.—Throughout this chapter it is assumed that the substitution method is used, *i.e*, that when two sources are compared, they are each measured by comparison with a third source which may be assumed to remain constant throughout the experiment

The slit or slits of the spectrometer should, whenever possible, be illuminated by diffused light. This may generally be arranged by causing the source to illuminate either a reflecting surface, such as magnesium carbonate, or a transmitting surface, such as depolished opal glass, and then forming an image of a portion of this surface on the collimator slit Slight selectivity of the diffusing material is unimportant so long as strict substitution between the test source and the sub-standard is arranged

It should be noted that the measurements made in spectrophotometry are nearly always relative, *i e*, it is the form of the spectral distribution curve that is required, and not the absolute intensity at any given frequency If absolute values are needed it is often convenient to obtain these by comparing the integral candle-power of the test source, obtained by ordinary photometric methods, with the candle-power obtained by weighting the energy distribution

curve in accordance with the sensitivity curve of the eye (see p 294). If this method cannot be used, as, for example, when a comparison is made at one region of the spectrum alone, the diffusing surface should preferably be a reflector, and should be at a sufficient distance from the source to ensure that the brightness of the reflecting surface bears a constant ratio to the candle-power when one source is substituted for another.

The accuracy attainable in photometric measurements is naturally less than that generally expected in ordinary photometric measurements under good conditions ([58])

Slit-width Correction.—It has been pointed out already that, owing to the finite width of the slit of a spectrometer, the light reaching any point of the spectrum is not homogeneous, but consists of a mixture of waves covering a given range of frequencies This lack of purity of the spectrum does not introduce any serious error into a spectrophotometric comparison so long as the spectra compared are continuous and not very different in energy distribution When, however, an irregularly distributed spectrum, such as that given by a Welsbach mantle, is compared with a black-body spectrum, or when two black-body spectra corresponding to widely different temperatures are compared, the correction for the finite width of the slit cannot be neglected ([59]) The mathematical treatment of the problem is somewhat lengthy, and for it the original papers should be consulted ([60]) The method of applying the correction will best be understood from an example Let the energy distribution curve of the standard source be represented by $f(\nu)$, and that of the test source by $\phi(\nu)$. Further, let the luminosity curve of the impure spectrum of the standard source as seen in the spectrophotometer be represented by $F(\nu)$, and let the ratio of the intensities at any wave-number ν be $p(\nu)$. In Fig 184 let the curve ABC represent the function $p(\nu)F(\nu)$, and let OP represent a given wave-number ν. Let PM and PN each represent a wave-number interval equal to $\frac{1}{2}(a+b)$, where $2a$ and $2b$ are respectively the breadths of the collimator and telescope slits in terms of the wave-number scale of the spectrometer Let $BD = \delta(\nu)$. Then it may be shown that if $n \equiv b/a$,

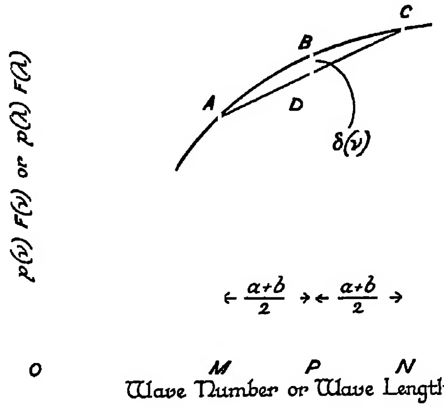

FIG 184 —The Slit-width Correction in Spectrophotometry

$$\frac{\phi(\nu)}{f(\nu)} = \frac{\{p(\nu)F(\nu) - l\delta(\nu) + m\delta'(\nu) - \quad \cdot \ \}}{\{ \quad F(\nu) - l\delta_0(\nu) + m\delta'_0(\nu) - \quad \}}$$

where $\qquad l = (1 + n^2)/6(1 + n)^2$

and $\qquad m = (6 + 5n + 10n^2 + 5n^3 + 6n^4)/180(1 + n)^4$.

$\delta_0(\nu)$ is found in the same manner as $\delta(\nu)$ from the curve for $F(\nu)$, and $\delta'(\nu)$ and $\delta_0'(\nu)$ are found in an exactly similar manner from the curves for $\delta(\nu)$ and $\delta_0(\nu)$ It should be noticed that $\delta(\nu)$ is positive when the curve is convex towards the ν-axis When $a = b$ the expression reduces to

$$\frac{\phi(\nu)}{f(\nu)} = \frac{\{p(\nu)F(\nu) - \delta(\nu)/12 + \delta'(\nu)/90 - \ldots\}}{\{F(\nu) - \delta_0(\nu)/12 + \delta'_0(\nu)/90 - \ldots\}}$$

$F(\nu)$ may be calculated from $f(\nu)$ if the dispersion curve of the spectrometer prism be known, for $F(\nu) \propto K_\nu f(\nu)/\Delta(\nu)$, where $\Delta(\nu)$ represents the dispersion (see p 275) and K_ν the luminosity function.

It is to be remarked that in the case of an instrument in which the spectrum is viewed by means of an eyepiece, $2b$ represents the width of the aperture limiting the spectral region viewed by the eye. In the case of an instrument in which the Maxwellian view is employed $2b$ represents the width of the pupil in the eyepiece, or the natural pupil, whichever be the smaller. The above treatment holds, *mutatis mutandis*, if wave-lengths be worked with instead of wave-numbers.

The Use of Colour Filters for Approximate Work.—Rough spectrophotometric determinations may be carried out with any ordinary form of photometer by placing over the eyepiece coloured media having comparatively narrow transmission bands in the different parts of the visible spectrum ([61]). The relative intensities of the two lights compared are thus obtained for the regions of the transmission bands of the media. Special glasses or gelatine filters suitable for this purpose have been prepared ([62]). By using a modified form of direct-vision spectroscope in the eyepiece of an ordinary photometer it is possible to obtain photometric comparisons at different parts of the spectrum ([63])

The spectral transmission curve of a medium may often be determined conveniently by using sources of light from which homogeneous radiations at different parts of the spectrum may be obtained by means of suitable coloured glasses The mercury, hydrogen, and helium tubes are particularly convenient for this work ([64]), or, where very intense light is required, a suitable form of electric arc may be used ([65])

Unsteady Sources.—In the spectrophotometry of unsteady sources, such as the arc, it is generally necessary to use an ordinary photometer as a control instrument, readings on the spectrophotometer being made only when an auxiliary observer at the ordinary photometer signals that the arc is of normal intensity and sufficiently steady for observation ([66])

Measurement of Reflection and Transmission Factors.—From the description, given in the last chapter, of the use of colour filters in heterochromatic photometry, it will be clear that a very important application of spectrophotometry is to the measurement of the transmission factor of a coloured transparent medium throughout the visible spectrum. The modifications necessary in order to make this type of measurement with the apparatus which has been described above will generally be obvious, but a further treatment

of the subject will be found in Chapter XIII (pp. 387 *et seq.*). In the same chapter the methods used for measuring the reflection factors of surfaces for light of any frequency are described ([67])

The Plotting of Spectrophotometric Data.—It is often convenient to plot spectral reflection or transmission curves in such a way that, in addition to giving the values of ρ_ν or τ_ν throughout the spectrum, they also show graphically, by their areas, the values of $\int K_\nu \rho_\nu d\nu$ or $\int K_\nu \tau_\nu d\nu$, *i e.*, the integral reflection or transmission factors for an equal energy spectrum This can be done by plotting the values of ρ_ν or τ_ν on specially prepared co-ordinate paper on which the scale of abscissæ is such that $x_\nu = a \int_0^\nu K_\nu d\nu$ ([68]) The principle may clearly be extended so as to exhibit the values of ρ and τ for light of any given spectral distribution by making $x_\nu = a \int_0^\nu K_\nu E_\nu d\nu$, where E_ν is the energy per unit wave-number interval at wave-number ν (see p. 36) for the light adopted An example of this method of plotting is given in Fig. 185 (*a*), where curve G represents the spectral transmission of a certain blue-green medium The ordinate at any value of ν gives the value of τ_ν, while the total area gives the value of τ for light having an equal energy spectrum on the wave-number scale, *i e*, equal amounts of energy in equal wave-number intervals throughout the spectrum A similar chart for light having an equal energy spectrum on the wave-length scale is shown in Fig 185 (*b*) Tables have been prepared to enable the integral transmission or reflection factor of a coloured medium to be calculated readily for lights of certain defined spectral distributions of practical importance ([69]).

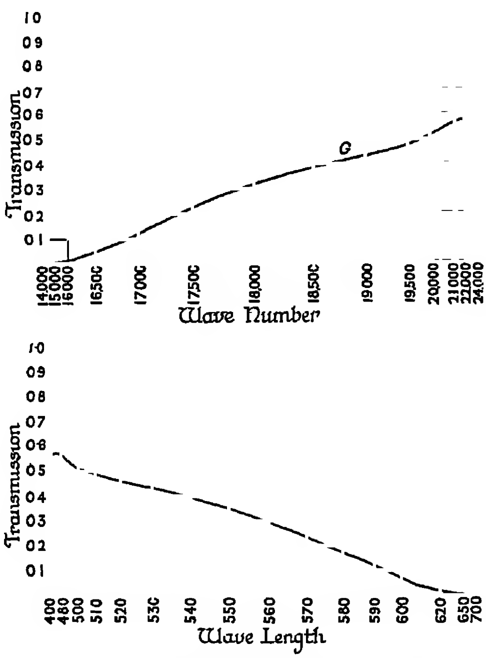

FIG 185 —Spectrophotometric Plot with Luminosity Abscissæ
(*a*) Wave-number Basis.
(*b*) Wave-length Basis

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$$\log_{10} L/W = 23\ 31_8 - 5\ 1 \log_{10} T - 185 \log_{10}(1 + 155/T)$$

where L/W is uncorrected for end losses For a carbon lamp they find

$$\log_{10} L/W = 21\ 51 - 4\ 58 \log_{10} T - 185 \log_{10}(1 + 155/T).$$

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CHAPTER X

THE MEASUREMENT OF COLOUR

In the last chapter a description was given of the instruments and methods used for comparing two lights in any desired portion of the spectrum, and so obtaining the spectral distribution curve of one light by comparison with another, standard, light of known distribution

This process does not demand of the eye anything more than a judgment of simple equality of brightness, the complicating factor of colour difference having been eliminated from the problem The results of a spectrophotometric comparison of this kind, while they give complete data as to the relative *energy* distributions of the two integral lights compared, give no information regarding the relative effects which these lights will produce on the eye, as regards either intensity or colour, unless the relative luminosity values which the eye assigns to equal amounts of radiant energy in waves of different frequencies be accurately known ; for, as has been said already in Chapter III. (p. 64), the eye responds very variously to lights of different colours, being most sensitive in the yellow-green and least sensitive at the ends of the spectrum, with the result that equal amounts of radiant power concentrated, for example, in the yellow-green and in the blue-violet would not be equal from the point of view of luminosity It follows that an essential link in the chain connecting visual photometry with the physical measurement of energy is the determination of the "sensitivity curve" for the average eye, *i.e*, the curve giving the value in photometric units of a given amount of radiant energy in the form of light waves of any frequency within the visible spectrum. This curve is frequently referred to as the "visibility curve." Better names are (i.) "sensitivity curve," when the response of the normal eye to radiation of different frequencies is being considered, and (ii) "luminosity curve," when it is desired to consider the capability of radiant energy of different frequencies to produce the visual sensation

The Sensitivity, or Luminosity (Visibility), Curve.—This curve (1) has been accurately determined by several workers, mostly in America, who have used different numbers of observers and worked with different apparatus and, usually, different photometric conditions. Most work, to the present time, has been done with the flicker photometer. E. Thurmel (2), H Bender (3), H E Ives (4), P G. Nutting (5), W. W. Coblentz and W. B. Emerson (6), P. Reeves (7), and M. So (8) all used this method, the last five under the standard conditions suggested by Ives (see p 259). Both Ives and Coblentz used also the steady comparison method, measuring the light at each frequency by direct comparison with the integral light from the source, but the large colour differences involved make their results unreliable, and until recently the best data obtained

by the steady comparison method were those of E. P Hyde, W. E Forsythe and F E. Cady (9), who used a step-by-step method in which large colour differences were avoided

A recent careful redetermination (10) by both the steady comparison and the flicker methods, using a field of 3° diameter, has shown that these two methods give practically identical results, and it is these results which have been provisionally adopted for general

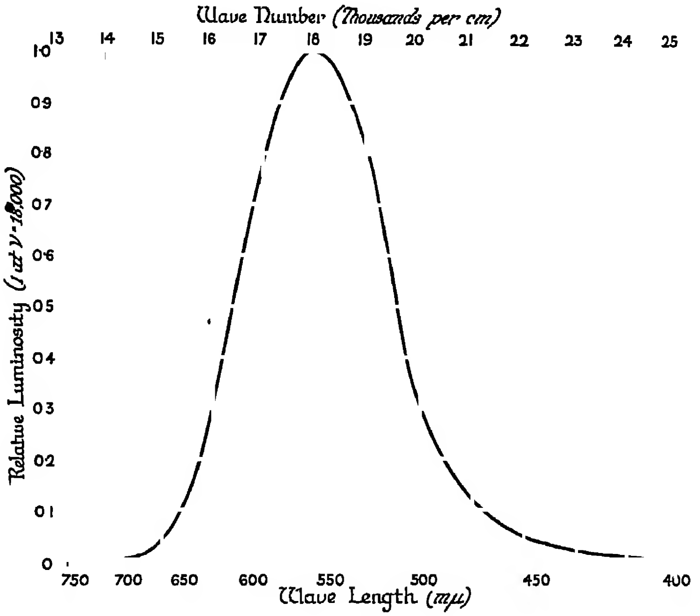


FIG. 186 —The Sensitivity Curve of the Normal Eye The Luminosity (Visibility) Curve of Radiant Energy.

use (11) and which are listed in Appendix IV. These have been used for plotting Fig. 186

Several attempts have been made to find an empirical expression which will represent the luminosity curve to the accuracy of experiment (12). Of these the latest forms are those of Kingsbury, as revised by Ives (13), and Coblentz (14), the former of which fits the curve obtained by Ives and Kingsbury using the flicker method, while the latter fits the curve obtained by the comparison of brightness (step-by-step) method. Both of these expressions have the same general form, *viz* ,

$$K_\nu = \Sigma A(R\nu e^{1-R\nu})^n$$

there being three or four terms in the summation The values of the parameters given by these workers, and the best values required to fit the mean luminosity data shown in the table of Appendix IV , are as given on p. 296

The symmetry of the sensitivity curve is greater if allowance be made for the change in the transmission factor of the ocular media with alteration of frequency of the light, so that the curve exhibits the response of the *retina* to radiation of different frequencies (15).

	A_1	$R_1 \times 10^7$	n_1	A_2	$R_2 \times 10^7$	n_2	A_3	$R_3 \times 10^7$	n_3	A_4	$R_4 \times 10^7$	n_4
Kingsbury	0 977	556	200	0 04	470	200	0 085	600	1300	—	—	—
Coblentz & Emerson	0 999	556	200	0 035	455	400	0 180	610	1000	0 084	525	2000
International *	0 995	556	220	0 055	475	350	0 105	610	700	0 096	523	2000

The Mechanical Equivalent of Light.—It is clear that, since the energy distribution of the radiation given by a black body at any temperature is known, its relative candle-powers at different temperatures can be at once obtained from the sensitivity curve of the eye (16), for clearly the luminous flux $F = A\int_0^\infty K_\nu E_\nu d\nu$, where K_ν is the value of the luminosity function and E_ν is the rate of energy emission at wave-number ν. Further, if the factor relating the luminous flux and the rate of energy emission be known for radiation of any one frequency, the absolute candle-power of the black body at any temperature may be deduced Alternatively, if the absolute candle-power of the black body at any temperature be known, the factor relating luminous flux with energy emission may be found for radiation of any frequency The frequency usually chosen is that of maximum luminosity, *viz*, $\nu = 18{,}000$ ($\lambda = 556$ mμ), and the factor expressed in lumens per watt is usually known by the somewhat misleading name of the "mechanical equivalent of light" This term may, therefore, be defined as "the ratio of radiant flux (expressed in watts) to luminous flux (expressed in lumens) for the frequency of maximum luminosity" (17), and is the factor A in the equation given above It will be seen that this equation applies not only to the black body, but to any source of light of known spectral distribution. The value of the mechanical equivalent of light may therefore be found by measuring the values of F and of $\int_{\nu_1}^{\nu_2} K_\nu E_\nu d\nu$ for any convenient source, the limits of integration ν_1 and ν_2 being such that the whole of the visible spectrum is included (18).

Alternatively the ratio of the luminous flux to the radiant flux may be measured for monochromatic radiation of any convenient frequency, and this ratio may then be divided by the value of K_ν for that frequency (19). This method may be extended by using a special chemical solution for which the transmission factor throughout the visible spectrum is proportional to the sensitivity of the normal eye (20). Such a solution is composed of—

Cupric chloride ($CuCl_2$) .		61·25 gm
Cobalt ammonium sulphate ($Co(NH_4)_2(SO_4)_2$)		14·5 ,,
Potassium chromate (K_2CrO_4)		1·9 ,,
Water . . .	to	1 litre

* This four term expression gives figures which agree generally with the listed values to about 1 per cent between 450 and 630 mμ At the wave lengths outside this range the difference is a few units in the third place of decimals (J W T Walsh, Opt Soc Am, J, 11, *1925*, p 111, E P T Tyndall and K S Gibson, Opt Soc. Am, J., 9, *1924*, p 403).

and may be used in a cell of the form described on p. 245, the thickness of the solution being 1 cm. The radiation in the infra-red may be absorbed by a cell of clear water of sufficient thickness, say, 4 cm (see p. 322, *infra*). The energy transmitted by this solution being automatically weighted in accordance with the sensitivity curve of the eye, the ratio of the luminous flux given by any source to the energy radiated by that source through the solution is the value of the mechanical equivalent of light multiplied by the transmission factor of the solution at the frequency of maximum sensitivity.

The mean value of the mechanical equivalent of light as determined recently by various methods may be taken as 0 0016 watts per lumen (21)

Using Wien's form of the expression for the energy distribution of a black body (see p 135), and any one of the expressions given above for representing algebraically the sensitivity curve of the eye, it is possible to find an expression for the candle-power of a black body at any temperature (22), for

$$F = \int_0^\infty K_\nu E_\nu d\nu = \Sigma \int A(R\nu)^n e^{(1 - R\nu)n} C_1 \nu^3 e^{-C_2\nu/T} d\nu$$

$$= \Sigma\, C_1 A R^n e^n \int \nu^{(n+3)} e^{-(Rn + C_2 T)\nu}\, d\nu$$

$$= \Sigma\, C_1 A R^n e^n (Rn + C_2/T)^{-(n+4)}\, \Gamma(n + 4).$$

This may be written

$$\Sigma\, \alpha\, (1 + \beta/T)^{-(n+4)}$$

where $\alpha = C_1 A R^n e^n (nR)^{-(n+4)}\, \Gamma(n + 4)$

$\qquad = C_1 A (\beta/C_2)^4 \sqrt{10n}\, .\, (n + 4)(n + 3)(n + 2)$*

and $\beta = C_2/nR.$

In this expression T is the absolute temperature of the body, and C_1 and C_2 the values of the constants, in Wien's equation (see p. 135). From the above expression the luminous efficiency (ratio of total light to total radiated energy) may be found, and the temperature of maximum luminous efficiency deduced.

It is satisfactory that Hyde, Forsythe and Cady (23) have found an agreement to 1 per cent. between the observed brightness of a black body over a temperature range from 1,700° to 2,600° K., using a direct comparison of the integral lights, and the *computed values*, using Planck's formula and the luminosity curve obtained by them The calculated values of the brightness of a black body at temperatures between 1,800° and 9,500° K. are shown plotted in Fig. 187

The use of the luminosity curve in physical photometry will be discussed in the next chapter. A different application of this curve and allied data, *viz*, the specification of colour in terms of physical measurements, will be briefly described in what follows.

Colorimetry.—Colour is the psychological sensation resulting from the physiological action produced on the retina by that physical property of the light waves which is called "frequency." Colorimetry is concerned with the specification of light in such a manner that its colour may be reproduced uniquely from physical data, although, as

* By Stirling's theorem, to an accuracy of 0 1 per cent. since $n > 100$.

will be seen later, two lights which are quite different in spectral composition may be identical in colour (24).

It has been found experimentally that every light, whether composite or homogeneous, may be exactly "matched," *i e*, the colour sensation to which it gives rise may be precisely simulated,

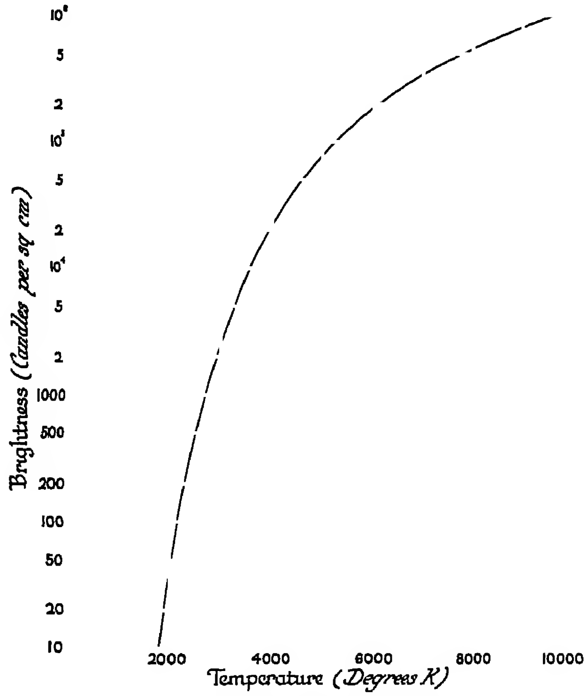


FIG 187—The Brightness of a Cavity Radiator

in two different ways, which give rise respectively to the "trichromatic" and the "monochromatic" systems of colour specification (25).

The Trichromatic System.—It was shown by Maxwell (26) that if lights of three different colours be chosen from the three principal regions of the spectrum, *viz.*, the red, green and blue-violet, then all other lights can be matched by mixing suitable (including negative) amounts of these three primaries. The primaries selected may be either homogeneous or composite, and may be produced spectroscopically or by means of colour filters. If R, G and B represent unit amounts of each of these three primaries then the colour of any given light may be represented by an expression of the form $rR + gG + bB$, where r, g and b are coefficients which may be either positive or negative The values of the units R, G and B are generally so chosen that for white light $r = g = b$ (27), *i.e.*, any mixture of an equal number of units of each of the three primaries gives sensation white.

It is clear that on this system the values of r, g and b must depend on the exact colour of the lights chosen for the three primaries, but

conversion from one set of primaries to another is easy if the primaries of one set be known in terms of primaries of the other set, for if

$$R = l_1R' + m_1G' + n_1B'$$
$$G = l_2R' + m_2G' + n_2B'$$
$$B = l_3R' + m_3G' + n_3B'$$

then a colour $(rR + gG + bB)$ on the one system becomes $\Sigma(rl_1 + gl_2 + bl_3)R'$ on the other system ([28]) so long as $R + G + B = R' + G' + B'$, *i e.*, equal numbers of the units on both systems make the *same amount* of white.

So far nothing has been postulated as to the mechanism of vision, but if the trichromatic theory be taken as a working hypothesis (see p. 71) it becomes evident that it should be possible to choose the primaries (which need not, in fact, be lights actually producible) in such a way that r, g and b may always be positive For if the primaries be the pure sensations postulated on the trichromatic theory of vision, it is clear that any real light can only stimulate these sensations positively.

By a consideration of the observed phenomena of colour mixture and other experimental data which cannot be considered here ([29]) it is possible to analyse the sensitivity curve shown in Fig. 186 into three component curves such that the ordinate of each at any frequency gives the contribution of the corresponding colour sensation to the total luminosity of energy at that frequency ([30]). These

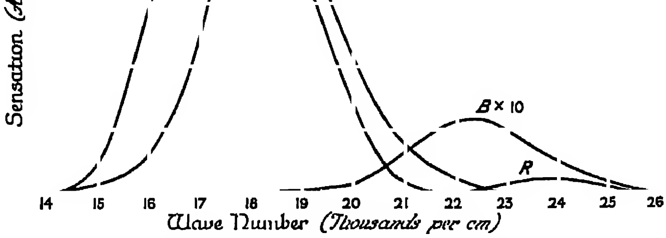


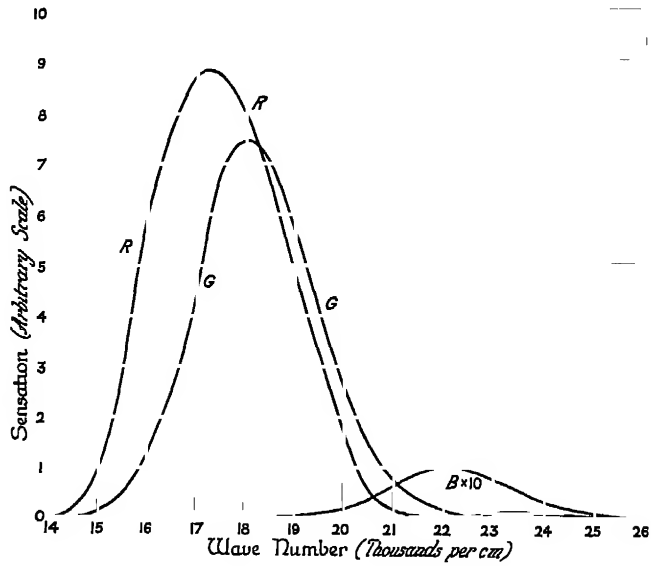
FIG 188.—The Sensation Curves for an Equal Energy Spectrum.

curves are shown in Fig. 188 for an equal energy spectrum, *i e*, for a total luminosity curve identical with that of Fig. 186.*

* It should be noted that the term "equal energy spectrum" on the wave-number basis signifies that the energy radiated within a given *wave-number interval* is the same throughout the spectrum: on the wave-length basis it implies that the energy within a given *wave-length interval* is constant. The actual energy distribution in the spectrum is therefore not the same in the two cases In Figs 186 and 188 if the wave-number scale be used the equal energy spectrum assumed is that of equal energy per unit wave-number, while if the wave-length scale be used, the spectral distribution is assumed to be such that there is equal energy in equal wave-length interval

It follows that the sensation curves for light of a given spectral distribution must

Similar curves may be drawn for light of any other spectral distribution by multiplying each ordinate of the sensation curves



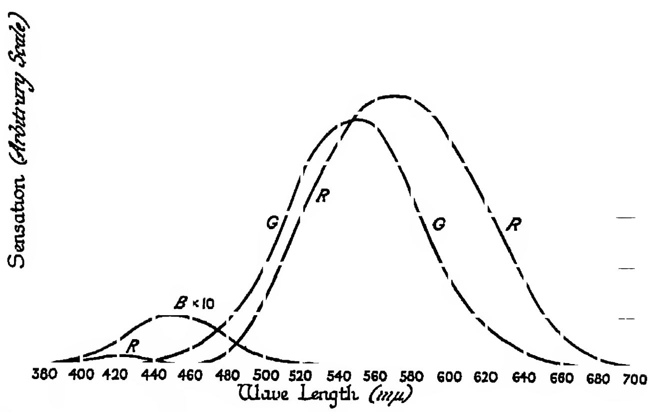


FIG 189 —The Sensation Curves for Light from a Black Body at 5000° K (*a*) Wave-number Basis. (*b*) Wave-length Basis

by the corresponding ordinate of the spectral distribution curve of the light considered This has been done for black-body radiation

necessarily be different according as they are calculated on the wave-number or the wave-length basis Both sets of curves are given in Fig. 189, and it will be seen that although the forms of the curves are different in the two cases, the relative areas remain the same, as, indeed, is evidently necessary *a priori*. The same considerations apply to the mixture curves (Fig 190)

at 5,000° K. in Fig. 189, where curves R, G and B show the respective contributions of the three primary sensations at each frequency

The areas of the three sensation curves of Fig. 189 are in the ratios

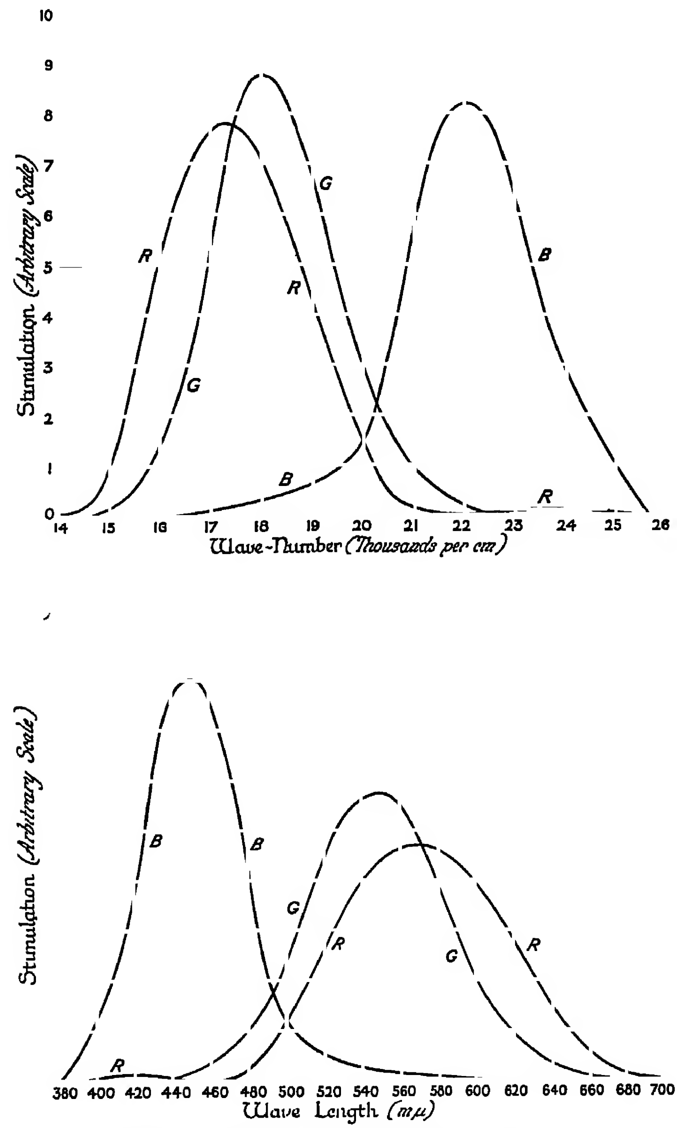


FIG 190 —The Mixture Curves (a) Wave-number Basis (b) Wave-length Basis.

$R . G : B = 0\ 568 \quad 0{\cdot}426 \quad 0{\cdot}006$ Now if it be agreed to take for the unit of stimulation of each sensation a magnitude such that an equal number of units of each of the three sensations will, when combined, give the sensation white (5,000° K), it follows that the luminosities of these units will be in the ratios 0 0103 0 0137 0·976. The curves of Fig 190 have been drawn with the ordinates multiplied

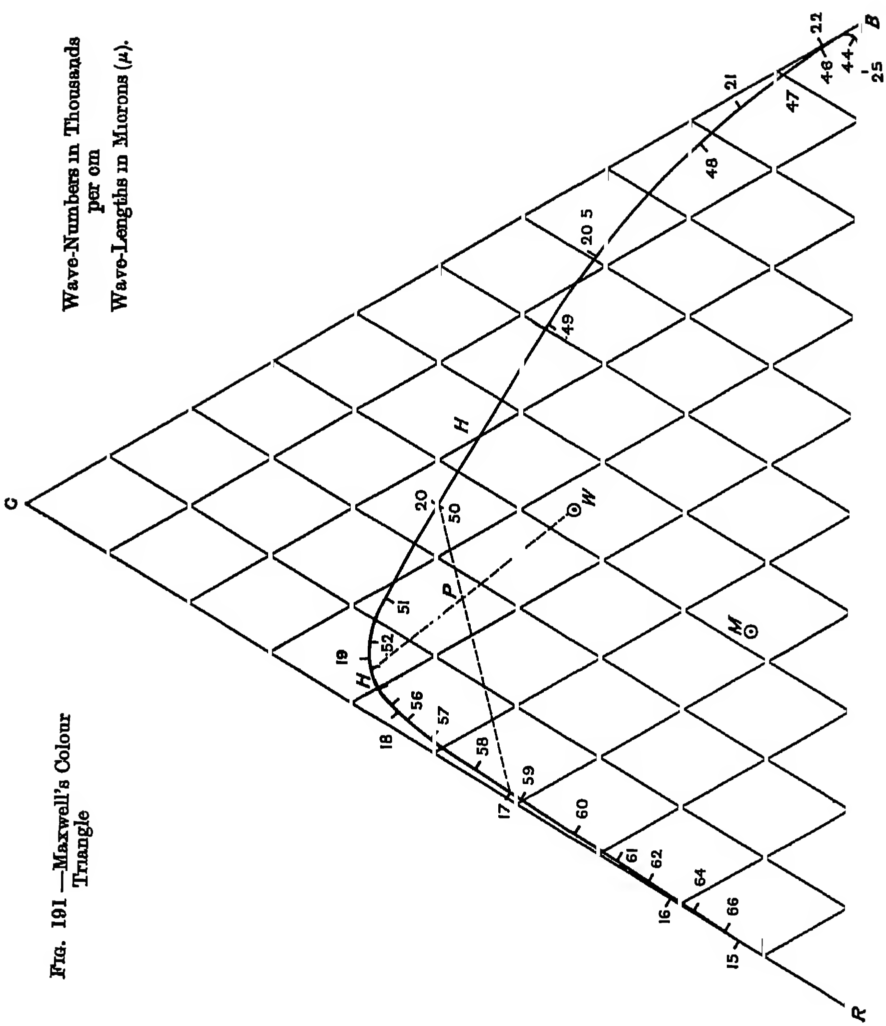


Fig. 191 —Maxwell's Colour Triangle

respectively by these numbers, so that the areas are equal. The ordinates of these curves at any given frequency show the relative stimulations of the three sensations for light of that frequency. These curves, termed the "mixture curves," may be used for obtaining the relative values of r, g and b for light of any known spectral distribution. The ordinates of each of the mixture curves are first multiplied by the corresponding ordinates of the spectral distribution curve, and the areas of the resulting three curves give the values required for r, g and b on the trichromatic system with the fundamental colour sensations as primaries and white as 5,000° K.

The colour of a light may be expressed graphically on the trichromatic basis by means of the Maxwell colour triangle shown in Fig. 191 (31) Every point P within the equilateral triangle RGB represents light whose colour is given by the expression $rR + gG + bB$, where r, g and b are the lengths of the perpendiculars from P to the sides of the triangle. It will be noticed that $r + g + b$ is the same for every point in the triangle, and that, therefore, every point represents a light of equal total stimulation. Further, the centre of the triangle W represents white, since for this point $r = g = b$ The curved line from R to B represents the light of a spectrum of constant stimulation, for it will be found that the trilinear co-ordinates of any point on this line are proportional to the ordinates at the corresponding point of the three curves of Fig. 188.

The colour of the light obtained by mixing two given lights in a definite proportion may readily be found from the colour triangle by joining the points representing the colours of the two components and dividing the line of junction in the ratio of the mixture. Thus the point P (Fig. 191) represents the colour of the light obtained by mixing in the ratio of 2 to 1 the spectrum lights having frequencies 20,000 and 17,000 respectively (32) The line PW cuts the spectrum line in the point 18,850. It follows that the compound light P may be matched by mixing white light with spectrum light of frequency 18,850 in the ratio 1 to 1·47 (33). Wherever the line passing through W cuts the spectrum line in two points, those points represent lights of complementary colours, for if mixed in the right proportion these lights will give sensation white (34). It will be noticed that lights in the region 20,300 to 17,500 have no complementaries in the spectrum. The complementaries of these, the yellow-greens, are mixtures of red and blue in various proportions, *i e.*, purples and magentas, represented by points such as M These conclusions will be referred to again when the monochromatic system of colour specification is considered.

In the case of a composite light, *i e.*, one composed of a number of different homogeneous radiations, it is quite simple, from a knowledge of the spectral distribution, to express the colour in terms of the three sensation primaries.

The method may be described most clearly by reference to Fig 192, where curve E represents the energy distribution of the given light (that from a black body at 2,220° K) Curve R is obtained by multiplying each ordinate of curve E by a factor proportional to the corresponding ordinate of curve R in Fig. 190 *a* (p. 301). Curves G and B are obtained similarly. The coefficients r, g and b of the given lights are then proportional to the respective

areas of these three curves, i.e., to the relative stimulations of the three primary sensations.

The Trichromatic Colorimeter.—Several instruments have been devised for finding more directly the values, absolute or relative,

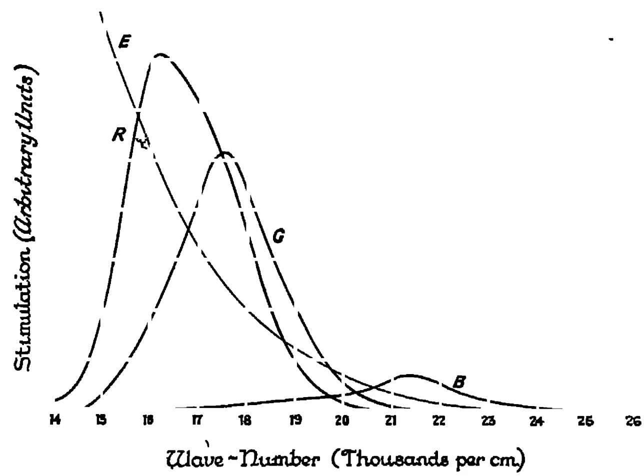


FIG. 192.—Curves of the Colour Sensations for a Black Body at 2220° K. (See Fig 168.)

of the three coefficients in the expression for the colour of any given light by enabling a match to be made between the light in question and a mixture, in measurable proportions, of three arbitrary red, green and blue lights, which may be termed the instrument primaries.

The Ives colorimeter ([35]) is shown in plan in Fig 193. The light to be examined is admitted at L, and forms one-half of the field F which is viewed by the eyepiece E. The light from the comparison source is admitted at K and passes through three adjustable slits, which are respectively covered with red, green and blue media. The light passing through these slits is mixed by persistence of vision, a number of lenses arranged in a circle A being rotated by means of a small motor so that the other half of the field at F is occupied by light from the three slits in rapid succession. The substitution method is employed in making measurements "White" light is first matched, and the scales on the slits are each adjusted to read 100. The light to be measured is then substituted for the white light and the slits are altered until a match is obtained The scale readings at the slits then give the colour of the measured light in terms of the instrument primaries, i e, the coefficients in the trichromatic expression for the colour of the light in question.

Other instruments, similar in principle, have been devised ([36]). That used at the National Physical Laboratory is shown in Fig. 194 ([37]) A 500 c p. "pointolite" lamp is placed at the focus of a large condensing lens A, which throws a beam of light normally on one end of the colour-mixing apparatus. This end is shown on the right of the plan. In it are cut three sectorial openings (shown shaded), each of about 60° extent, which are backed by sheets of ground glass and coloured gelatine filters. These latter are respec-

tively red, green and blue, the particular filters chosen being selected from a complete set of the available gelatine filters. Inside the box a prism CD of shape shown is mounted in such a way that it is capable of rapid rotation about an axis DE. During rotation the end C passes each of the sectorial openings in turn. When C is opposite an opening, light enters the prism and, after two internal reflections at the inclined faces of the prism, emerges along DE. A lens E is placed so that the effective stop (a circular hole in the mount of the prism CD) is at its focus. F is a Lummer-Brodhun photometric prism giving a triple rectangular field. G is a lens with its focus at H. An observer with his eye at H sees the reflecting section of the photometer field illuminated by light from the particular sector which the end of the rotating prism is opposite. Thus the colours red, green and blue alternate in the field as the prism rotates. If the speed is sufficiently rapid the sensations mingle and a mixed colour is observed. Each sector opening is fitted with a sectorial shutter, not shown in the diagram, by which the relative durations of the red, green and blue stimuli may be varied, thus varying the proportions of these colours in the mixture. The colour to be matched is situated outside the aperture I. Light from this aperture fills the transmitting portion of the photometer field. J is a plane glass plate, its purpose, in conjunction with the lens K and prisms L and M, is to make it possible to add a little white light to the coloured light to be matched. This is sometimes necessary when matching very saturated colours in certain regions of the spectrum. The amount of this white light is regulated by a lamp-black gelatine annulus N, of varying density. With this addition it becomes possible to specify any colour whatever in terms of the instrument primaries.

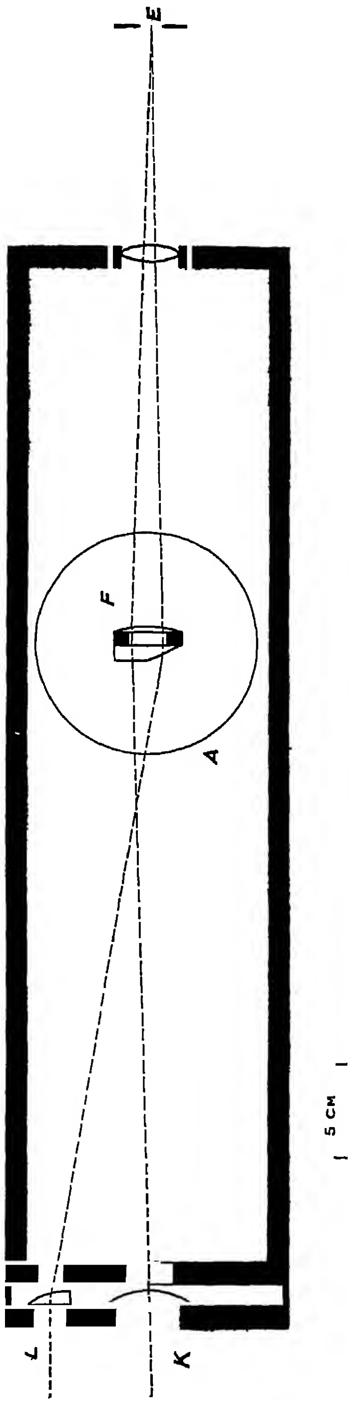


FIG 193 —The Ives Colorimeter

As pointed out above, the values of the colour coefficients obtained with an instrument of this type necessarily depend on the exact colours of the lights used to form the mixture This, however, is of little importance, since it is in any case impossible to obtain a pure green, and a reduction is therefore always required if the results are to be expressed in terms of the sensations as primaries. The easiest method of performing this reduction is to mark on the fundamental colour sensation triangle the three points which represent the primaries used in the colorimeter (see Fig. 195) and then to obtain the position of the light under examination by a simple graphical construction. If ρ, γ, and β be the measured coefficients, the side $R'G'$ of the instrument triangle is divided at L in the ratio $\gamma . \rho$. LB' is then divided in the ratio β $(\rho + \gamma)$, and the point of division P is the point representing the measured light, so that the required reduced colour coefficients can be at once obtained It is to be noticed that if the instrument primaries be so chosen that $R' + G' + B'$ represents white, the centroid of the instrument triangle coincides with W, the centre of the fundamental triangle.

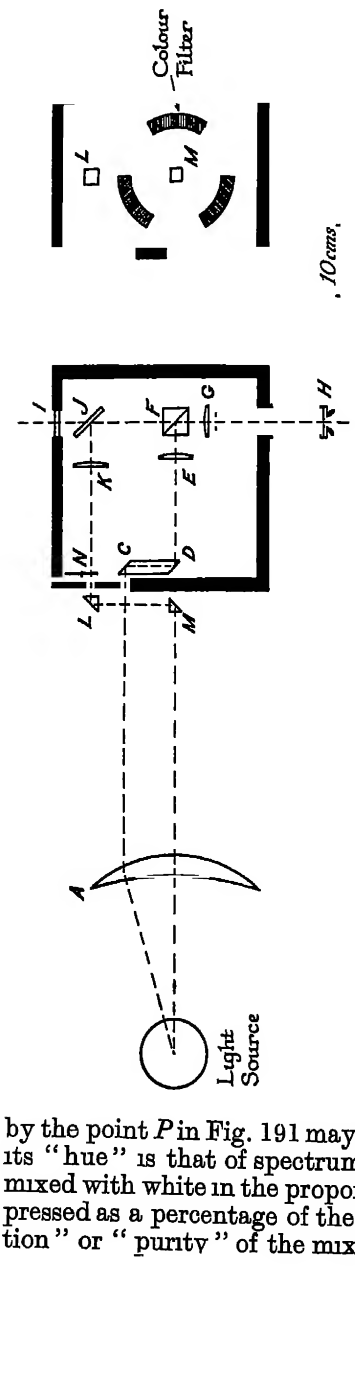


Fig 194 —The N P L Trichromatic Colorimeter

The Monochromatic System.—As has been said already, every colour, except certain mixtures of blue and red, can be matched by a correctly proportioned mixture of white light with light of a suitably chosen spectrum colour ([38]). For it is clear that any point on the Maxwell triangle, unless it be situated in the region approximately represented by RWB, lies on a line joining the point W to some point on the spectral line Hence the colour of a light may be defined by (*a*) the frequency and (*b*) the percentage amount of the homogeneous spectrum light which, if mixed with white, will match it in colour ([39]) For instance, the colour represented by the point P in Fig. 191 may also be described by the statement that its "hue" is that of spectrum light of frequency 18,850, and that it is mixed with white in the proportion of PW . HP. This proportion, expressed as a percentage of the spectrum light, is termed the "saturation" or "purity" of the mixed light ([40]). Similarly, a purple light,

represented by a point such as M, may be expressed on the monochromatic system by the hue of its complementary, H', and the amount of this complementary which has to be subtracted from white in order to produce a match.

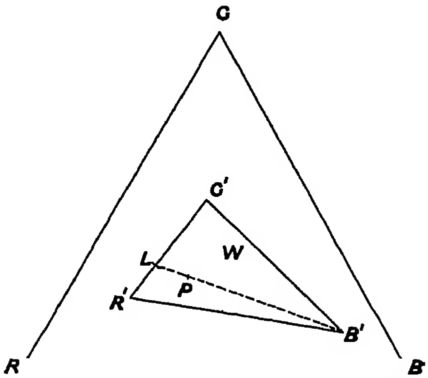


FIG 195 —The Transformation from Instrument Primaries to Fundamental Primaries on the Trichromatic System

The Monochromatic Colorimeter.—The instrument shown in Fig. 196 has been designed for the specification of colour on the monochromatic basis ([41]). The spectral component is supplied by the source N and the constant deviation spectroscope system SPE. The white component is supplied from an illuminated white surface R by reflection at the sectored mirror M, which is rotated by an electric motor The proportions of the mixture might be changed by having adjustable openings in M, but in practice it is more convenient to keep these openings constant and to vary the intensities independently by means of the Nicol prism pairs shown at N_1, N_2 The light used to illuminate R must be that adopted as white if the instrument is to read directly. The light to be measured is reflected from a second white surface R', and the colour of this light is compared by means of the Lummer-Brodhun cube L with the colour of the mixture reflected and transmitted by M. When a purple light has to be analysed by this instrument, it is placed so as to illuminate R, while R' is then illuminated by white light The results of spectrophotometric analysis may be expressed on the monochromatic system either by the intermediary of the Maxwell triangle or directly by calculation

Conversion from One System to Another.—It will be noticed that the instrument just described measures the relative amounts of the spectral and white components of the light in terms of *luminosity*. This makes conversion to the trichromatic system difficult, because the magnitudes of the units employed for the sensations on that system are such that an equal number of units of each primary will, when combined, give the sensation of white, and the units so arrived at have very different luminosity values. In fact, their luminosities are in proportion to the areas of the three curves of Fig. 189, L_R L_G L_B say It follows that if a light of dominant hue H and saturation p excite the three fundamental colour sensations in the ratio $\rho : \gamma : \beta$, and if the spectrum light H excite the sensations in the ratio $r : g : b$, then the equation for the red sensation is

$$100\rho L_R/\Sigma\rho L_R = prL_R/\Sigma rL_R + (100 - p)L_R \text{ *}$$

* For light exciting the sensations in the ratio $\rho : \gamma : \beta$ has a luminosity $\kappa(\rho L_R + \gamma L_G + \beta L_B)$, where κ is a constant which, for unit luminosity, has the value $(\Sigma\rho L_R)^{-1}$ Hence, light of this colour having unit luminosity excites $\rho L_R/\Sigma\rho L_R$ units of red sensation Similarly for the light $r : g : b$, and for white (where $r = g = b$ and therefore $r/\Sigma rL_R = 1$).

Whence it follows that

$$100\rho/\Sigma\rho L_R = pr/\Sigma r L_R + (100 - p) \quad . \quad . \quad . \quad . \quad . \qquad \text{(i)}$$

or

$$\rho \,.\, \gamma : \beta = (pr\sigma + 100 - p) : (pg\sigma + 100 - p) : (pb\sigma + 100 - p)$$

where $\sigma = (rL_R + gL_G + bL_B)^{-1}$.

The reduction of a colour analysis expressed on the trichromatic

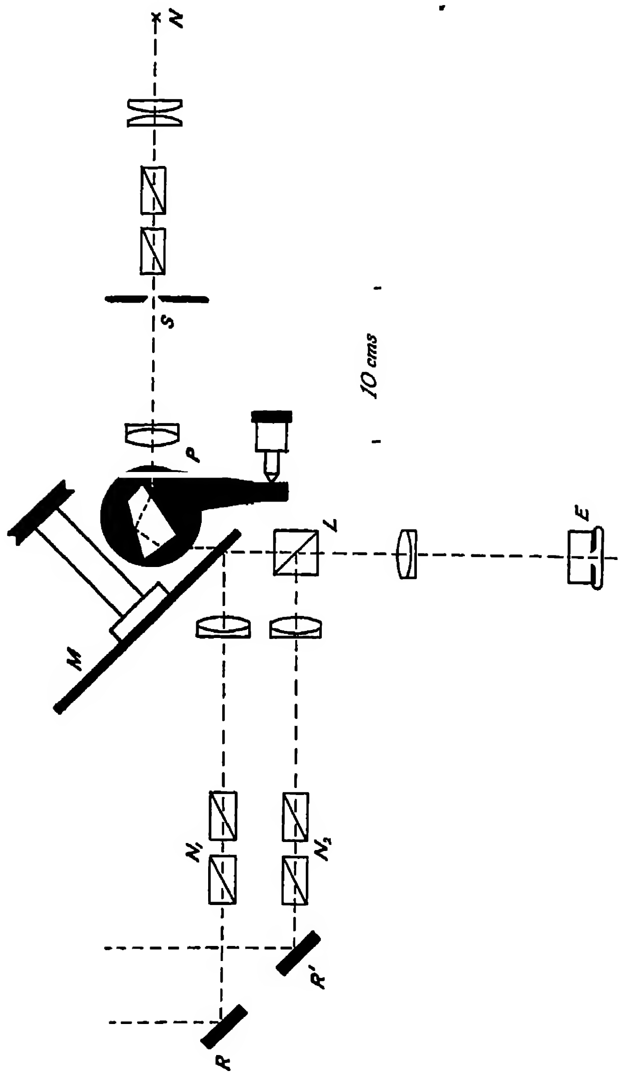


FIG. 196.—The Nutting Monochromatic Colorimeter

system to its equivalent on the monochromatic system is most easily performed by means of the Maxwell triangle [42]. The line from W which passes through the point P representing the light under consideration, gives at once, by its intersection with the spectrum line, the dominant hue. Thus r, g and b are known. Since ρ, γ and

β are also known, any one of the sensation equations, such as (i.) above, will give p.

When the light considered is purple, the equation corresponding to (i.) is

$$100 - pr'/\Sigma r'L_R = (100 - p)\rho/\Sigma\rho L_R \quad . \quad . \quad \text{(ii.)}$$

where r' is now the red excitation of the spectrum light complementary to the purple considered The values of σ for lights at intervals along the spectrum are given in the following table :—

TABLE of Values of $(rL_R + gL_G + bL_B)^{-1}$ at different Parts of the Spectrum

$\nu \times 10^{-3}$	σ	λ (mμ)	σ
14	1·82	400	27
15	1 82	420	36
16	1·87	440	78
17	1 98	460	42·5
18	2·08	480	9 53
19	2·18	500	2 82
20	2·82	520	2 24
21	15·5	540	2·12
22	52·5	560	2 07
23	58	580	2·01
24	34	600	1·93
25	27	620	1·88
—	—	640	1 85
—	—	660	1·83
—	—	700	1 82

Tables and diagrams enabling the excitation values to be found directly for any light expressed on the monochromatic scale have also been calculated (43).

In all that has been said above it has been assumed that the spectral composition of a light uniquely determines its colour, so that the relative excitation values of a light, as well as its dominant hue and saturation, remain invariable at all intensities. That this cannot be strictly true must follow from the alteration in the shape of the luminosity curve which takes place at low intensities (see p. 65) Further, it has been found that increasing the intensity of a light tends to decrease its saturation (44). It is impossible here to discuss these problems, which necessarily complicate colorimetric measurement unless they can be avoided by choosing conditions for the measurements at which these effects are absent or unimportant (45). For a fuller consideration of the whole subject one or more of the books referred to in the bibliography at the end of this chapter should be consulted.

Colour Charts.—Other methods of colour specification have been used for various purposes. The colour temperature scale has already

been described (see p. 270), and the leucoscope (see p. 244) can clearly be adapted to give a colorimetric scale ([46]).

Comparison with a set of coloured media, or pigmented papers, of definite, and as far as possible reproducible, tints, arranged so as to form a rough step-by-step scale, is sometimes useful ([47]) The number of standards required to cover the whole range of luminosity, hue and saturation is, however, very great (over 3,000), and the most useful field for this system would appear to be one in which the range to be covered is severely restricted by the nature of the problem, as, for example, in the grading of a particular class of oil.

Conclusions.—It should be remarked that, of the three methods of measuring the colour of a light, both the monochromatic and the trichromatic colorimeter depend on the conformity of the observer's sight to the average known as "normal colour vision," since the colour match obtained is a *sensation* equivalence and not a true physical identity. The only measurement giving full information as regards the colour of a light, and more especially its behaviour when transmitted or reflected by colour media, is the determination of the spectral distribution curve ([48])

The labour involved in the full determination can, in many cases, be avoided by making a comparatively small number of observations, each covering a patch of the spectrum which is just narrow enough to avoid the detection of any significant colour difference by the eye of the observer ([49])

By choosing these patches so that the breadth of each is inversely proportional to the chromatic sensitivity (or hue discrimination) of the eye at the part of the spectrum concerned (see p 66 and Fig 37), it is possible to divide the whole visible spectrum into about eighteen adjacent parts, and therefore to obtain a very complete specification of the colour of any particular light by means of eighteen measurements on the spectrophotometer, the patch of spectrum included at each measurement having the limits given in the following table :—

Patch Number —	1	2	3	4	5	6	7	8	9	10
Frequency. $(\times 10^{-8})$	14·9	15·4	15·9	16·4	16·6	16·8	17·1	17·3	17·7	18·2
Wave-Length. (mμ)	670	649	628	611	601	594	586	577	564	550

Patch Number —	10	11	12	13	14	15	16	17	18
Frequency. $(\times 10^{-8})$	18·8	19·3	19·7	20·0	20·4	20·9	21·5	22·3	23·3
Wave-Length (mμ)	531	517	507	499	491	479	466	449	430

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(40) On the sensitivity of the eye to change of saturation, see Chap III, p 67
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The instrument shown in Fig 196 is the modified type made by Adam Hilger, Ltd
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(45) Conditions which have been suggested are (*a*) an intensity of at least 100 photons (see p 52) and (*b*) a field size of 3 degrees See L T Troland, Opt Soc Am, J, 6, *1922*, p 527
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E Brodhun. Wied Ann, 34, *1888*, p 897
L Arons Ann d Phys, 33, *1910*, p 799, and 39, *1912*, p 545; E T Z, 32, *1911*, pp 729 and 738
I G Priest Phys Rev, 14, *1919*, p 264
(47) J W Lovibond. "The Measurement of Light and Colour Sensations" (Gill, London, *1893*) Brit Patents Nos 12,867 (*1886*), 3,859 (*1887*), and 14,926 (*1910*) Chem News, 65, *1892*, p 250, Nature, 46, *1892*, p 93
Société Française des Chrysanthémistes, "Répertoire de Couleurs," *1905*. [1,450 colours]
R. Ridgway. "Color Standards and Color Nomenclature" (Washington, *1912*) [1,115 colours]
A H Munsell "A Color Notation" (Boston, *1916*), and "Atlas of the Munsell Color System" (Boston, *1915*)
I G Priest, K S Gibson and H. J. McNicholas Bureau of Standards, Technol Paper No 167, *1920* [The Munsell System].
J Lawrance Oil and Colour Chem Assn, J, 2, *1919*, No 6, p 1
F W Ostwald "Die Farbenfibel" (Leipzig, *1917*), "Farbenatlas" (Leipzig, *1918*) [2,500 colours] See also Z phys Chem, 91, *1916*, p 129, and Phys Z, 17, *1916*, pp 322 and 352, 22, *1921*, pp 88 and 125, Z f techn Phys, 1, *1920*, pp 173 and 261, and 2, *1921*, p 145, "Die Grundlage d. messenden Farbenlehre" (Barth, Leipzig, *1921*)
I G Priest Cotton Oil Press, 4, *1921*, p 43
A Ames Opt Soc Am, J, 5, *1921*, p 160
W Seitz Phys Z, 23, *1922*, p 297 [On Ostwald's system.]
A P Harrison. Science, 57, *1923*, p 716
(48) I G Priest has found (Opt Soc Am, J, 4, *1920*, p 388) that if any two lights, however different in spectral distribution, excite colours of the same quality (hue and saturation) the frequencies of the centres of gravity of their spectral distribution curves are coincident
For the computation of dominant wave-length and purity from spectral distribution, see I G Priest, K S Gibson and A E O Munsell, Opt Soc Am, J, 8, *1924*, p 28
Slide rules to assist in these computations are made by Messrs Keuffel and Esser, Hoboken, N J (U S A)
(49) H E Ives Opt Soc Am, J, 5, *1921*, p 469.

CHAPTER XI

PHYSICAL PHOTOMETRY

The Psychological Aspect of Visual Photometry.—In all the practical methods of photometry so far described the eye judges the equality or inequality of brightness of two surfaces which are presented to it in such a manner that they form adjacent images on the retina It has been pointed out already (p. 146) that vision involves a chain of physical, physiological and psychological processes, and it is important that in any study of the subject too much attention should not be devoted to the physical process without corresponding regard to the physiological and psychological phases which so strongly influence the final visual perception resulting from any given physical stimulus produced by irradiation, *i e*, the incidence of radiant energy, at the surface of the retina

The threshold between the physical and physiological regions is fairly clearly marked out, but at present it is almost impossible to separate the physiological and psychological regions, so that the phenomena described in Chapter III. can only be regarded as the effects of a combined action involving three processes, of which only one, the physical stimulus, is accurately known and under control. It follows that the results must be liable to considerable variation according to (*a*) the particular individual making the observations, (*b*) factors quite unconnected with the physical stimulus, which may affect either the physiological or (*c*) the psychological characteristics of that individual at the time of observation For example, it is well known that (*a*) different individuals behave differently as regards their relative evaluation of different coloured lights. Further, (*b*) and (*c*), the same individual may vary markedly from day to day in his relative evaluation, either from definitely physiological causes (*e.g.*, excessive tobacco smoking), from purely psychological causes (such as unconscious change of mental criterion of equality), or from a physiologico-psychological action, such as bodily or ocular fatigue

Similar effects undoubtedly take place to a greater or less extent in the more ordinary processes of photometry Thus in homochromatic photometry it is the perception of brightness contrast that is important. The experience of all those who have done much observational work in photometry is that this perception is impaired by anything which tends to diminish the power of concentrating the attention Thus unfamiliarity with the instrument used, or discomfort while observing due to a cramped attitude or to excessive heat or cold, and other circumstances of a similar nature, tend to diminish the accuracy of photometric observations

It has been pointed out already (see p 57) that the glare which results from looking at an object brighter than the photometric field, even for a short period, will vitiate readings for a length of time dependent on the brightness of the object and the time during which

it has been looked at (1). Similarly it was stated in Chapter VI. (p. 148) that every photometer should be so designed that the point of photometric balance can be passed through from one side to the other as rapidly as possible and with the smallest amount of manual effort (2). This is of great importance if the best results are to be obtained The unconscious mental bias which may result if an observer becomes aware of any progressive tendency in his readings is avoided in most laboratories by arranging that the observers shall work in pairs, each one noting down the readings obtained by the other. In accurate work not more than about a dozen settings should be made by a single observer without at least a brief period of rest. A very definite "settling-in" effect is often noticeable in photometry. At the beginning of a day's work the first few readings made by an observer may differ noticeably and in a definite direction from his subsequent observations, owing, apparently, to a preliminary uncertainty of criterion which is not resolved until after the first half-dozen or more observations have been made. This effect is naturally more pronounced when the colour difference worked with is considerable. Some observers, however, find that with an equality of brightness photometer a small colour difference is more troublesome than one which is large enough to be immediately apparent The same is sometimes said when working with the contrast type of head, but in that case the difficulty is generally found to arise from the fact that the observer has, consciously or unconsciously, used as his criterion of balance the disappearance of the central dividing line instead of the equality of contrast between patch and background in the two halves of the field , in fact, the head has been used as an equality of brightness instead of as a contrast head.

Physical Photometers.—It is due in large measure to the uncertain factors involved in visual photometry, such as those which have been mentioned above, that workers in this branch of radiometry have for many years sought for some physical instrument which may be used instead of the eye for the measurement of light, and which is not subject to the limitations inseparable from the employment of any physiological organ of special sense. In every case the instrument used for the comparison gives, in some form or another, a measure of the energy received by a certain surface exposed to the light to be measured (3). Physical photometers, therefore, depend on the comparison or measurement of *illumination*, regarded as a rate of energy reception.

The Classification of Physical Photometers.—It was pointed out at the beginning of the chapter on Visual Photometry (Chapter VI) that the eye is quite unreliable as an instrument for *measuring* brightness, although it can *compare* the brightnesses of two adjacent surfaces with considerable precision. It is the ultimate aim of research in methods of physical photometry to produce an instrument which, while giving exactly the same result as visual methods for the relative values of different illuminations, will at the same time serve as a means for the direct *measurement* of any illumination without simultaneous comparison with a standard illumination This aim is still far from realisation with certainty and to the necessary degree of sensitivity except by means of delicate laboratory apparatus of high precision, used with great care and with a number of important

precautions. On the other hand, a sensitive *comparison* of illuminations produced by light of the same spectral distribution can be achieved conveniently by means of fairly simple apparatus, and the various methods that have been used with promising results will be described here.

Physical photometers may be divided into two classes in either of two ways (i.) according as they are suitable for *measuring*, or only for comparing illumination; (ii.) according as they are suitable for *heterochromatic* or only for strictly *homochromatic* photometry. In practice these two methods lead to an identical classification of the existing instruments

Absolute Physical Photometers.—In the case of an instrument which is to be used for measuring illumination the following requirements should be fulfilled, the first two absolutely, and the others as far as possible .—

(*a*) The indication must be proportional to the illumination, or bear some definite relation to it which can be determined by calibration.

(*b*) The calibration curve must remain constant during a reasonable period of time, particularly if the law connecting indication with illumination be not a simple one.

(*c*) The sensitivity should be at least as great as that attainable by visual methods.

(*d*) The instrument should be capable of use under ordinary laboratory conditions, and without the need for special precautions involving complicated apparatus or highly skilled operation

(*e*) The instrument should be one capable of giving a continuous record if required.

As regards (*d*) it has to be remembered that most instruments first devised in the laboratory are made fit for general use only after a more or less gradual development along the lines of simplification and robustness of construction, while it is the possibility expressed in (*e*) that makes the subject of physical photometry so attractive in connection with many problems met with in the measurement of modern light sources

In addition to the above five requirements, an instrument which is to be used in the measurement of any but strictly homochromatic sources, *i.e.*, sources giving light of exactly the same spectral distribution as that with which it is calibrated, must, as a first essential, respond to light of different frequencies in exactly the same way as the eye [4], in other words, the curve connecting indication and frequency must be identical with the sensitivity curve of the normal eye, as shown in Fig 186 (p 295). A first necessity, therefore, in physical photometry is the accurate determination of the sensitivity curve for as many observers as possible, working under normal conditions as to brightness, field size, etc. [5]. This determination has been carried out with considerable care, and the results arrived at are given in Appendix IV (p. 471).

Detectors for Heterochromatic Comparison.—The sensitivity curve of the eye having been determined, it remains to find some detector whose response to radiant energy of different frequencies will follow either the same curve or some other well-defined curve which is capable of reduction to conformity with the sensitivity curve by

some selective transmitting medium, such as a coloured solution. As might be expected, the latter of these alternatives is the only one practically worth consideration.

This requirement at once rules out the most sensitive detectors, *viz.*, selenium and similar light-sensitive materials, the photo-electric cell, and the photographic plate, for these detectors not only show a response to light which is very different from that of the eye, but they are also liable to alter their "sensitivity curve" either with the intensity of illumination or with small variations in construction which at present are quite uncontrollable They will be considered later in the class of instruments best suited to the comparison of homochromatic sources

There remain, then, the detectors of radiant energy which depend simply on the measurement of the total amount of energy reaching them, and which are therefore quite non-selective, so that when used with a medium having its spectral transmission curve identical in form with the sensitivity curve they give at once an exact reproduction of the normal eye. While theoretically perfect, the chief practical difficulty attending the use of this method is the exceedingly small portion of the total energy given by most light sources which lies within the limits of the visible spectrum As stated above, the mechanical equivalent of light is approximately 0·0016 watt per lumen, so that a lamp operating at an efficiency of 1 candle per watt has a luminous efficiency of only 2 per cent Much of the remaining 98 per cent is radiated in frequencies lying outside the visible spectrum, mainly in the infra-red, and this introduces the further difficulty that the detector has to be very carefully shielded from invisible radiation, to which it naturally responds as readily as to that within the visible spectrum.

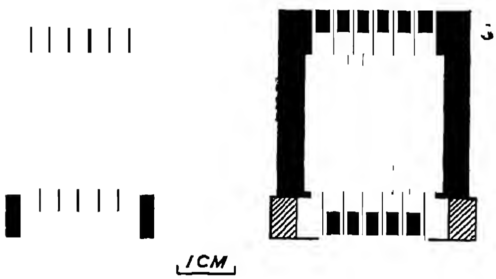

FIG 197—The Bolometer

The detectors most suitable for the measurement of the very low intensities of radiation met with in ordinary photometry are all electrical (6) The bolometer depends on the fact that when radiation is absorbed by a long and very thin strip of blackened platinum (see Fig. 197) its electrical resistance is altered owing to the rise in temperature which takes place (7) If, therefore, four bolometers be arranged in a Wheatstone bridge, as shown in Fig. 198, a very sensitive means for measuring the difference in the radiation

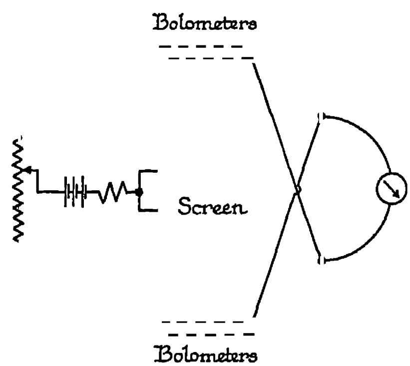

FIG 198—The Measurement of Radiation by the Bolometer

reaching the two sides of the apparatus is obtained For use in spectral distribution measurements a single narrow strip is employed.

The thermopile and the radiometer both depend on the fact that when a circuit is composed of two dissimilar metals a difference of temperature between the two junctions of these metals in the circuit results in a flow of electricity round the circuit. For example, if junction A (Fig 199) be at the temperature t_1, while junction B is at temperature t_2, an electromotive force of $\alpha\,(t_1 - t_2)$ volts is generated in the circuit, so that if a galvanometer be included, and the total resistance of the circuit (including the galvanometer) be R ohms, a current of $\alpha(t_1 - t_2)/R$ ampères will flow round the circuit For the metals tin and bismuth α, the thermo-electric power, is about 7×10^{-5}, so that if $t_1 - t_2 = 10°$ C. and $R = 10$ ohms, the current will be 0 07 milliampère, which is easily measurable with a sensitive milliammeter In Boys' radiomicrometer, shown in Fig 200 ([8]), the thermo-junction T is connected to the small loop of wire which forms the circuit and is suspended by means of a fine quartz fibre between the poles of a magnet N, S, so that when radiation is absorbed by T the difference in temperature produced causes a current in the loop, and therefore a deflection due to the magnet This deflection is measured by means of the small mirror m, which reflects a beam of light as in the case of an ordinary galvanometer. Deflections of the order of 10 mm. per metre for 1 sq. mm. of surface exposed to 1 metre-candle may be obtained. In the radiomicrometer only one junction is used, but the thermo-electric voltage may be increased by employing a number of junctions in series. A form of thermopile used for spectral distribution measurements is shown in Fig 201. It is of linear form and contains ten metal squares, often of tin, to each of which is soldered a pair of thermo-junctions, which may be of bismuth-silver The breadth of the exposed surface is reduced as much as possible in order that it may occupy only a narrow strip of the spectrum when used in the eyepiece of a spectrometer. It is generally enclosed in a cell of the form shown on the right of the figure. The front surface of the thermopile is covered with a matt

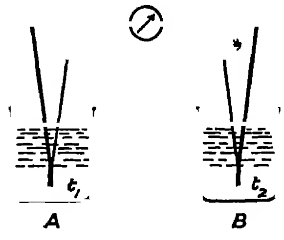

FIG 199 —The Thermo-electric Circuit

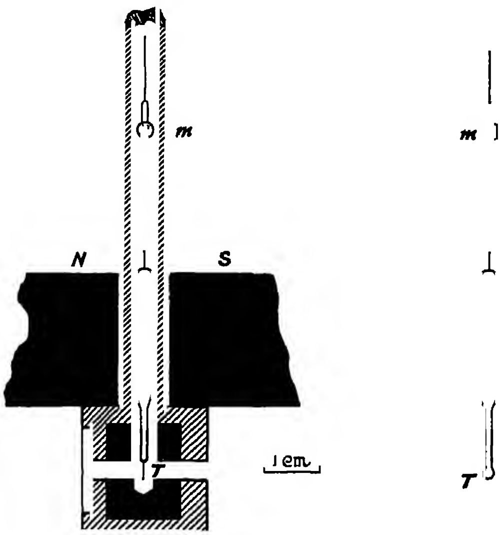

FIG 200 —Boys' Radiomicrometer

black coating of soot or platinum black to obtain as complete a absorption as possible of the incident radiation, and the heat capacit

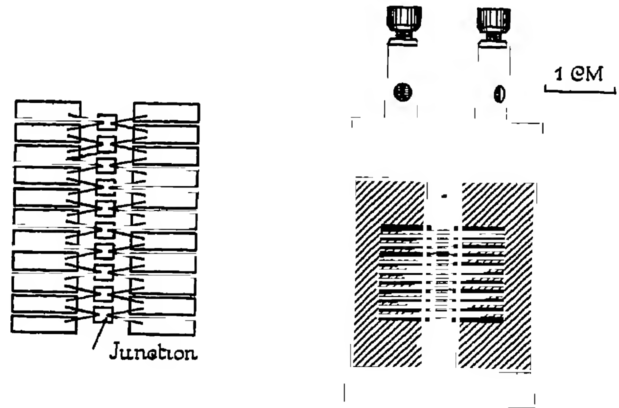

FIG 201.—The Linear Thermopile for the Study of Spectral Energy Distribution.

is a minimum, so that a given amount of energy produces th greatest possible rise of temperature ([9]).

The Thermopile and Visibility Solution ([10]).—While all the abov instruments may be used for measuring the energy distribution o the radiation given by a source, they are not immediately suitabl

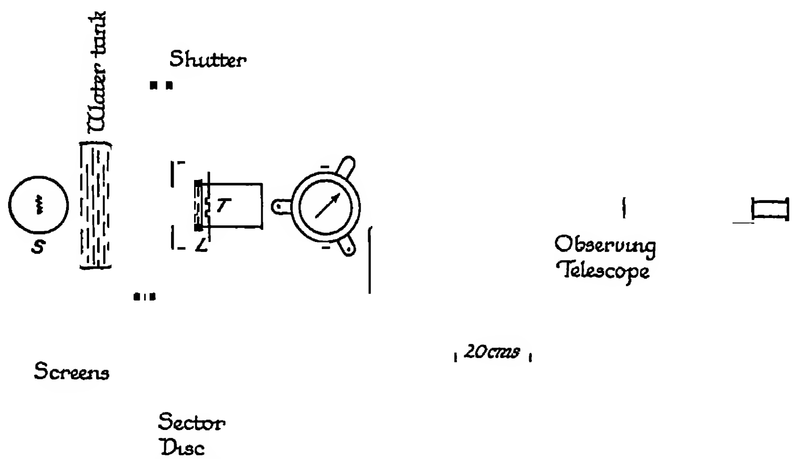

FIG 202.—The Thermopile and Visibility Solution for Physical Photometry

for use as physical photometers to give an absolute measurement o candle-power ([11]). For such a purpose a practical arrangement which has been used by Ives and Kingsbury ([12]), is that shown in Fig. 202, where the radiation from the source S passes through a medium L, such as the chemical solution described on p 296 above which weights each frequency to the same extent as the normal eye so that the energy reaching the thermopile T is directly proportional to the visual effect, *i.e.*, to the luminous intensity of the source.

The chief difficulties attending this method of physical photometry are common to all measurements necessitating the evaluation of very small electric currents, viz., the susceptibility of the very sensitive galvanometer to mechanical disturbance, drift of zero, and lack of exact proportionality between current and deflection. There is also the effect of variation in room temperature, which causes uncertainty in the zero of the thermopile-galvanometer system. This effect may be practically eliminated, if serious erratic fluctuations of room temperature be avoided, by making two zero readings at equal periods of time before and after the deflection reading. Ives and Kingsbury found that, under the conditions of experiment above described, an exposure of thirty seconds was sufficient for a reading (13).

The Thermo-junction and Visibility Template.—An alternative method which has been proposed (14) is the use of a luminosity template, which is placed in the path of the light dispersed by a prism, so that the amount of radiation of each frequency which is allowed to pass bears a constant ratio to the luminosity (K_ν) at that frequency, and the energy of the recombined beam is therefore exactly proportional to its luminosity. The apparatus used may take several forms. That involving the use of a rotating sector is shown in Fig. 203, where L is the light source, S the slit, and P the

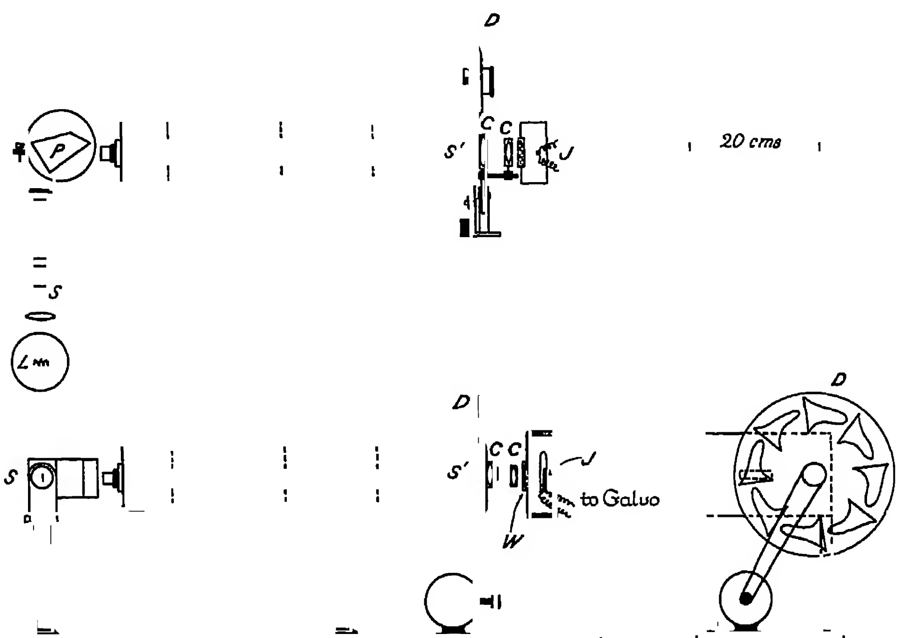

FIG. 203.—The Thermopile and Visibility Template.

prism of a spectrometer producing a spectrum at S'. Over this spectrum passes a rotating sector disc D, the apertures in which are of such a form that the exposure of each part of the spectrum is in proportion to the luminosity value of that part. Close to the tem-

plate is a lens system C, C, the function of which is to recombine the spectrum, forming an image of the prism face in the colour of the recombined light upon the thermo-junction J. To shield this junction from infra-red radiation it is placed inside a protective case, the opening of which is covered by a glass cell W containing water. Even with a water cell in place, some residual infra-red radiation causes trouble, and a 3 per cent solution of copper chloride is preferable, due allowance being made for its selective absorption in the visible spectrum by an appropriate modification of the curve used in the template. In the work actually described by Ives ([15]) an iron-clad Thomson galvanometer of 5 1 ohms resistance and a sensibility of from 2 to 5×10^{-10} amps. per mm. was used Trouble due to mechanical vibration was overcome by the use of a Julius suspension ([16]) The thermo-couple was of bismuth-tin and bismuth-antimony alloys *in vacuo*, as this form has been found to have five times the sensitivity of the ordinary bismuth and silver couple ([17]) The 45-candle carbon lamp used in the experiments on the solution method gave deflections of about 20 cm under these conditions.

The same apparatus may be used with a fixed template in front of S', care being taken to ensure that the height of the spectral band is sufficient to cover the template opening completely The form of the template is then exactly that of the curve of Fig 186, allowance being made for the effect of prismatic dispersion and for the selective absorption of the copper chloride, if this solution be used in place of water at W.

Selective Receptors.—The chief disadvantage attached to both the above methods of physical photometry is the smallness of the currents available and the consequent necessity for using galvanometers of very high sensitivity. In the methods now to be described the sensitivity is greater, but unfortunately it varies with the frequency of the radiation according to some complicated relation which is, in many cases, variable by causes which are at present but little understood or under control

Selenium and Photo-Sensitive Substances.—One of the first substances suggested for use as an approximate physical photometer was selenium. It had long been known that this substance in the crystallised form was sensitive to light, in that its electrical resistance changed enormously when exposed to radiation in the visible spectrum ([18]). The substance is most conveniently used in the form of a selenium cell, better termed a " selenium bridge " ([19]) The chief object to be attained in the design of such a cell is maximum sensitivity, constancy and quickness of response to radiation Most bridges are now so designed that a very long and narrow strip of selenium is obtained between two metallic electrodes Many such designs have been proposed. One form of bridge, originally due to S. Bidwell ([20]), may be made by taking a small sheet of ground glass, mica or slate, and spreading this with a very thin layer of purified amorphous selenium by means of a hot glass rod. If now four strands of fine bare nickel or platinum wire are wound round the plate so that the whole of the selenium surface is covered, and two alternate strands are then removed, the other two strands are left separated for the whole of their length by a space equal to the diameter of a wire. To make the bridge sensitive to light it is heated

in an oven to a temperature of 180° C for about five minutes, when the transformation from amorphous to metallic selenium should be complete. The dark resistance of such a bridge, formed on a plate $1 \times 3 \times 0.1$ cm has been found by Pfund ([21]) to be of the order of 2×10^7 ohms, and the sensitivity such that the resistance is decreased to 10 per cent. of this value by an illumination of 150 to 200 metre-candles The bridges should be protected from moisture by being placed in a vacuum vessel or waxed to a sheet of glass or mica Another form is shown in Fig 204 ([22]) A thin layer of

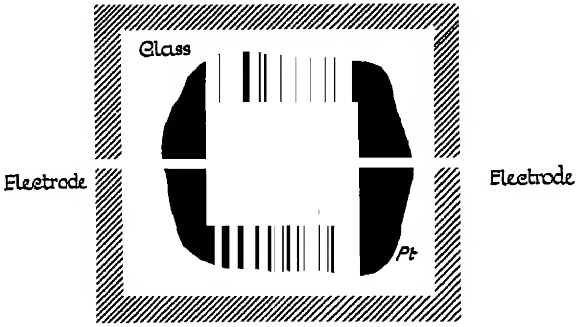

FIG. 204 —A Selenium Bridge.

platinum deposited on a glass plate is scratched with a fine graving tool along a line of the zigzag form shown Selenium in the plastic form is spread over the platinum surface and is then heated to convert it to the crystalline form Electrodes are attached to the two platinum surfaces so that the zigzag line of selenium which separates them forms a bridge of comparatively low resistance.

Much work on the characteristics of selenium bridges has been done by many workers ([23]), and the most sensitive is capable of detecting an illumination of 10^{-5} metre-candles ([24]), a limit which approaches the sensitivity of the human eye (see p 68) It is still, however, inferior in sensitivity to the best type of photo-electric cell, which, with a tilted electroscope capable of measuring currents of 10^{-15} amps , should be capable of detecting the light from a candle at a distance of 2·7 miles ([25])

The chief objection to selenium is that its sensitivity curve is not only very different from that of the eye, but it varies in form with the method of preparation of the cell ([26]). Moreover, the law connecting response with illumination is not the same throughout the spectrum, being linear in the deep red, while the response varies as the square root of the stimulus for the remainder of the visible spectrum ([27]) The resulting change of form of the sensitivity curve with change of illumination is shown in Fig 205. It follows, therefore, that selenium can only be used for the comparison of sources giving radiation of the same spectral distribution, and even then the cell must be calibrated, since the law connecting stimulus and response is not a simple one ([28]).

There is, moreover, a further disadvantage attending the use of selenium, and that is its slowness in recovering its resistance after

exposure to illumination ([29]), the period taken for recovery increasing with the intensity of the illumination to which it has been

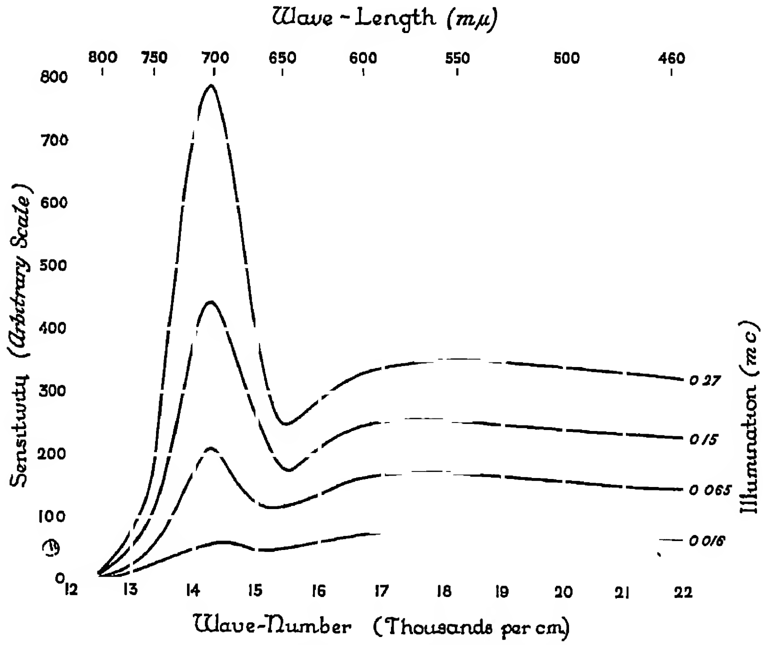

FIG 205.—The Sensitivity Curve of a Selenium Bridge

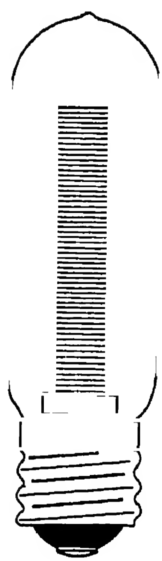

FIG 206 —A Selenium Bridge mounted *in vacuo*

exposed ([30]). Spontaneous change of sensitivity with lapse of time can largely be overcome by mounting the bridge *in vacuo* ([31]). A bridge so mounted is shown in Fig. 206 The sensitivity of a selenium bridge decreases, in general, with rise of temperature ([32]). Any voltage up to 100 may be used on all the ordinary types of bridge, for it has been found that the sensitivity of a selenium bridge is comparatively slightly affected by change of voltage, at any rate up to this limit ([33]) Talbot's law (p 58) holds for selenium, at least for frequencies between ten and sixty per second ([34])

A form of photometer has been devised in which light from the sources to be compared is alternated on the selenium, the criterion of equality being absence of "flicker" in the current through the cell at a minimum speed of alternation ([35])

A H. Pfund summarises as follows ([36]) the conditions under which selenium may be used for photometry —

"(*a*) Monochromatic light must be used.

"(*b*) An accurate sensibility curve [curve of sensitivity to light of different frequencies] must have been established.

" (c) Exposures to light must be made automatically and must be of short duration."

The last-named condition is met in a form of selenium photometer designed by T Torda ([37]) In addition it is desirable that the curve connecting stimulus and response should have been found by means of a previous calibration under the above-named conditions

Many other substances besides selenium have been found to behave similarly under the action of radiant energy ([38]), but most of them are much less sensitive, at any rate in the visible spectrum. One, however, a thallium oxy-sulphide termed " thalofide," has been found notably to surpass selenium in the visible spectrum ([39]). The sensitive material is fused on a quartz plate and mounted in an evacuated bulb to prevent oxidation The most sensitive bridges made of this material change resistance by 50 per cent. for an illumination of less than 1 metre-candle. The dark resistance ranges from 5 to 500 megohms The potential used on the bridges must not exceed 50 volts These bridges possess a large temperature coefficient of resistance The spectral sensitivity curve of a bridge so enclosed is shown in Fig 207 This substance possesses the

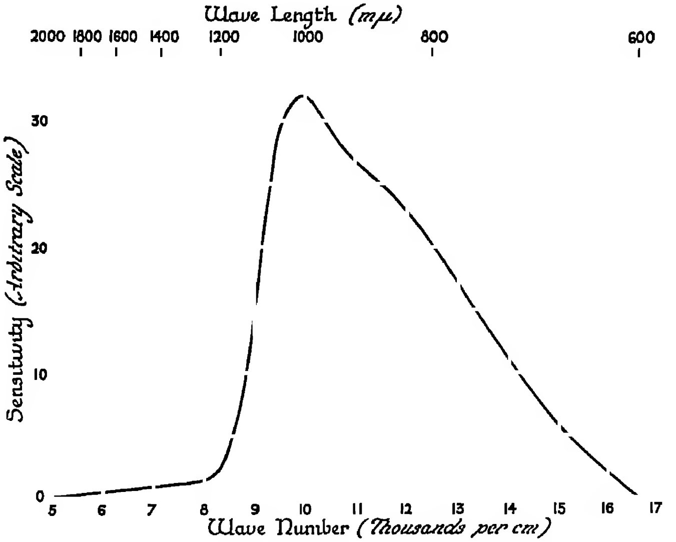

FIG 207 —The Sensitivity Curve of a Thalofide Bridge

advantage over selenium that the response to light and recovery after exposure are both rapid

The Photo-electric Cell.—The photo-electric effect has already been described briefly in Chapter II (p. 42). The application of this effect to photometry has received considerable attention ([40]), and much work has been done on the construction and design of photo-electric cells in which a specially prepared metal surface is illuminated and the resulting electronic emission measured.

Normally the sensitivity of a photo-electric cell should increase with the frequency of the incident radiation (see p. 42), but in 1889 Elster and Geitel showed that the alkali metals had a selective

sensitivity in the visible spectrum, and subsequent work by these experimenters and by others has led to the development of the cell, the general form of which is shown in Fig 208. It consists of a

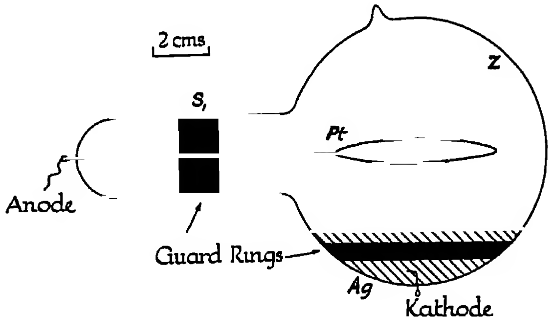

FIG 208—The Elster-Geitel Photo-electric Cell

glass globe Z, into the centre of which projects a loop of gold or platinum wire Pt, which acts as the anode of the cell, while the cathode consists of a surface of alkali metal (Na, K, Rb or Cs ([41]) or a K—Na alloy) which makes contact with a silver film Ag and a piece of platinum wire sealed into the glass A guard ring S_1 connected to earth prevents leakage over the surface of the glass between the two electrodes

A great increase in sensitivity (of the order of 100 times) may be obtained by special treatment of the kathode metal surface ([42]). This metal is distilled first *in vacuo*, and then in an atmosphere of hydrogen at about $\frac{1}{8}$ atmospheric pressure. The surface is then composed of hydride and is insensitive to light The hydrogen having been pumped out, the kathode surface is bombarded with kathode rays until it shows a marked coloration (blue-violet for potassium, brownish yellow for sodium, and pale blue-violet for rubidium) due to the formation of free metal which is extremely sensitive photo-electrically The permanence of the cell is improved by removing the hydrogen set free during the bombardment and replacing it with helium or argon at very low pressure ([43]) Even when this precaution is taken, however, a slight "fatigue" effect, or secular change of sensitivity, is always liable to occur in gas-filled cells. Where permanence is of more importance than great sensitivity the vacuum form of cell is preferable on this account ([44])

When used for measuring illumination the cell may conveniently be mounted as shown in Fig 209 The light enters by the tube R, which is closed with a cap when the photometer is not in use, so that the cell is completely protected from the light except when in operation. I is an iris diaphragm and M a plate of matt uviol glass, which cuts off all ultra-violet radiation. The box is blackened inside and is capable of rotation about the axis defined by Pt. Thus the whole instrument is mounted like a theodolite and the tube R can be oriented in any desired direction This is essential, since the sensitivity of the cell depends on the angle of incidence of the light and, in the case of oblique incidence, on its plane of polarisation (see p. 43). The light should, therefore, always be

incident normally on the sensitive surface The electrodes of the cell are connected to the terminals K_1 and K_2, which are in circuit

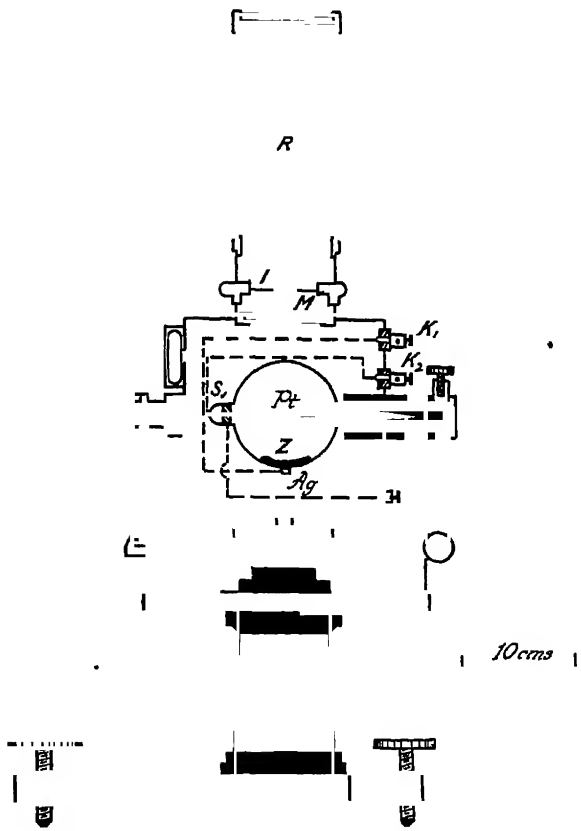

FIG 209 —The Photo-electric Cell in its Mounting

with a battery of dry cells or small-capacity lead accumulators and a sensitive measuring instrument

In such a cell it has been found that the current produced by sunlight illumination is of the order of 10^{-6} amps The current due to an illumination of 1 metre-candle at the cell is of the order of 10^{-11} amps, and it is claimed that if proper precautions be taken to eliminate possible disturbing factors, an accurate proportionality exists between current and illumination over a very wide range (0 07 to 6,000 metre-candles) ([45]) Other workers, however, have found ([46]) that the response of such cells to light is not always directly proportional to the intensity, and that the form of cell shown in Fig 210 is preferable on this account ([47]) The bulb S is silvered on the inside and provided with a small window W carried on an extension tube E. S forms the anode ([48]) The kathode is a small centrally-placed glass bulb K, which is silvered, and on which is distilled the alkali metal which has first been distilled on to the walls of the large surrounding bulb from a side tube. G is

the guard ring [49]. Other forms of cell have been devised for special purposes

The photo-electric current may be measured by means of either

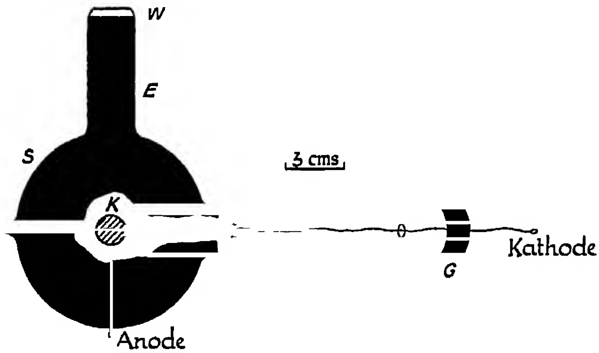

FIG 210 —The Ives Photo-electric Cell

a galvanometer or an electrometer In the former case the connections are as shown in Fig 211 The galvanometer may advantageously be of a high resistance type [50] When an electrometer is used the connections may be as shown in Fig 212 [51], the instrument employed being of the Dolezalek or other suitable type [52]. B is a light-tight metal box containing a drying agent to stop leakage due to moisture on the surface of the photo-electric cell, T is a guard tube surrounding the leads Both B and T are earthed The needle is charged to a potential of 80 to 100 volts The guard ring of the cell is at a potential of about 200 volts below that of the anode in order to avoid a drift of the electrometer due to leakage current. Either the

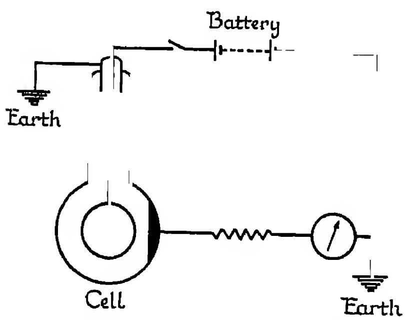

FIG 211 —Connections of the Photo-electric Cell (Central Kathode) with a Galvanometer

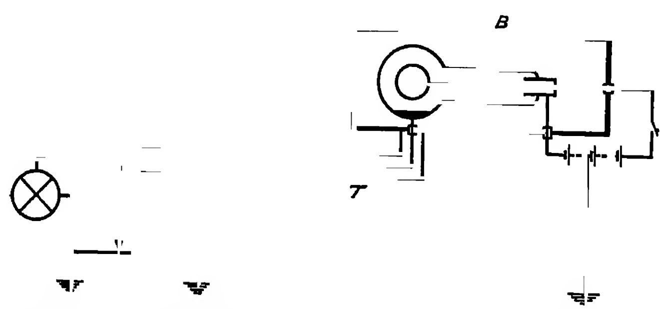

FIG 212 —Connections of the Photo-electric Cell with an Electrometer

rate of increase of deflection may be measured [53] or the total deflection produced in a given time may be taken, allowance being

made for the deflection produced in the same period of time when the cell is unexposed [54] Alternatively, the potential drop across the cell may be measured by means of the arrangement shown in Fig 213 [55], where R is a high resistance (*e.g*, a capillary tube of absolute alcohol with adjustable wire immersion), while S is an adjustable high resistance The contact on S is so placed that the electrometer gives a zero reading when the cell is not illuminated

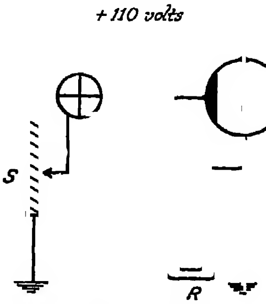

FIG 213—Connections of the Photo-electric Cell (Central Anode) with an Electrometer and Leak

In order to limit the current through the cell, a high resistance of the order of 1 to 10 megohms is placed in series with the cell This resistance may be of liquid type or may consist of a fine graphite (lead pencil) line on porcelain or some similar form of carbon resistance [56]. When a very high illumination is being measured the voltage on the cell must be reduced, as if the current passing through the cell exceeds a certain limit (of the order of 10^{-6} amps) the subsequent behaviour of the cell becomes erratic [57]

The photo-electric current rises very rapidly with the voltage applied (see Fig 214) until a certain value, known as the critical

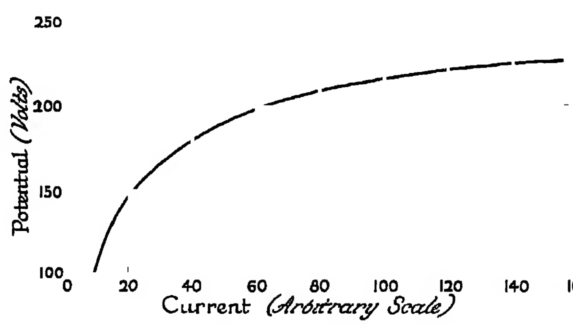

FIG 214—Typical Voltage-current Curve for a Photo-electric Cell

voltage, is reached. The most convenient voltage for ordinary work is about half the critical voltage, as then the cell is not unduly sensitive to slight voltage variations.

As might, perhaps, be expected, the sensitivity of any given type of cell varies enormously from cell to cell and with the age of the cell [58] and its previous history, but the most fundamental objection to the photo-electric cell as a physical photometer is the fact that its spectral sensitivity curve is quite arbitrary in form and, what is worse, it is different for different cells of the same type, and even for a single cell with lapse of time [59]. Most alkali cells show a marked maximum of response between $\nu = 20{,}000$ and $25{,}000$ [60].

It follows that no solution can be found to give the cell the same sensitivity curve as the normal eye, and therefore the most useful field for this instrument is in the comparison of lights of the same

spectral composition. A null method devised by Richtmyer ([61]) is shown diagrammatically in Fig 215 Two cells, C_1 and C_2, of the same type are connected in a circuit resembling a Wheatstone bridge, with two sources of variable potential e_1 and e_2, and a Dolezalek electrometer Q The circuit is earthed, permanently at b, and when desired at b' L_1 and L_2 are the two sources to be compared With the cells unexposed, e_1 and e_2 are adjusted so that the dark currents through C_1 and C_2 are equal as shown by zero deflection of Q With the cells exposed to radiation from L_1 and L_2, the distances L_1C_1 and L_2C_2 are adjusted until there is again no deflection of Q. The illuminations at C_1 and C_2 are then equal. This method depends on the condition that the two cells obey exactly the same current-illumination law, but it does not demand that this law shall be linear By a substitution method in which one lamp, say L_2, was fixed throughout the experiment, two other lamps *giving light of the same spectral distribution* could be compared to an accuracy which would probably exceed that of visual photometry, due care being taken to shield the cells from stray radiation ([62]). A similar method has been used for the determination of spectral transmission curves (see Chapter XIII, p. 389), and might be adapted to the spectrophotometric comparison of sources

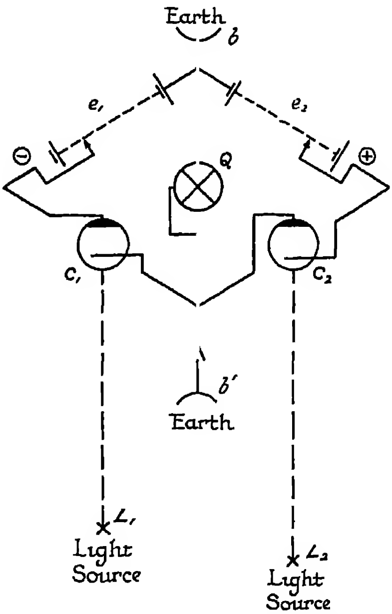

FIG 215 —The Photo-electric Null Method

Alternatively, a single photo-electric cell may be used, and the lamps to be compared may be successively placed so as to produce equal illuminations of the cell as judged by equality of the deflection in the two cases Instead of varying the distance of the source from the cell, crossed nicol prisms (p 173), a variable rotating sector (p. 177), or a neutral wedge (p 179) may be used to vary the illumination according to a known law ([63]). Talbot's law is rigorously obeyed, since the response of a photo-electric cell is practically instantaneous ([64]).

For approximate photometry colour filters are sometimes used to give a photo-electric cell a sensitivity curve which in some measure approximates to that of the eye ([65]), but for accurate work it is essential that the sources to be compared shall give light of exactly the same spectral composition

A photo-electric method of adjusting lamps, such as the substandards described on p 137, to colour match has been devised ([66]). Each lamp in turn is placed so that it illuminates simultaneously two cells of different alkali metals, *e g*, sodium and rubidium. Since these cells have widely different spectral sensitivity curves, if the

photo-electric currents through the two cells be adjusted to equality for one lamp (by means of a diaphragm, neutral wedge or other device), this equality will be disturbed if another lamp be substituted unless the spectral distribution of the light is unchanged, *i.e.*, unless the lamps be adjusted to colour match. It is desirable that the illumination of the cells should be approximately equal with both lamps, since the law connecting intensity of illumination with photo-electric current may not be exactly the same for the two cells.

There is one effect in the photo-electric cell which has so far only been referred to incidentally This is the "dark current," *i.e*, the current passing through the cell when it is not illuminated Part of this current may be due to leakage over the surface of the bulb between the electrodes, and this can be eliminated by the use of a guard ring The remainder, which is a true dark current effect, has either to be compensated by some electrical means (as in the arrangement shown in Fig 213) or it may be subtracted from the observed current when illumination measurements are made. It is of considerable importance in the measurement of very faint illuminations (67)

There is no noticeable temperature effect, at any rate in vacuum potassium cells, above 0° C (68).

It has been found possible to increase the photo-electric current many thousands of times by means of a three-electrode valve (69). The connections are as shown in Fig. 216, where *P* is the photo-electric cell, *A* the valve, *G* a galvanometer, and B_1, B_2, B_3 batteries The degree of amplification naturally depends on the temperature of the valve filament *F*. It also depends on the photo-electric current through *P* (70), so that it is necessary to calibrate the system as a whole in order to obtain quantitative results. Other arrangements have also been used (71)

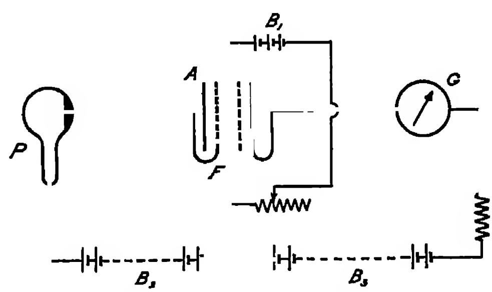

FIG 216 —The Use of a Valve with a Photo-electric Cell (Central Anode)

Metallic Oxide Photo-electric Cells.—It has recently been found that the filament of a high-vacuum valve shows a strong photo-electric effect if coated with one of the alkaline earth oxides A strontium or barium oxide cell gave a current of the order of 10^{-9} amps per metre-candle illumination (72) This form of cell has recently been developed (73).

Chemical Photometers The Photographic Plate.—The suggestion has been made repeatedly to use as a physical photometer some form of apparatus in which light is measured by the amount of chemical action it produces (74). The only practical apparatus of this kind is the photographic plate (75), incidence of radiation on which causes a chemical reaction resulting, on development, in a deposition of silver particles. The opacity thus produced in the plate may be measured by other photometric means (see p 392), and under

certain conditions is nearly proportional to the total energy reaching the plate during the period of exposure ([76]) If this period be known accurately, the illumination if steady, or its mean value if fluctuating, can be deduced ([77]) The photographic plate, therefore, is the sole instrument capable of performing a time integration for an illumination. Its sensitivity is, however, variable (*a*) from one plate to another even under otherwise identical conditions, (*b*) according to the particular emulsion used on the plate, (*c*) according to temperature and other conditions of development ([78]), and (*d*) according to the spectral distribution of the light ([79]). Normally the plate is most sensitive to radiation in the violet and ultra-violet, but, by the use of sensitisers, its response to light of the lower frequencies may be increased as shown in Fig 217 Even in these cases, however,

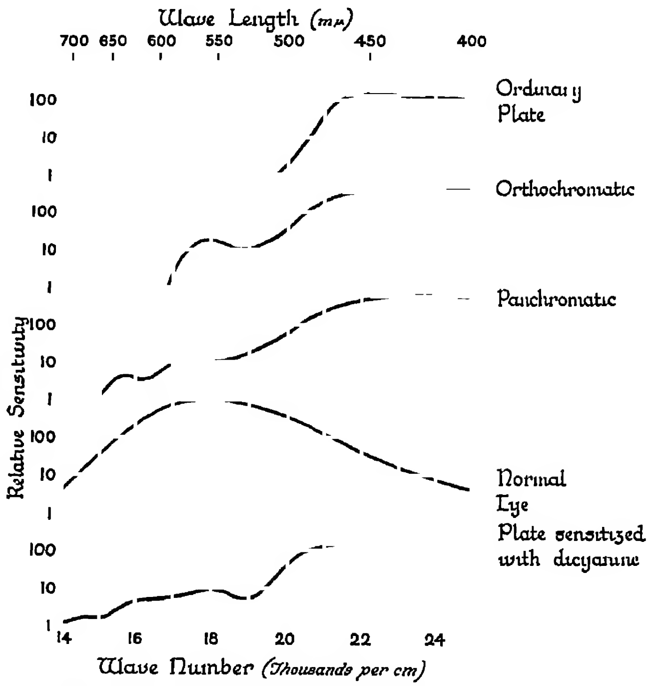

FIG 217 —The Relative Sensitivities of Photographic Plates

the sensitivity curve is irregular and variable, and it seems to follow that the chief sphere of usefulness of the photographic method of photometry is in strictly homochromatic comparisons or in the measurement of very faint illuminations, such as those met with in stellar photometry (see p. 427), where the eye, or any instrument depending on the measurement of luminous *flux*, is of insufficient sensitivity, and where the time integration of the photographic plate (*i e*, the fact that it measures energy, and not power) enables it to give an approximate measurement if due care be taken to use standard conditions in both the preparation and the development of the plate ([80]) The photographic method is also useful for obtaining a continuous record of light intensity ([81]), especially in inaccessible places, such as the upper atmosphere ([82]) or below sea level ([83]).

Probably the most important application of photography to photometric measurement at the present time is in the special field of spectrophotometry, where the comparison is strictly homochromatic, and where the great sensitivity of the photographic plate makes it possible to use much smaller slit widths than are practicable in visual or even in radiometric work ([84]). It is particularly useful for determining the approximate relative intensities in the different parts of the spectrum of a source, such as a flame arc, where there are narrow spectral bands superposed on a continuous background It may also be used for finding the relative intensities of the lines in a pure line spectrum ([85]). In such work the spectrum to be studied and a spectrum of known intensity distribution are photographed side by side, one of the two being photographed several times with different exposures of known duration By subsequent comparison of the records in any particular region of the spectrum it is possible to make an estimate of the respective times of exposure to the two lights which would give equal densities in that spectral region. Schwartzchild's law, or one of its modifications, may then be used to find the relative intensities. A similar method may be applied to the determination of the spectral transmission curve of a coloured medium ([86]). It is clear that, instead of using different exposure times to obtain the necessary scale of intensities of one spectrum, a number of exposures of equal duration may be made with different densities of a neutral absorbing medium over the slit of the spectrometer.

A rough comparison of the densities of photographic images may be made by eye if the densities be approximately equal. For accurate work, and for the comparison of unequal densities, however, some form of densitometer or microphotometer must be used (see p 393) ([87]). In making an approximate comparison of the intensities of two line spectra, it is sometimes convenient to split each line into a series of dots by means of a diffraction grating placed between the prism of the spectrometer and the camera, the rulings being at right angles to the length of the slit In the resulting photograph, made either directly or through a fine process screen, the number of dots just visible at each line gives an approximate measure of the intensity of that line ([88]).

The same device is sometimes useful in the case of continuous spectra, for since the relative luminous intensities of the different diffraction bands for light of any given frequency are known from theoretical considerations, the relative densities of the photographic images of these bands provide a scale by means of which it is possible to compare images of unequal densities for light of the same colour

It is important to notice that a sector disc cannot be used as a means of reducing the light intensity by a known factor in the case of photographic photometry, since it has been found that the plate does not obey Talbot's law An intermittent exposure gives less photographic effect than the same amount of luminous energy acting continuously ([89])

The Determination of Polar Distribution Curves and of Intensity Fluctuations.—It is clear that the disadvantage of irregularity in the "luminosity curve" of a physical photometer does not affect its usefulness for comparing lights of exactly the same spectral com-

position and, in particular, for measuring the relative candle-powers of a light source in different directions All the physical photometers described in this chapter may therefore be used for obtaining polar curves of light distribution or illumination curves, but so far little use has been made of them for this purpose ([90]) They have, however, been used for obtaining records of the fluctuations of a source of light ([91])

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Eng, 1, *1908*, pp 239 and 343, Lum El, 2, *1908*, p 149, El World, 51, *1908*, p 396 [Thermopile]

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See also references in note (39), p 374

CHAPTER XII

ILLUMINATION PHOTOMETRY

THE instruments and methods described in Chapters VI and VII. are all designed primarily for the measurement of the candle-power of a source of light, and the illumination produced by the flux reaching a surface from that source is only considered in so far as it is a necessary intermediary in the process of candle-power measurement For many purposes, however, the illumination of a surface at a point is of more interest than the sources of the light flux producing that illumination For instance, in a general specification of the lighting of factories and schools it is the minimum illumination at the desk or point of work which is stated, the sources of light being ignored except as regards such general considerations as glare, position of shadows, etc (1).

There are several factors which have to be considered carefully in the design of instruments for illumination photometry Chief of these are (a) the frequent necessity for making measurements in positions where bulky apparatus cannot be accommodated, or in which it is impossible to use delicate instruments or those requiring accurate positioning, (b) the fact that the illumination at a point is generally derived from a number of sources, so that the flux reaching it comes from many different directions and must not be obstructed by the measuring apparatus itself or by the person of the observer. The first of these considerations may, for practical purposes, be summed up under the name of "portability," and the aim of illumination photometer design is to obtain the maximum of sensitivity and accuracy with the minimum of size and weight.* The second consideration is extremely difficult of fulfilment, and it will be noticed that with many instruments, particularly those of older design, it is impossible to obtain an accurate measurement of illumination when the flux reaches the surface from all directions.

Since, as has been said already (p. 147), all accurate photometry ultimately depends on a comparison of the brightness of two surfaces, it follows that in order to measure the illumination at a point it is necessary to place at that point a standard surface whose reflection factor ρ is known, so that a measurement of its brightness B gives at once the illumination $E = \pi B/\rho$, assuming that the surface is a perfect diffuser (see p 101) This at once introduces another requirement in illumination photometry In candle-power photometry the light is arranged to be incident on the comparison surface at an invariable angle, which is preferably 0° (see p 152). Further, this surface is always observed at a constant angle, so that any slight departure of the comparison surfaces from the state of perfect diffusion is immaterial In the case of the illumination photometer

* See the British Standard Specification for Portable Photometers, No 230, September, *1925* Illum. Eng., 18, *1925*, p 298

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THE instruments and methods described in Chapters VI. and VII. are all designed primarily for the measurement of the candle-power of a source of light, and the illumination produced by the flux reaching a surface from that source is only considered in so far as it is a necessary intermediary in the process of candle-power measurement For many purposes, however, the illumination of a surface at a point is of more interest than the sources of the light flux producing that illumination For instance, in a general specification of the lighting of factories and schools it is the minimum illumination at the desk or point of work which is stated, the sources of light being ignored except as regards such general considerations as glare, position of shadows, etc [1]

There are several factors which have to be considered carefully in the design of instruments for illumination photometry. Chief of these are (*a*) the frequent necessity for making measurements in positions where bulky apparatus cannot be accommodated, or in which it is impossible to use delicate instruments or those requiring accurate positioning, (*b*) the fact that the illumination at a point is generally derived from a number of sources, so that the flux reaching it comes from many different directions and must not be obstructed by the measuring apparatus itself or by the person of the observer The first of these considerations may, for practical purposes, be summed up under the name of "portability," and the aim of illumination photometer design is to obtain the maximum of sensitivity and accuracy with the minimum of size and weight * The second consideration is extremely difficult of fulfilment, and it will be noticed that with many instruments, particularly those of older design, it is impossible to obtain an accurate measurement of illumination when the flux reaches the surface from all directions.

Since, as has been said already (p 147), all accurate photometry ultimately depends on a comparison of the brightness of two surfaces, it follows that in order to measure the illumination at a point it is necessary to place at that point a standard surface whose reflection factor ρ is known, so that a measurement of its brightness B gives at once the illumination $E = \pi B/\rho$, assuming that the surface is a perfect diffuser (see p 101) This at once introduces another requirement in illumination photometry In candle-power photometry the light is arranged to be incident on the comparison surface at an invariable angle, which is preferably 0° (see p 152). Further, this surface is always observed at a constant angle, so that any slight departure of the comparison surfaces from the state of perfect diffusion is immaterial. In the case of the illumination photometer

* See the British Standard Specification for Portable Photometers, No 230, September, *1925*. Illum. Eng, 18, *1925*, p. 298

the light may be incident at any angle or at many different angles, while the angle at which the test surface is viewed may also vary It is therefore necessary that this surface should, as nearly as possible, behave as a perfect diffuser.

The Standard Surface or Test Plate.—The errors which arise from a lack of perfect diffusion of the incident light by the standard surface are among the most troublesome which have to be considered in the design and use of illumination photometers, particularly those in which the surface is detached from the photometer, so that the angle of view depends on the whim of the observer In most instruments the surface is viewed by reflected light, and for this purpose plain white blotting paper, bristol board depolished by rubbing its surface with fine pumice powder (2), white enamelled iron depolished by etching with hydrofluoric acid (3), plaster of Paris (4), compressed powders such as magnesium oxide or carbonate (5), depolished opal glass (6), opaque white celluloid rendered matt by sandblasting (7), porcelain (8). and other substances (9) have been studied.

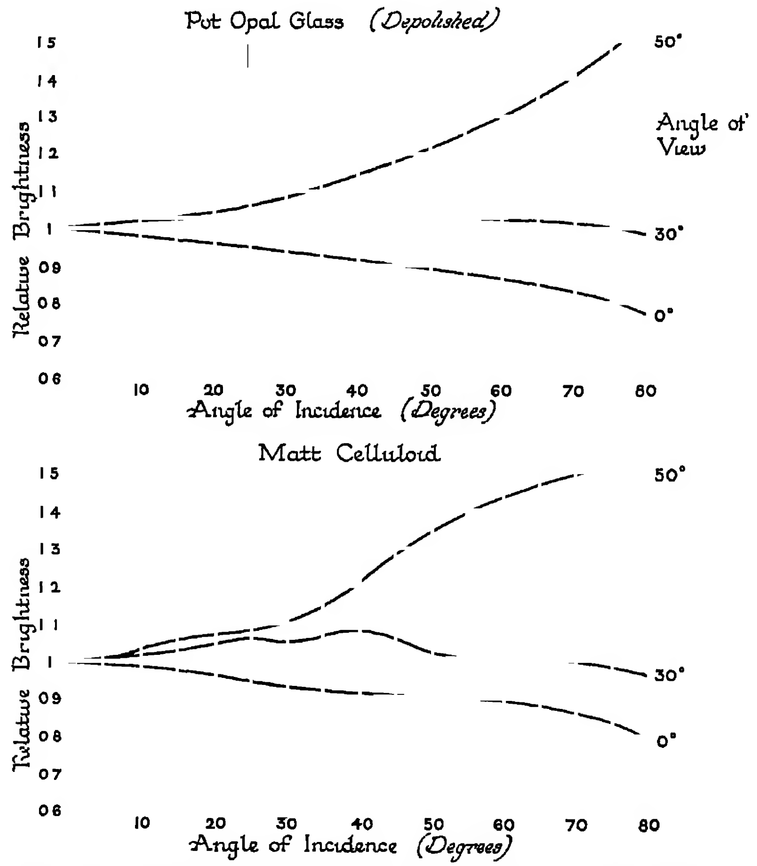


FIG 218.—The Characteristics of Diffusing Surfaces for Use as Test Surfaces

The results obtained with matt celluloid and depolished opal glass may be considered, as these surfaces are fairly representative

of the general behaviour of approximately diffuse reflectors The curves of Fig. 218 ([10]) show the departures from the true cosine law which are exhibited by a depolished opal and a matt celluloid surface viewed normally and at 30° and 50° from the normal, the line of view being in the same plane with, but on the side opposite to, the incident light. The abscissæ represent angles of incidence, and the ordinates the percentage differences from the theoretical values, the value at 0° incidence being assumed correct. It will be seen that so long as the plate is viewed at an angle of 30° the errors in the case of the glass do not exceed about 3 per cent for angles of incidence up to 80°, while for matt celluloid the error may rise to 8 per cent.

The two surfaces here chosen as examples of diffusion have the very desirable quality of permanence, and both can readily be cleaned by simply wiping them with a damp cloth. In the case of celluloid, however, repeated wiping in this way results in a partial polish, and the surface should be occasionally renewed by rubbing it gently with a paste of fine pumice powder and water.

A very good matt white surface may be obtained by smoking a plate of metal or other material over burning magnesium ribbon. Although the surface of magnesium oxide thus obtained is very easily damaged, and is therefore most suitable for use in enclosed apparatus, it is very readily renewed when it becomes dirty or scratched.

For a transmitting standard surface, such as is required for the Sharp-Millar photometer (p. 348) where the test plate is viewed from below, opal glass is almost universally employed It may be either polished or depolished The former possesses the advantage of retaining a clean surface for a longer period without washing. Curves *A* and *B* respectively of Fig. 219 show the behaviour of such

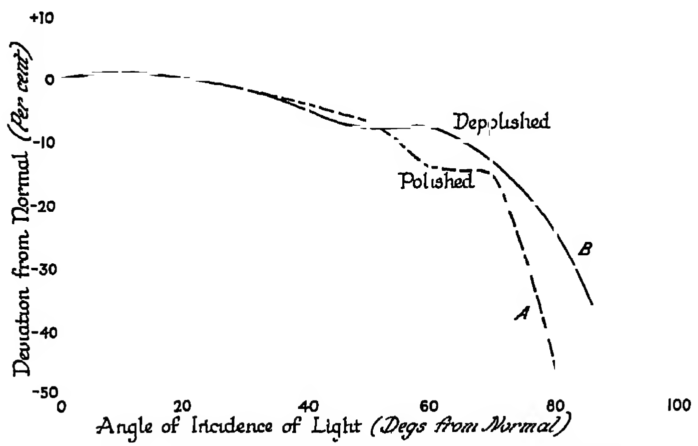


FIG 219.—The Characteristics of Transmitting Test Surfaces.

plates at different angles of incidence of the light, the plate being viewed normally in every case ([11]) It will be seen that at angles of incidence greater than about 50° the errors introduced are very large, and Sharp and Little ([12]) have developed a form of test plate in which this negative error at large angles of incidence is com-

pensated. The principle of the plate will be seen from the section in Fig 220 P is a sheet of polished opal glass, R is a ring of opal glass in which the diffusing layer may be quite thin (e g , a light flashing of opal on a clear glass), C is an opaque ring, and S is an

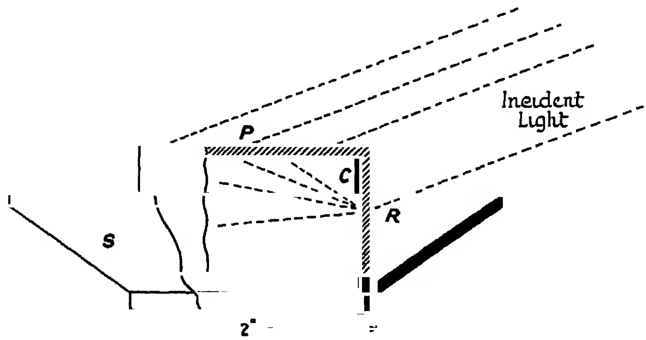


FIG 220 —The Compensated Transmitting Test Surface

opaque saucer-shaped screen, so arranged that its upper edge is at the same level as the lower edge of C. This screen has also a vertical portion surrounding the lower part of R. When the light is incident normally, R is not illuminated, and the brightness of the under surface of P is due entirely to transmitted light At large angles of incidence, however, light falls on R, is transmitted through the portion between C and S, and by diffuse emission from the inner surface of R adds to the brightness of P, so that the defect in the transmission of this latter plate is compensated As will be seen from curve A of Fig 222, by properly adjusting the breadth of the exposed band of R to the transmission curve of P very accurate compensation up to angles of incidence of 80° may be obtained

A reflecting test surface on the same principle is shown in Fig 221 In this case P is of depolished opal glass, the negative error of which

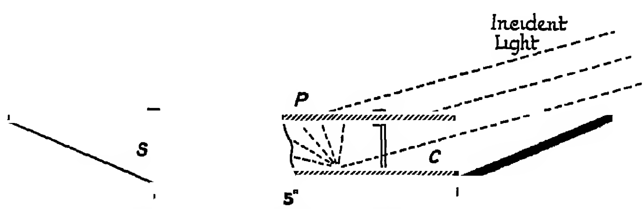


FIG 221.—The Compensated Reflecting Test Surface

at oblique incidence is compensated by the light reflected to it from a second similar plate C. The performance of this plate is shown in curve B of Fig 222. A disadvantage of this plate is that it has to be viewed normally, whereas matt celluloid may be viewed at angles not exceeding about 30° without serious error, especially if the light be not incident too obliquely Although this last requirement is not always easy to comply with when the illumination is derived from many sources situated at all distances from the surface, it must be

remembered that in such cases the oblique light is frequently quite a small fraction of the whole, and that the least important from the point of view of the resulting illumination

The whole position may, perhaps, be summarised thus. (*a*) for work of the highest accuracy under all conditions a transmitting

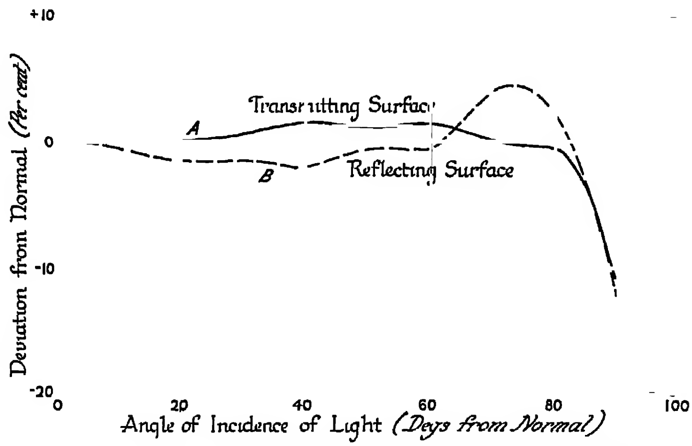


FIG. 222.—The Performance of Compensated Test Surfaces

test plate of the compensated type described above gives the best performance, if it be viewed always in the normal direction, (*b*) for ordinary accuracy (3 to 5 per cent.), and in cases where it is not possible to use a transmitting surface, the best performance is obtained from depolished opal glass or a matt celluloid surface. When the illumination is obtained from light coming in but few directions the surface should be viewed at an angle not greater than 30° When a large number of sources contribute to the illumination, it is inevitable that errors will be made in the measurement of the obliquely incident light owing to the imperfect behaviour of the surface at large angles of incidence. As, however, this part of the illumination is derived from distant sources, the inverse square law combines with the cosine law of illumination to make this by far the least important part of the whole illumination, so that even large errors here will produce but a small effect on the measured total.

The absolute value of the reflection or transmission factor of a test surface is of importance, particularly in the measurement of low illuminations, where as bright a comparison field as it is possible to obtain is most desirable. The values of these factors for certain important materials are given in Appendix VII It is clear that the material used for a test surface should be as non-selective as possible.

Illumination Photometers based on the Inverse Square Law.—Illumination photometers, like candle-power photometers, may be classified according to the method adopted for altering the brightness of the comparison surface within the instrument Among those in which the inverse square law is used the ordinary Weber photometer, already described on p 172, must be included, since it is readily adapted to the measurement of illumination by the removal of the opal glass plate M_1 (Fig 98, p. 172) The tube T_1 is then directed

towards a test surface placed in the position at which it is desired to measure the illumination, and the brightness of this surface is compared with that of the surface M_2 (13). Photometric balance is then obtained, as in the measurement of candle-power, by moving M_2 backwards and forwards, and the illumination of the surface is calculated from the equation given above and the brightness of M_2 Preferably, the instrument may be calibrated by using it to measure the brightness of the standard surface when illuminated by a standard lamp of candle-power I placed at a known distance d from it in the direction of the normal, so that the point of balance of M_2 under these conditions corresponds to an illumination of I/d^2.

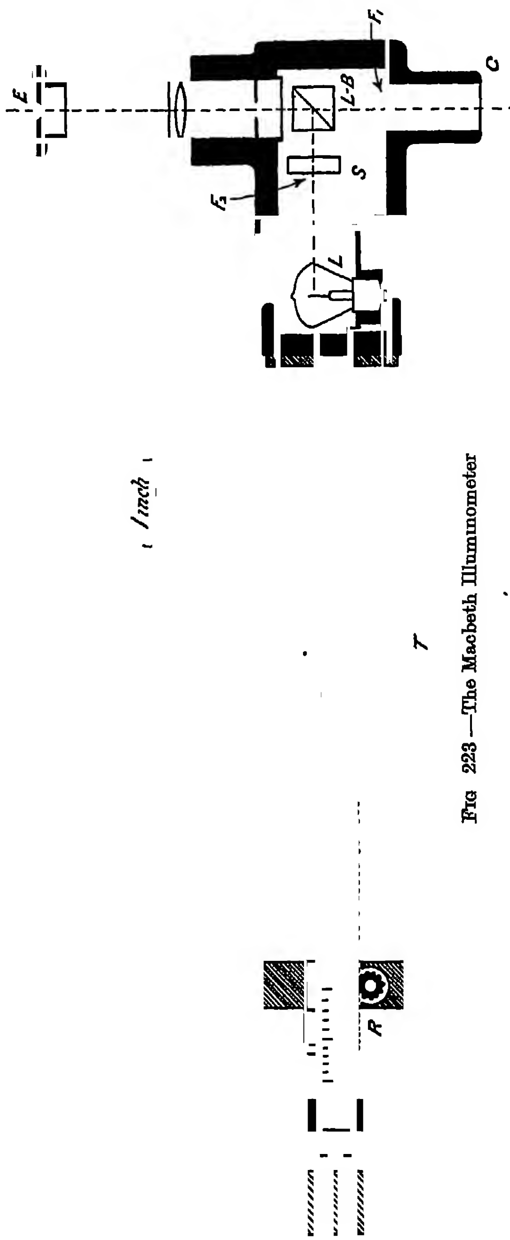


FIG 223 —The Macbeth Illuminometer

Since brightness is independent of distance, the only restriction on the position of the photometer is that it must be sufficiently near the test surface for the latter to fill the field of the prism C completely. The general remarks on angle of view of the test surface (p. 345) apply to this photometer as to all others in which the angle of view is not fixed by the instrument itself This angle may be made invariable by the provision of an arm rigidly attached to T_1, and carrying the test surface at its end

Many other photometers depending on the inverse square law have been designed (14). A serious disadvantage of some of these—for example, the Burnett and the Martens —is that the test surface is fixed in such a position that it can

receive light only from above and in front of the observer Among the most accurate and convenient instruments depending on the inverse square law are the Macbeth illuminometer (15) and the Sharp-Millar photometer (16)

The Macbeth Illuminometer.—The principle of this instrument will be seen by reference to Fig. 223. The opal glass comparison surface S is illuminated by the lamp L, which, in its diaphragmed enclosure, is moved along the tube T by means of a rack and pinion R A foot-candle scale is marked on the rod carrying the lamp enclosure. The lamp is supplied from a dry battery by means of leads plugged into the end of this rod The battery is carried in a separate control case, which also contains a rheostat and milliammeter by means of which the current through the lamp L is adjusted to the value at which the foot-candle scale has been found, by previous calibration, to read correctly The test surface is of depolished opal glass and is detached from the instrument, although a transmitting test plate may be used instead. The brightness of the test plate is compared with that of S by means of the Lummer-Brodhun cube L-B viewed through the eyepiece E. A reflecting elbow-piece, to fit on the end of C, is provided for use where convenient, particularly in making measurements in directions above the horizontal Neutral filters may be inserted either at F_1 or F_2 to extend in either direction the normal scale of the instrument which is from 1 to 25 foot-candles The instrument is shown in use in Fig. 235. In general it will be found convenient to use an accumulator (4 volts) in place of the dry battery whenever accurate measurements are required

The instrument calibration is checked from time to time by means of the reference standard shown in Fig 224 This is placed with its base C on the test surface, the lamp M being supplied from the same battery as that providing the current for the illuminometer lamp L. A second rheostat is provided in the control box for adjusting the current through M, and the ammeter is provided with a double-pole switch to enable it to be used on either the illuminometer or reference standard circuit. The illuminometer is sighted through the aperture A, and the current through M is adjusted to a stated value The milliammeter is then put over to the illuminometer circuit and the current in L is adjusted until the photometric balance point is attained at a stated value of illumination corresponding to the stated value of the current through M

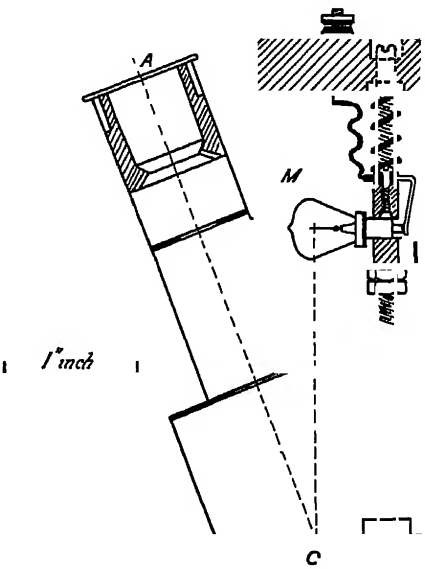


Fig 224 —The Portable Reference Standard for the Macbeth Illuminometer

The Sharp-Millar Photometer.—This instrument is shown in plan and elevation in Fig 225. The electric lamp L is moved backwards

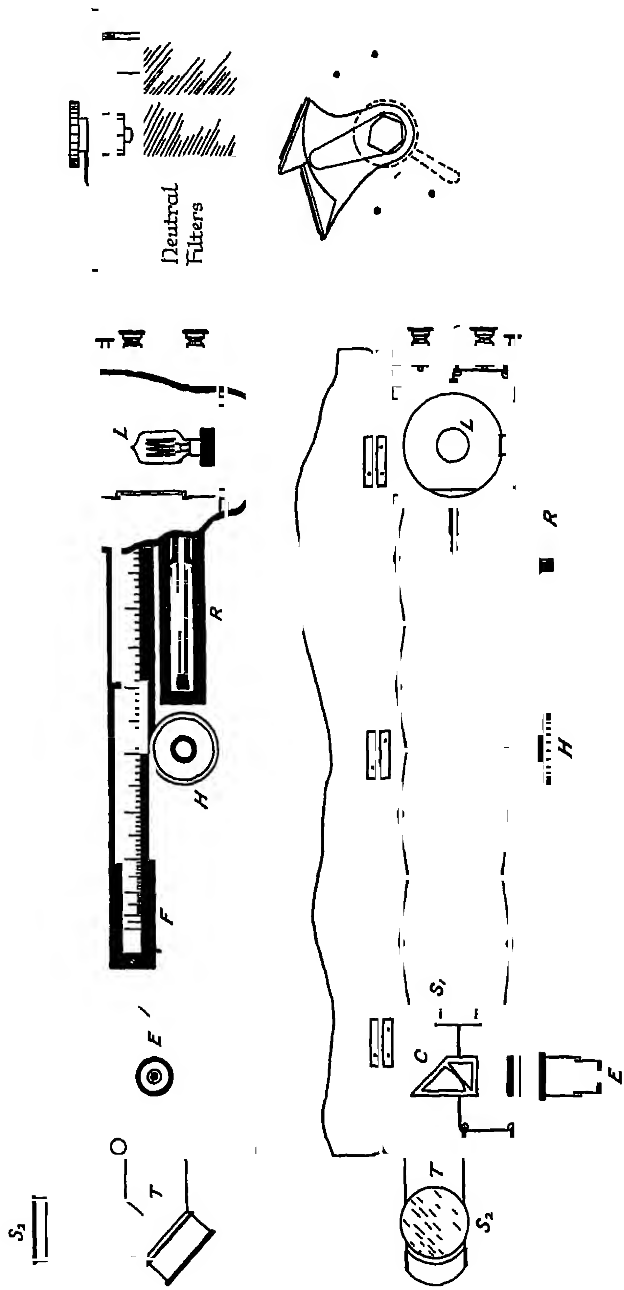


FIG 225 —The Sharp-Millar Photometer

and forwards by means of an endless cord which passes over a wheel attached to the handle H. It illuminates the diffusing glass screen S_1, which forms one of the comparison surfaces The other comparison

surface, S_2, is of matt opal glass and is carried at the end of an elbow-tube T which has at its angle a mirror, so that S_1 and S_2 may be compared by means of the Lummer-Brodhun type cube C viewed through the eyepiece E. The tube T may be swivelled about its collar F is a translucent celluloid scale upon which is cast the shadow of an index attached to the lamp holder, so that when a photometric balance has been obtained the illumination may be read off directly from the position of the shadow on the scale In order to avoid the entrance of stray light through this scale, it is covered on the inside by a shutter, which is raised by means of a handle at the end of the box when a setting has been made In order to avoid errors due to reflected light, a system of black screens of light fibre is placed between the lamp and the screen S_1 These screens are supported on two light rods and are attached to one another and to the lamp carriage by cords, so that when the lamp advances its housing pushes the screens successively in front of it, and when it recedes the cords pull the screens, one after another, into their original positions The normal range of the instrument (0 4 to 20 foot-candles) is increased by the provision of neutral filters with transmission factors of 1/10 and 1/100, placed in front of either of the comparison surfaces, so that the scale is extended by a factor of 100 in each direction. The current through the lamp is adjusted by means of the slides on the rheostat R until it gives the correct value on an auxiliary ammeter Alternatively, a Wheatstone bridge with a telephone and interrupter instead of a galvanometer is used for regulating the current through the lamp, the correct current being indicated by absence of sound in the telephone circuit When the instrument is calibrated the bridge is adjusted to be in balance when the current through the lamp is such that the true illumination of S_2 agrees with the reading of the pointer on F

It will be noticed that this form of photometer has a test surface which is viewed by transmitted light, so that there is no possibility of error due to shadow of the observer or the instrument Further, the test surface is always viewed normally, so that errors due to variable angle of view are avoided The instrument also possesses the advantage that the scheme of indication enables readings to be taken in the dark, a considerable convenience in certain kinds of illumination work The accuracy of the scale depends on the extent to which the inverse square may be assumed to hold as the lamp approaches the comparison surface, and even more on the exact alignment of the lamp filament with the scale index The instrument as originally designed is somewhat bulky, being 23 inches long by 3¾ by 3¾ inches, and weighing about 8 lb It is generally used on a tripod A smaller model, measuring 14½ by 2½ by 2½ inches, has since been designed on exactly the same general lines, but with slight modifications in constructional details ([17]) When it is impossible to place the whole instrument in the position at which it is desired to measure the illumination, the tube T is removed and a detached test surface (such as that described on p 344) is viewed through the opening thus left For the measurement of candle-power, S_2 is removed and a diffusing surface is substituted for the mirror at the elbow of T. The plain opal glass may be

removed, and the compensated test surface described on p. 343 may be substituted.

Illumination Photometers with Tilting Screen.—A number of illumination photometers have been devised in which the variation of brightness of the comparison surface within the instrument is achieved by tilting this surface so that the light from the comparison lamp reaches it more or less obliquely. The first of these instruments was devised by Trotter in 1891 ([18]) It has since been modified several times ([19]), and in its present form is shown in Fig. 226 ([20]). The light from a small metal filament glow lamp *L* is reflected by a mirror *M* to a white diffusing surface *C* made of matt celluloid. This surface is rigidly attached to a mount which is capable of easy rotation about an axis perpendicular to the plane of the paper. This mount carries a pin and roller *D*, which bears on the surface of

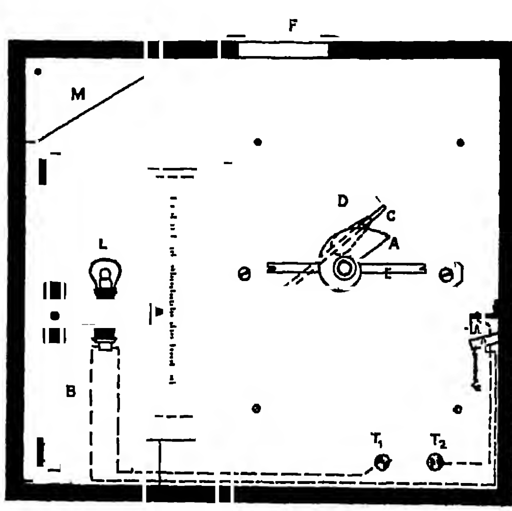
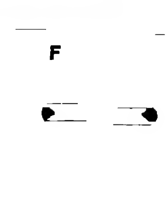


Fig 226 —The Trotter Photometer

Appearance of *F* and *C* when correctly viewed



the cam *A* and is held in contact with it by a light coiled spring (not shown). *A* is moved about an axis perpendicular to the plane of the paper by means of a shaft which passes through the case of the instrument and bears a pointer, which moves over a graduated scale. A light leaf spring *E* bears on *A* so that it retains any position to which it may be set The leads from a 4-volt portable battery are attached at T_1 and T_2, and a small knife switch *S* is depressed to make the circuit through the lamp when a reading is being taken. The surface *F*, which also is of white celluloid, has a transverse slot cut in it as shown separately in plan, and through this slot the

surface C is viewed, A being rotated until the brightness of the slot is equal to that of the surrounding surface. The position of the pointer attached to the shaft of A then gives, by direct reading on the scale, the illumination of F This scale has to be calibrated empirically by means of a standard lamp placed at given distances from F so as to produce known values of illumination The cam is necessary in order to open the scale of the instrument at its lower end, for since the change of illumination of the comparison surface is produced by change of inclination of this surface to the incident light, the scale would be a cosine scale if the pointer and the surface were to have equal angular motions throughout, and if the surface behaved as a perfect diffuser Actually the cosine scale would not be accurate on account of imperfect diffusion, and therefore there is nothing lost, even in theoretical accuracy, by the employment of a cam which reduces the angular velocity ratio at the lower illuminations, and so gives a scale which, though arbitrary, is much more uniform than a cosine scale throughout its length. When the instrument is calibrated, the carriage bearing L is moved up and down upon the vertical rod B. This carriage bears a pointer, which moves on a vertical scale, so that its correct position can be noted for purposes of reference The scale of the instrument, normally 0 01 to 4 foot-candles, may be increased for greater illuminations by using a standard surface which has a reflection factor one-tenth that of the surface ordinarily used, and for low illuminations by introducing a series resistance into the lamp circuit so that the candle-power of the lamp is reduced to one-tenth of its normal value The box is painted a dead black inside to avoid errors due to stray light

The instrument is somewhat bulky, measuring 9 by 7½ by 4½ inches and weighing about 4 lb. (without battery or tripod) It is generally used on a tripod stand, and provision is made for levelling it accurately or for tilting it through any desired angle

A portable form of the Trotter photometer is the "Luxometer," shown in Fig 227 ([21]) In this the principle of the tilting comparison

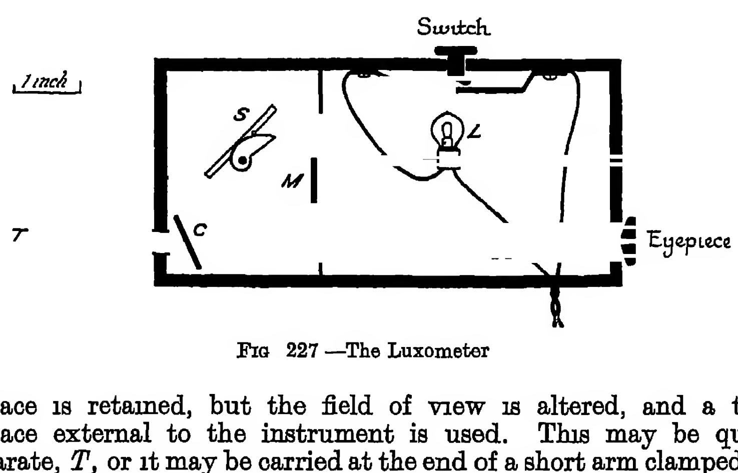


FIG 227 —The Luxometer

surface is retained, but the field of view is altered, and a test surface external to the instrument is used. This may be quite separate, T, or it may be carried at the end of a short arm clamped to the case. It fills the lower half of the circular field seen through C, which is a glass plate silvered over the upper part of its surface, while

the lower part is clear The tilting surface S, by means of the mirror M, fills the upper half of the field formed of the silvered part of C To increase the range of the instrument neutral filters of known transmission factors are mounted in a disc framework which moves over the aperture of the instrument so that the brightness of the external surface is reduced by the factor, τ_1, τ_2, τ_3, etc. It follows that when the filter of transmission τ is in use, the reading of the instrument has to be multiplied by the factor $1/\tau$. Alternatively, a standard surface of low reflection factor may be used, as in the case of the ordinary Trotter photometer

The Harrison photometer (22) is designed mainly for street lighting work The tilting screen method is employed for varying the brightness of the comparison surface, but a distinctive feature of the instrument is its use of the flicker principle (see p. 250) for eliminating colour difference. A rotating test surface which has the form shown in Fig 165 of Chapter VIII (p 262) is mounted at an angle of 45° with the horizontal, and is pneumatically set spinning in front of a mirror, which reflects the comparison surface. The system is shown diagrammatically in Fig. 228, and it will be seen that the field of view at A is alternately occupied by B and by the image of the tilting surface in M

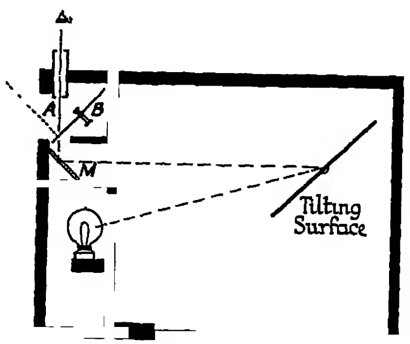


FIG 228 —The Harrison Photometer

Photometers depending on change of Candle-power of the Comparison Lamp. — Probably the earliest form of illumination photometer was that of Preece (23), who used as his comparison source a glow lamp in circuit with a variable resistance, so that a photometric balance could be obtained by varying the current through the lamp The candle-power variation with change of current was known by previous calibration The same principle is used by H T. Harrison for his portable "lightometer" (24), by J T Marshall in the "luminometer" (25), and by W J Dibdin in the "hand photometer" (26). A scale attached to the sliding rheostat which is used for varying the current gives a direct indication of the illumination in each case The same system, but with a benzine lamp as source, was originally used by A. Wingen (27), but in later forms of his instrument, as modified by H Kruss, the tilting screen and cam device of the Trotter photometer has been substituted (28) A Blondel uses an electric lamp with a straight filament, in front of which is placed a variable slit so that the length of filament exposed can be varied at will (29)

Photometers depending on the use of Diaphragms and other Devices.—From the descriptions given in Chapter VI it will be seen that the Mascart, Blondel and Broca (note (89), p 192) (30), Brodhun (p. 178), Martens polarisation (p 173), Bechstein (p. 251), and other photometers can readily be adapted to the measurement of illumination (31). The instrument of E. J. Houstoun and A. E. Kennelly (32),

and others depending on visual acuity, need not be described here. The diaphragm method of control of the standard brightness, used in the Mascart and Blondel instrument, has been the basis of several illumination photometers ([33]), notably a very portable instrument, termed the lumeter, which is shown in Fig 229 ([34]). The lamp L,

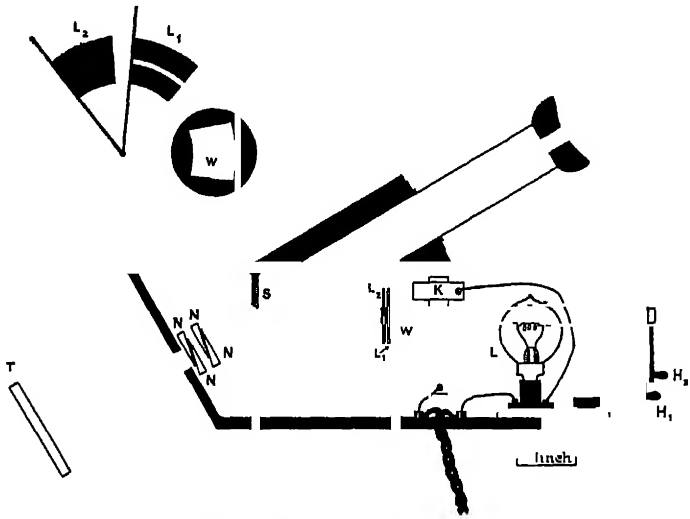


FIG 229 —The Lumeter (First Form)

controlled by a press key K, is enclosed in a whitened box and illuminates a depolished opal glass window W. This window in turn acts as a source illuminating the outer opaque white annulus of the screen S, while the test surface T is seen through the central transparent part of S In front of W move two opaque screens L_1 and L_2, which have the forms shown in the detail at the top of the figure, and which are rigidly attached to independent shafts and are separately controlled by the handles H_1, H_2 The width of the band in L_1 is exactly one-tenth of the radial width of W, so that as L_1 moves across W the area of the latter which illuminates S is reduced from a to $a/10$ As L_2 then moves across, the area is further reduced from $a/10$ to 0 It follows that, if the surface of W be uniformly bright and the screens carefully made, then, assuming that a photometric balance is obtained with W completely exposed when the illumination at T is 1 foot-candle, as L_1 moves across, the illumination of T which gives a balance decreases at a uniform rate to 0 1 foot-candle The subsequent movement of L_2 gives a second uniform range between 0 1 and 0 foot-candle, so that by attaching pointers to H_1 and H_2 the instrument may be made to give the illumination of T, from 1 to 0 foot-candle, by direct reading on one of two uniform scales A precaution which it is important to notice in the use of this instrument is the necessity for having H_1 at its minimum position whenever a reading is being made with H_2. Similarly, it is necessary to have H_2 at its maximum position whenever readings are being taken with H_1 The range of the instrument is extended upwards by means of neutral filters N, N Each of

these consists of a double wedge of neutral glass, so that the transmission factor can be adjusted to 10 and 1 per cent. exactly. Either or both of these screens can be introduced at will, so that the upper limit of the instrument is 1,000 foot-candles

In a slightly modified form of this photometer, shown in plan in Fig 230 (35), the lamp is contained in a cylindrical enclosure, and

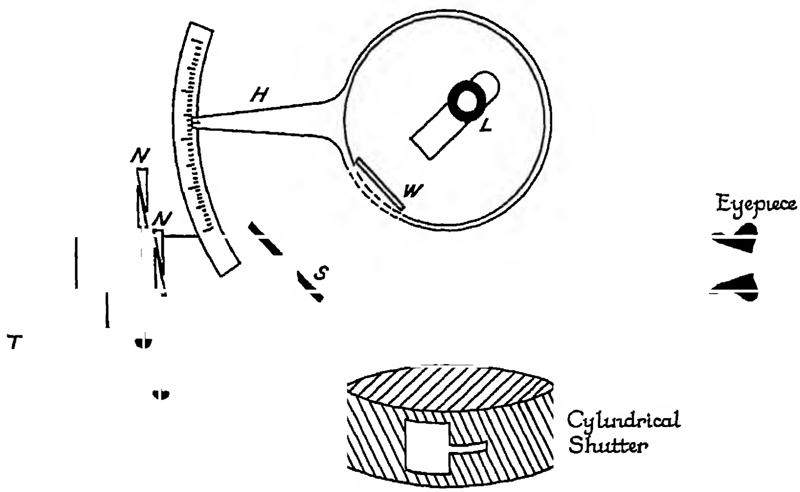


FIG 230 —Second Form of Lumeter

instead of the screens L_1 and L_2 an opaque cylinder having an aperture of the shape shown in Fig 231 moves across the window A single handle with a scale of the kind shown can then be used Unless the aperture is very carefully cut, the readings in the neighbourhood of 0 2 foot-candle, where the change of scale takes place, are liable to be inaccurate, and for this reason a slot with three steps has been adopted in the latest models The instrument is calibrated by means of a standard lamp, placed so as to give a known illumination at T, the lamp L being moved back and forth until a photometric balance is obtained when the true reading is given by H_1 or H_2

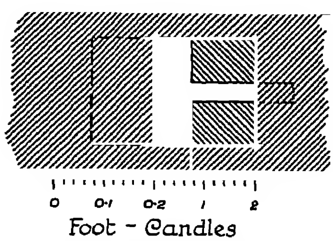


FIG 231—Modified Form of Lumeter Screens and Window.

In some types of portable photometer a neutral wedge is used instead of a diaphragm of variable opening (36)

The "Foot-Candle Meter."—Although scarcely to be classed as a "photometer," and probably better described as an "illumination gauge," the instrument shown in Fig. 232 is convenient for rough measurements of illumination where the use of a photometer is impossible or inconvenient (37) The lamp L illuminates the under side of a long screen S, partly by direct light and partly by reflection

from a strip of mirror M, which is tilted as required when the instrument is re-adjusted from time to time. S consists of a strip of opaque white paper with a row of Bunsen spots in it Since the brightness of these spots decreases progressively from the end of the screen which is nearest to the lamp, any given illumination of the screen from above will cause one of the spots to match its surroundings in brightness, while all those on the right appear bright, and those on the left appear dark. An empirically graduated scale printed on the screen enables the illumination to be read directly in foot-candles The lamp is supplied with current from a dry battery, the rheostat R being turned until the correct indication is given by the voltmeter V. To extend the range of the instrument downwards, R may be increased until the voltmeter needle indicates a division marked 1/10. The candle-power of the lamp is then one-tenth of its normal value, and the instrument therefore measures from 0 1 to 4 foot candles instead of from 1 to 40, as when the full current is passing through the lamp [38]. An instrument of this kind can be used for making instantaneous measurements of a rapidly fluctuating illumination (see p 185)

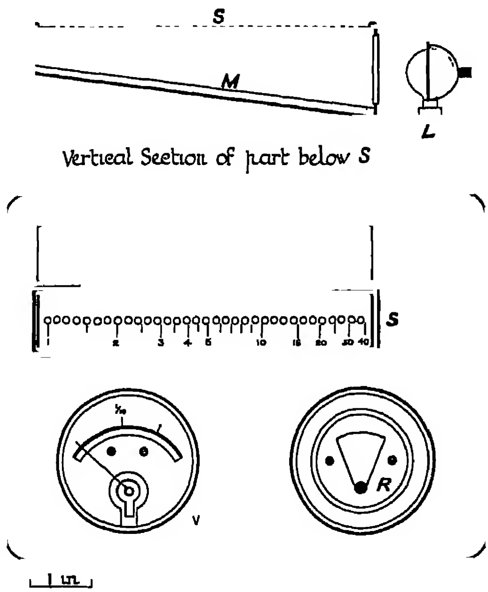


FIG 232 —The Foot-candle Meter

Illumination Measurement by Physical Photometry.—Some of the types of physical photometers described in Chapter XI have been applied to the measurement of illumination [39] In particular, various kinds of light-sensitive chemical papers have been used for measuring the daylight illumination in rooms [40]. The difficulties introduced by the special nature of the sensitivity curves of such instruments make this method valueless, however, except for obtaining either the *relative* values of illumination at different points in a room, or the *fluctuations* of illumination which take place at any particular point. In these cases the measurements are comparable so long as they refer to light of identical spectral distribution. It is to be remarked, however, that the spectral distribution of daylight is very far from constant [41]

Precautions in the Use of Illumination Photometers [42].—From the descriptions of illumination photometers which have been given in the preceding sections it will be noticed that almost without exception these instruments depend on a more or less arbitrary scale for indicating the value of the illumination which is being

measured Even in instruments of the Weber type, which nominally depend solely on the inverse square law, the distance at disposal for the movement of the lamp is so restricted that, unless very great care is taken in the design and manufacture of such instruments, quite appreciable errors are likely to arise from the use of scales based absolutely on this law Even supposing this difficulty overcome, the impossibility of securing absolute symmetry on the two sides of the photometric device still makes the use of a substitution method indispensable, quite apart from the impossibility of accommodating a standard lamp in the instrument

It follows that an illumination photometer should be standardised before every period of use, either to ensure that the instrumental constant is as near unity as may be necessary in the work which is to be undertaken, or, failing this, to determine the correct value of the constant so that a correction factor may be applied to the instrument readings The latter scheme is inconvenient, and provision is made in most modern illumination photometers for easy adjustment of the lamp to give a constant of unity Change of constant with lapse of time may be due to (*a*) reduction of reflection factor of the test surface due to dirt or discoloration, (*b*) ageing of the lamp inside the instrument, (*c*) change, due to dirt or other causes, in the transmission factor of glass plates or other devices through which any part of the light has to pass; (*d*) change in the voltage of the battery supplying current if the standard source is an electric lamp. Of these four causes, the first three may be assumed to take place progressively and at a comparatively slow rate, the fourth will be dealt with in detail later ([43]).

It is necessary, then, to provide a known standard illumination with which to check the photometer before commencing work with it Quite the most accurate means of doing this in a photometric laboratory is to mount a standard lamp on the photometer bench in the usual way (see p. 151), and to place the test surface of the photometer in such a position on the axis of the bench that it is normal to the incident light, while its plane passes through the mark on the bench which indicates that the illumination has the value at which a check reading of the instrument is desired, *e g*, 10 metre-candles or 1 foot-candle ([44]) The instrument pointer is set to this reading, and the lamp adjustment is altered until a photometric balance is obtained. It is important that, in the case of an instrument with a detached test surface, the actual surface to be used in subsequent work should be used for the standardisation

The process of calibrating an instrument is carried out in a similar manner, except that the lamp adjustment is arranged to give a minimum error over the greater part of the scale, or its more important part, and the actual magnitude of the error at every part of the scale is then determined In photometers which include neutral filters or similar devices for extending the range of the instrument, the accurate determination of the transmission factors of these filters is included as part of the calibration of the instrument. This determination may be carried out by checking the instrument carefully at a convenient value of illumination E, say, with no filter in use. The filter (of nominal transmission factor τ) is then inserted

and the illumination of the standard surface is raised to E/τ. The difference between the old and new readings of the instrument gives at once the error in τ In portable instruments errors of scale reading amounting to $\pm$ 2 to 3 per cent over the working parts of the scale are regarded as admissible, since this is within the accuracy aimed at in work carried out with these instruments.

In cases where a photometer bench and sub-standard are not available for checking a portable photometer it is necessary to use some form of portable standard or "calibrator" The apparatus designed for this purpose in connection with the Macbeth illuminometer has been described already (see p 347 and Fig. 224) In the case of the Sharp-Millar photometer a short tube containing an aged and standardised lamp fits over the end of the transmitting test surface, and is registered in such a position that it gives the latter a known definite illumination

It is now necessary to consider the fourth of the sources of error in illumination photometers which were enumerated above, *viz*, the alteration of battery voltage. An essential part of every illumination photometer (disregarding acuity and physical instruments) is its comparison lamp, and in all modern instruments this is an electric glow lamp supplied with current from a portable battery ([45]). The requirement as to portability results in two practical difficulties of design and sources of error in use. The first of these, and the one more easily remedied, is the possibility of an imperfect contact somewhere in the electrical circuit. The great importance of this arises from the fact that a portable battery is essentially of a low voltage, one of 2 or 4 volts being employed in most cases The remedy is extremely simple All permanent contacts should be soldered, terminals making semi-permanent connection should have clean surfaces and should be screwed down as tightly as possible, and the lamp should have a screw cap and be firmly seated A screw cap sometimes exhibits a tendency to become loose in its socket after a short period of use Its firmness should be ascertained at frequent intervals In many instruments, especially those of the most portable type, where the battery has to be as small as possible, a switch is provided in the lamp circuit so that the lamp is alight only while the measurement is actually being made A press switch must be very firmly held during a reading and the contacts must be kept absolutely clean A knife switch, such as that used in the Trotter photometer, is much to be preferred A switch of more robust type, such as an ordinary tumbler, may be put in parallel with the press switch, for use when a large number of readings have to be taken in a short period. A small ammeter in the lamp circuit is useful for giving an immediate indication of contact trouble, and in this respect it is preferable to a voltmeter ([46]) A faulty contact will frequently manifest itself unmistakably by a flicker in the photometer field, or by sudden changes of reading quite outside the ordinary experimental error

The second source of trouble due to the portability of the battery is the gradual diminution of voltage as the battery is discharged. In some instruments no attempt is made to regulate the current, the lamp circuit being simply connected straight across the battery terminals. The instrument reading therefore depends on the

battery voltage, and will be in error by 4 per cent or more * for every 1 per cent. of difference between the actual battery voltage and the voltage which was supplied to the instrument when it was being calibrated. In the case of a lead accumulator of fairly large capacity and of suitable design, the voltage during discharge is very constant after the first half-hour, as shown by the discharge curve of Fig 233.

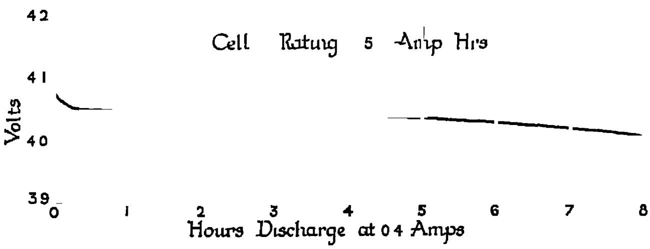


FIG 233 —Discharge Curve for a Lead Accumulator

The capacity of the battery should, however, be such as to give a total period of discharge, with the current taken by the lamp, of at least twelve hours, *i e*, a 0 5-amp lamp requires a battery of at least 6 amp-hours capacity, while it should not be used for more than half this period, *i e*, the total time during which it is actually supplying current should not exceed six hours The preliminary excess voltage after charging must be avoided by discharging the cell for at least half an hour at its normal rate before it is connected to the photometer. It should not be forgotten that accumulators become discharged gradually even when not used

In accurate work an ammeter or voltmeter should always be used in the lamp circuit, so that the current may be regulated to the correct value (47). In the case of small lamps, such as are generally used in portable photometers, the rate of change of candle-power with current or voltage is higher than in the case of normal tungsten filament lamps, a change of 1 per cent in current may cause a change of as much as 10 to 11 per cent in candle-power For this reason the accuracy of setting is higher with a voltmeter than with an ammeter When a voltmeter is used, it should be so connected that it is always in circuit when the lamp is alight

In some instruments dry cells are used in place of accumulators The discharge curve for these is much less flat, so that frequent regulation of the lamp current is a necessity in such instruments (48) Their freedom from the unpleasant consequences arising from the accidental spilling of acid, apparently inevitable at some time or another when lead accumulators are used, give them, however, a considerable practical advantage

An ingenious method of voltage regulation which has been suggested (49) depends on the difference between the voltage/candle-power characteristics of a tungsten lamp and an under-run carbon lamp (see Fig 234) Provision is made for substituting for the standard surface in the photometer a surface illuminated by a combination of an under-run carbon lamp with a blue filter This

* The ordinary volt/c p characteristic of tungsten lamps of normal type does not apply to very low voltage lamps of the type used in portable photometers

lamp is in parallel with the tungsten comparison lamp, and the voltage of the two together is adjusted by means of a resistance until the correct value is obtained, as indicated by a brightness balance in the photometer. This device was originally suggested for the provision of a portable light standard ([50]).

There is one source of error in illumination photometry which so far has not been considered, *viz.*, colour difference. When the

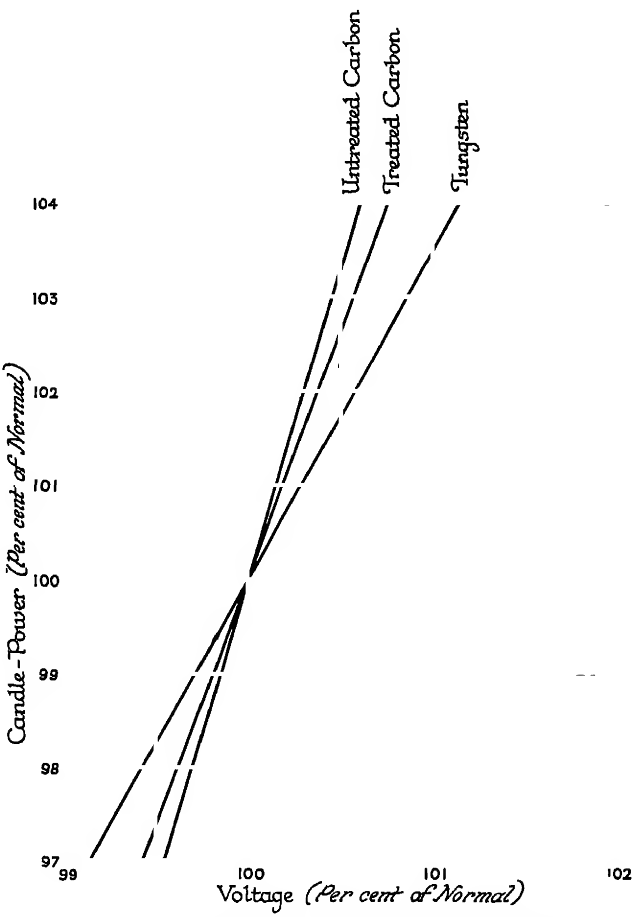


FIG 234 —The Characteristic Curves for Carbon and Tungsten Lamps

comparison source is a metal filament glow lamp operating at or near its normal efficiency, little difficulty is generally experienced due to colour difference when artificial illumination is being measured, except in the case of lighting by arc lamps or mercury vapour lamps ([51]) Further, the accuracy aimed at is usually not so high as in the case of candle-power photometry, so that a small colour difference does not invalidate the results obtained. For daylight measurements, however, the difference of colour is enormous, and it is usual to employ a colour filter to obtain an approximate match ([52])

Since the transmission factor of this filter can be accurately determined in the laboratory, all that is necessary is to multiply the readings of the instrument by a correction factor when the filter is in use. Since the transmission factor varies with the colour of the light passing through the filter (see p. 249), it follows that a blue filter should be inserted in front of the tungsten lamp, as the light given by this is of constant spectral distribution, while daylight is not. Unfortunately this reduces the brightness of the comparison surface, which is generally already at least as low as is desirable for accurate work, so that it has become customary to employ a yellow filter in the path of the light coming from outside the instrument, and to ignore the variation of transmission factor as between, say, sunlight and the light from a blue sky.

The Method of Measuring Illumination.—Whatever be the instrument employed, the general principles of the method of making a measurement of the illumination at a point are the same Unless otherwise specified, it is usual to assume that the illumination measured is that of a horizontal surface at the point under consideration. The test surface must, therefore, be put as nearly as possible in the horizontal position. It generally happens that the greater part of the illumination is due to light which is incident at an angle of 45° or less, so that a tilt of 2° in any direction on the test surface does not introduce an error of more than $3\frac{1}{2}$ per cent in the value assigned to the most oblique component When, however, this is not the case, and a considerable part of the light is incident obliquely, more care must be exercised in the exact positioning of the surface With a little care it is usually possible to adjust the level of the surface to within 2° by eye. A self-levelling device has been described by W F. Little ([53]) Occasionally it is of more importance to know the illumination of a vertical or a sloping surface (*e.g* , the wall of a picture gallery, the desk of a library, *etc*), and in such cases the position of the test surface should always be specified, *i e* , its angle of slope and the direction of its normal.

Frequently the exact position of the surface is determined by the nature of the illumination problem under investigation For example, the bed of a lathe, the plate of a sewing machine, the surface of a desk, *etc* , define the points where illumination is needed and where it must be measured. When this is not the case, however, and the illumination is treated generally, the test surface is placed at some definite height above floor level agreed upon as the height of the " working plane " It is regrettable that this height has not been the subject of more general agreement ([54]) Ordinary table height is about 2 feet 6 inches, but a very usual height for the working plane on the Continent, and a very convenient height for work which is carried on while standing, is the 1-metre level Sometimes the floor level is taken when there is no obstruction from objects in the room The *average* illumination is, clearly, the same whatever be the height of the plane (below the level of the lamps), except for the small amount of light absorbed by the extra strip of wall, but the *distribution* of the light may be very different, especially when the light sources are low

When the test surface has been placed in position, the observer and photometer must be arranged so that (*a*) the surface fills the

FIG 235 —The Method of Measuring Illumination

[*To face p* 360

field of view of the instrument, (b) the angle at which the surface is viewed is not too oblique (see p 345), and (c) no shadows are cast on the surface by either the observer or the instrument The importance of these requirements has been dealt with at length in an earlier part of this chapter Generally speaking, the angle which the line of view makes with the normal to the test surface should not exceed 20° to 30°. In the Trotter photometer special provision is made for ensuring that this condition is complied with The test surface F (Fig 226, p 350) has a single slot, while the tilting surface C has two small black pointers, one on each side, in the line of the shaft on which it turns The two surfaces F and C are so arranged that, when the angle of view is 20°, by moving the head a little sideways one of these pointers may be seen occupying each end of the slot as shown in Fig. 226. With most instruments having an attached test surface the difficulty does not arise. The third of the requirements mentioned above, *viz*, avoidance of shadow cast by observer or photometer, is difficult of fulfilment with some instruments, but it is nevertheless of considerable importance, especially in the case of an illumination by diffused light, or when the number of sources contributing to the illumination is large, so that light reaches the test surface from all directions In the case of the Sharp-Millar photometer, the test surface is above the level of the observer and the remainder of the instrument With instruments employing a reflecting test surface the best that can be done is to remove the photometer and the observer to as great a distance as possible from the test surface while still fulfilling requirement (a) above The solid angle obstructed is then as small as possible Judicious choice of position, when some sources do not contribute much to the total illumination, will also assist materially in reducing errors arising from this cause.

There is one special problem in illumination measurement which is of sufficient importance to need separate notice This is the measurement of street illumination It is a question long debated whether the illumination of a vertical or a horizontal surface is the more important, and therefore a fairer criterion of the efficiency of any system of street lighting ([55]) This is a question of illumination engineering, the discussion of which lies outside the scope of this book, but as the balance of opinion seems to lie rather in favour of the horizontal position, it is necessary to point out that in this case the light is frequently incident on the test surface very obliquely, so that an error of as little as 1° in the level of the card may produce a marked inaccuracy in the results, while lack of perfect diffusion in the test surface becomes very important. For this reason the illumination of the horizontal plane is frequently obtained by calculation from measurements of direct illumination The test surface is tilted so that the only light it receives is that which reaches it normally from one of the sources contributing to the illumination at the point in question The direct illumination thus measured is multiplied by the cosine of the angle which the light makes with the vertical, and the illumination of the horizontal surface due to this single source is thus obtained The total illumination is found by summing the components thus measured for each of the different sources contributing to the illumination at the point of observation.

A special form of test surface, designed to weight equally the light received from all directions, has been described ([56])

The Distribution of Illumination.—The method of calculating the illumination at different points on a given plane (*e g*, the working plane in a room or workshop, the road surface in a street, *etc*) due to the direct light received from a number of sources of known candle-power distribution, and placed in known positions with respect to the plane, has already been described (see p 97)

The carrying out of such calculations and the corrections to be made to the results on account of light reflected from surrounding objects—particularly the walls and ceiling in the case of a room ([57])—or on account of the interposition of obstructions, is the function of the illumination engineer, to whom belongs also the duty of designing installations on the data provided for him by the photometer For a description of the details which have to be considered in the design of a lighting installation, such as the illumination intensity required for a particular process, the use of shades and reflectors to avoid glare, the performance of the light sources commercially available, *etc*, reference must be made to some book dealing with illumination engineering, several of which are named in the bibliography at the end of this chapter

In an actual installation the distribution of illumination can be determined experimentally by means of measurements with an illumination photometer at as many points as may be desired on the working plane ([58]) These measurements include the effect of the reflected light above referred to, and if they are plotted on a plan of the area under investigation the points of equal illumination

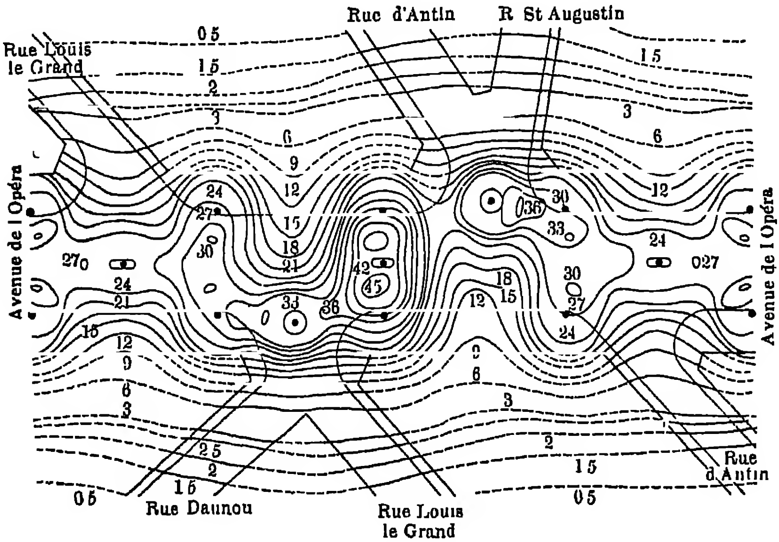


FIG 236 —Iso-lux Curves for the Avenue de l'Opéra (Paris) Figures indicate metre-candles

can be joined by curves, so that an iso-lux diagram is obtained (see Fig 236) ([59])

Although either an illumination curve or an iso-lux diagram

(see p 98) may be used to obtain a mental conception of the effect of a given system of light sources of known distribution, it is, nevertheless, desirable to have some definite figure by which the illumination performance of two lighting systems may be compared This is analogous to the comparison of the light-giving power of two sources in terms of their total luminous flux, instead of by the more detailed but comparatively cumbrous polar diagrams.

The Average Illumination.—Various suggestions have been made for a basis of comparison of different illumination systems On the whole the average illumination, or, what is equivalent to it, the average flux per unit area (generally expressed in lumens per square metre or per square foot), has been most generally adopted for indoor illumination This means that for a direct lighting system, where the reflection from walls and ceiling can be neglected to the approximation desired, the average illumination in metre-candles is equal to the total mean lower hemispherical candle-powers of all the sources in the room, multiplied by 2π and divided by the area of the room in square metres ([60]) In systems where the reflected light is considerable, the ratio of the total flux reaching the working plane to the total flux given by the sources is termed the "utilisation factor" or "coefficient of utilisation" of the installation ([61]) Thus the average illumination is equal to uF/A, where u is the utilisation factor, F the total flux from the sources, and A the area of the working plane.

The Variation Factor.—The description of a system by the average illumination gives no indication of the distribution of illumination over the room If there be only a few sources of high candle-power, the illumination may be concentrated in the regions underneath a source, leaving the other parts of the room in comparative darkness For this reason a second figure of merit, giving the ratio of maximum to minimum illumination in the useful area of the room, has been proposed as an addition to the average illumination This figure is termed the "variation range" *

Trotter's "Characteristic Curve."—Another method of describing an illumination system has been called by Trotter ([62]) a characteristic curve This is a curve having for its abscissæ areas, and for ordinates the values of the minimum illumination over those areas Thus, referring to the illumination diagram shown in Fig 46 (p. 97), it will be seen that the illumination at all points within a circular area of radius 5 metres equals or exceeds 2 1 metre-candles The area of this circle is 78·5 square metres, and therefore the point (78 5, 2 1) is a point on the characteristic curve which is shown in Fig. 237 Since the abscissæ of this curve represent areas, the mean ordinate of this diagram, as far as any given ordinate, is equal to the average illumination over the circle having an area represented by the corresponding abscissa Thus the average illumination over a circle 250 sq metres in area is equal to the mean ordinate of the curve to the left of BC, *i e*, to 1 68 metre-candles

The Measurement of Diffusion.—It is sometimes of importance in

* Occasionally "diversity factor" The ratio of the maximum (or the minimum) illumination to the average illumination is termed the "variation factor" See Report of Committee on Nomenclature, Illum Eng Soc N Y, Trans, 13, *1918*, p 517, C I E, Proc, 6, *1924*, p 178

practical lighting problems to know how much of the total light which reaches a certain point in a room is "direct," *i e*, received from the source without reflection from the walls or ceiling, and how much is "diffused" This can be determined by placing a small

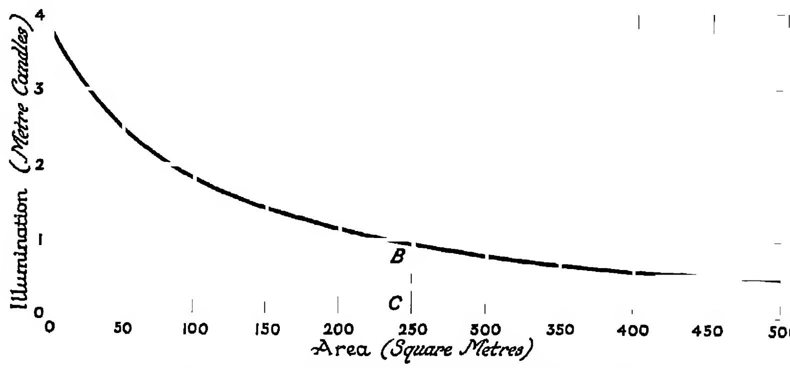


FIG 237 —The "Characteristic Curve" of Illumination

opaque screen at a short distance from the point, and in such a position that the umbral part of its shadow, cast by the light source illuminating the point, completely covers the test surface of an illumination photometer placed at the point If E be the total illumination, and E' the illumination when the surface is screened from the direct light, the fraction $100(E - E')/E$ may be termed the percentage of "direct" illumination, and $100\,E'/E$ the percentage of "diffused" or "indirect" illumination If more than one source contribute to the illumination at the point, a measurement must be made with each source screened in turn. The direct component is then $\Sigma(E - E')$, and the diffused component $E - \Sigma(E - E')$

A special piece of apparatus has been designed as an auxiliary to an illumination photometer for the purpose of making measurements of this kind (63).

The Measurement of Daylight.—While the artificial lighting of an area may be described quite definitely by measurements of the illumination at various positions within that area, since the sources of the illumination are, within practical limits, under control and constant from hour to hour, in the case of natural lighting the conditions are altogether different Owing, no doubt, to the wonderful ease with which the eye can adapt itself to quite large changes of brightness without any conscious effort, it is seldom realised how large and how rapid are the fluctuations which occur under ordinary conditions of daylight illumination. Large variations, such as those which have to be allowed for by the photographer, are perceived when the attention is specially directed to them, but in the majority of cases the degree of illumination is so much above the minimum required for comfortable vision that variations of 25 per cent or less are quite unnoticed It is only at dusk, when the lower limit of comfort is approached, that differences of this order become apparent Fig. 238 shows the daylight illumination of a horizontal surface in the open as actually measured on two typical equinoctial days, direct sunlight being shielded from the test

surface ([64]). It will be seen that in the upper curve variations of 25 per cent occurred within a period of five minutes, and yet to any one relying solely upon the eye for measurement the illumination appeared to be quite constant

It is clear that the daylight illumination at any point, indoors as well as out of doors, depends entirely on the brightness of that

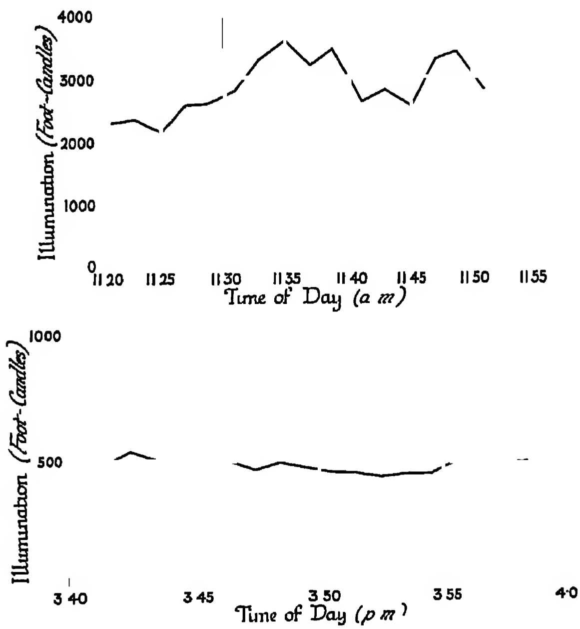


FIG 238 —The Fluctuation of Daylight Illumination

part of the sky from which the point in question receives its light Hence it follows that a measurement of the daylight illumination in a room, taken at any particular instant, is quite destitute of any permanent value unless related to the brightness of the sky at the same instant.

Daylight Factor Sill Ratio.—Several different methods have been used for expressing the relation between the daylight illumination at any point and the simultaneous value of the sky brightness, depending upon whether the basis of comparison taken is the average brightness of the whole sky or the brightness of that particular region which is most effective in illuminating the point considered

In the former case the illumination of a horizontal surface at the point in question is expressed as a fraction of the simultaneous illumination of a horizontal surface placed in the open with a completely unobstructed horizon ([65]). This fraction (usually written as a percentage) is termed the " daylight factor " for the point, or sometimes the " window efficiency " ([66])

It will be clear from what has been said in Chapter IV (p 102) that the illumination of a surface exposed to a complete hemisphere of sky is equal to πB if the sky be of uniform brightness B ([67]). If, however, the sky be not uniform in brightness, the illumination will

have an average value in which the sky brightness in the neighbourhood of the zenith is heavily weighted *

In order to measure the illumination of the open surface some position with an unobstructed horizon is necessary Sometimes the roof of the building in which the indoor measurements are being made is found to be suitable, but it is not infrequently difficult to obtain easy access to a spot which has an adequately unobstructed view of the whole sky Fortunately, as has been said above, the horizon contributes least to the total illumination, and for a uniform sky the percentage reduction due to obstructions on the horizon having an angular height θ is easily found to be $50(1 - \cos 2\theta)$,† which, as shown in Fig 239, is less than 7 per cent. so long as θ does not exceed 15°. In cases where a sufficiently open position cannot be found, the best that can be done is to use the brightness of the sky at the zenith instead of the average brightness of the whole sky This may conveniently be done by means of a tubular attachment devised by A P Trotter ([68]), and shown in section in Fig. 240 The tube T, which is blackened inside, is placed over the test surface P It has a side tube E, through which the surface is observed and its brightness measured The upper end of T bears an opaque diaphragm A, the opening of which is calculated, in relation to its height from the surface, to give the latter a brightness which is some definite fraction of the brightness it would have if

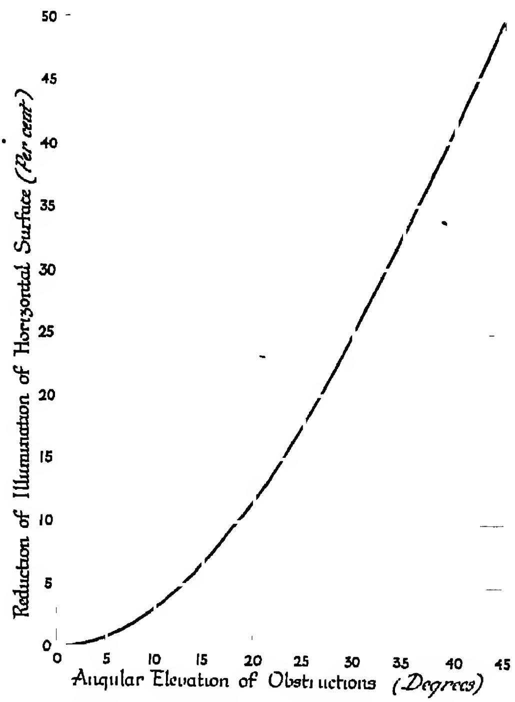


FIG 239 —The Effect of Obstructions on the Horizon

* It can easily be shown, for example, that if the sky be of brightness $B \cos \theta$ where θ is the angular distance from the zenith, then, while the true average brightness of the sky is $B/2$, the illumination of the horizontal surface is $\pi(2B/3)$ For the average brightness equals (see Fig 49, p 100) $(1/2\pi r^2)\int_0^{\pi/2} B \cos \theta \; 2\pi r \sin \theta \; r d\theta$, while the illumination of the surface is $\int_0^{\pi/2} (B \cos \theta/r^2) \cos \theta \; 2\pi r \sin \theta \; r d\theta$

† $100 \int_0^{\theta} \sin \theta . \cos \theta \, d\theta \Big/ \int_0^{\pi/2} \sin \theta . \cos \theta \, d\theta$ See, e g, J. Rheinauer, "Grundzüge d Photometrie" (Halle, *1862*), p 16.

exposed to a complete hemisphere of sky of the same brightness as that of the actual sky above the diaphragm. Assuming a sky brightness B, the illumination of the open surface is, as has been said, πB, while if the area of the diaphragm be a and its distance from the surface h, the illumination of the covered surface is aB/h^2, so that the fraction in this case is $a/\pi h^2$. If the aperture be round and of radius r, the fraction is clearly r^2/h^2 It is important to ensure that no light shall be admitted at E when the measurements are being made A "daylight attachment" embodying this principle has been designed for use with several of the portable photometers above described In these the tube is rigidly attached to the instrument, so that no stray light can be admitted. Diaphragms of various sizes for use under different conditions of sky brightness are provided

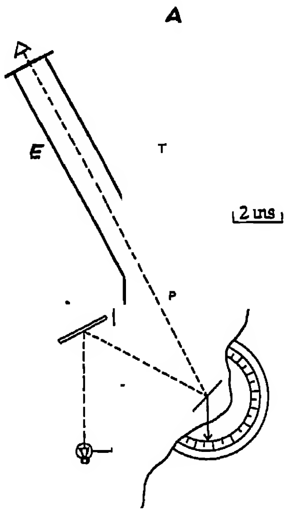


FIG 240—The Trotter Daylight Attachment

Although the direction in which the daylight attachment is pointed makes no difference when the sky is uniformly bright, under practical conditions, since the zenith brightness is the most heavily weighted in the average giving the illumination of an open surface, it is best to use some part of the zenith sky for the measurement, so that as close an approximation as possible to open surface measurements may be obtained The variation of sky brightness on a sunny day with a blue sky and white cumuli may be strikingly seen in the Trotter daylight attachment, the brightness of the test surface suddenly increasing by perhaps 100 per cent as a white cloud moves across the zenith

The rapidity with which the sky brightness is liable to change makes it necessary to ensure that the indoor and outdoor measurements are made as nearly as possible at the same instant This can be achieved fairly simply by arranging that the outdoor measurements shall be made every minute, while the indoor measurements are made only at exact minutes If the two observers synchronise watches before starting work, and each notes the times against his observations, a subsequent comparison of the two records enables each of the indoor values to be expressed in terms of the outdoor value taken at as nearly as possible the same instant.

Although the method of expressing daylight factor on the basis of the average sky brightness possesses some advantages, the non-uniformity of the sky brightness on most days ([69]) introduces a considerable element of uncertainty into the values thus obtained, and probably the most convenient method of expressing the daylight illumination at any point in a building is by reference to the simultaneous brightness of that part of the sky which is most effective in illuminating the point considered When part of the sky is visible from the point, there is no doubt as to the region in which the sky brightness is to be measured In other cases this region may be

taken as that which illuminates the lightest and most extens surface visible from the point where the daylight factor is bei determined

When this system or method of expressing daylight factors adopted, the factor is the ratio of the indoor illumination to t illumination of an imaginary horizontal surface receiving light fr a complete hemisphere (or quartersphere) of sky having a unifo brightness equal to the brightness of the particular part of the sl measured. It follows that if this latter brightness be B, while t indoor illumination is E, the daylight factor is $E/\pi B$ or $2E/\pi$ according as the hemisphere or quartersphere is used. The latt form of the factor is termed the "sill ratio," since a surface plac on the sill of an unobstructed window receives light from a quart sphere of sky.

With some portable photometers, particularly those having detached test surface, it is quite convenient to make a measureme of the illumination E at the point considered, and then to direct tl photometer towards the visible patch of sky and make a secon measurement E_s, regarding the sky as a test surface. If the tran mission factor of the window through which the measurement is ma be τ, while the reflection factor of the photometer test surface is ρ the daylight factor (sill ratio) is clearly $2\tau E/\rho E_s$.

Several very convenient instruments, called "relative phot meters" [70], have been designed for enabling the daylight fact to be obtained directl without the necessity fo using any auxiliary ligh source such as is require with an ordinary portabl photometer.

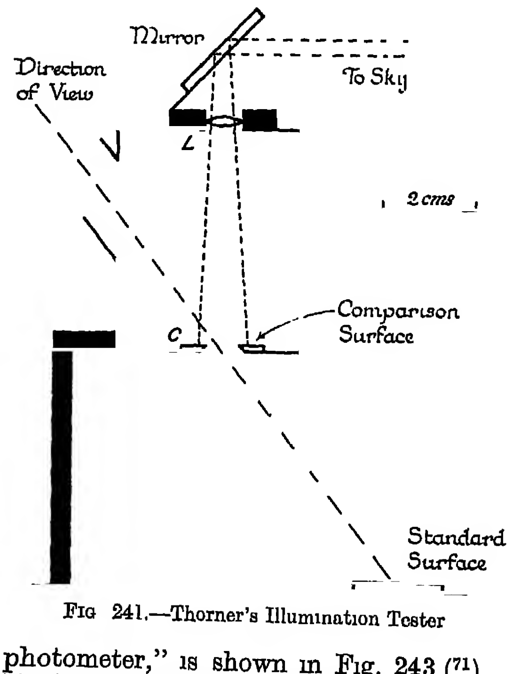


Fig 241.—Thorner's Illumination Tester

In the Thorner instru ment (Fig 241) the len L forms at C an imag of the portion of sky visible through the win dow towards which th mirror is directed. Th brightness of this imag is varied, by means of a diaphragm over L, until it matches the portion of the standard surface seen through the aperture in C. Weber's relative photo- meter (Fig 242) is similar in principle.

A simple instrument of this kind, the "hemera- photometer," is shown in Fig. 243 [71] The light metal tube T is blackened internally and fitted with diaphragms to eliminate stray light. When the instrument is in use the light from the sky passes through the aperture in the iris diaphragm I and illuminates the opaque matt white disc D. This disc has an aperture A, the edges of

which are sharply bevelled, and the eye at F views a detached test surface through A. This detached test surface is placed at the point

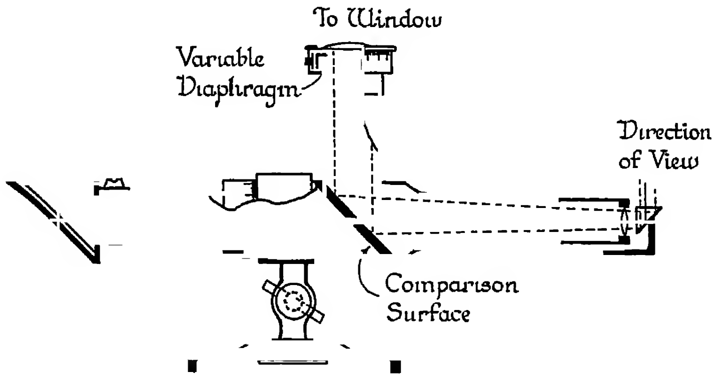


FIG. 242 —Weber's Relative Photometer

for which the daylight factor is to be measured, and photometric balance between the brightness of this surface and that of the disc D

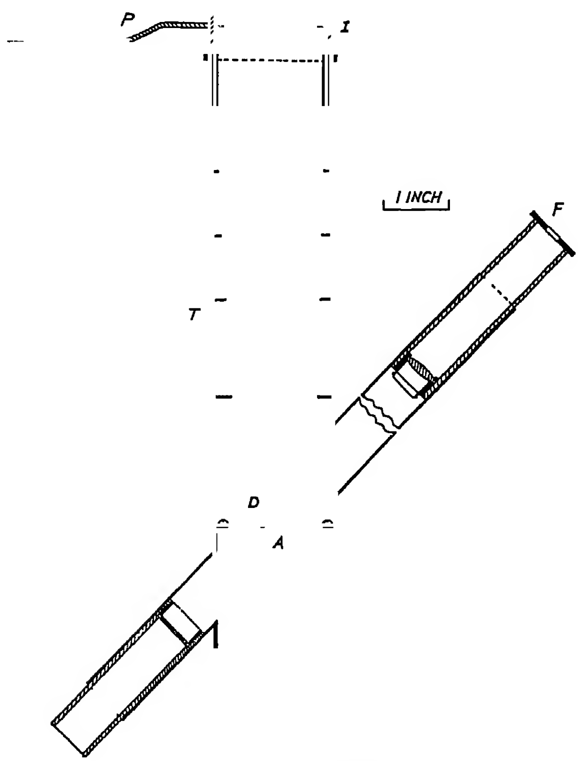


FIG 243 —The Hemeraphotometer

seen surrounding it is achieved by varying the aperture of I. P is a lever for effecting this variation, and carries a pointer by means of

which the daylight factor can be read directly on a previously calibrated scale. If D and the detached test surface be of the same material, there is no correction for the reflection factor of the surface but the correction for transmission of the window must still be made.

J Classen has used a system of mirrors by means of which it is possible to measure the illumination at any point in a room in terms of the illumination of some other arbitrarily chosen point ([72]). A measurement of this kind sometimes gives useful information as to the distribution of light in a room, but the same information can be obtained from values of the daylight factor measured in the manner described above

The Calculation of Daylight Factor.—The daylight factor at a given point in a building may be calculated with considerable accuracy by regarding the area of sky directly visible from the point in question as a diffusing surface of uniform brightness B, and calculating the illumination at the point due to this surface If this illumination be E, the daylight factor (sill ratio) is, as has been said, $2E/\pi B$ When the patch of sky is of some regular shape, *e g* , circular or rectangular, the value of E may be calculated from one of the formulæ given in Chapter IV (p 106) ([73]) In general, however, the shape of the visible sky is quite irregular, and recourse must then be had either to approximate formulæ or to some graphical method

In Pleier's formula ([74]) the light from the patch of sky is treated as if it all came from a point in the centre of the patch, and the results are therefore somewhat inaccurate if the patch be large A theoretically correct graphical solution has been given by P J and J. M Waldram ([75]) The outline of the patch of sky visible from a point is traced on the diagram shown in Fig. 244, in which the

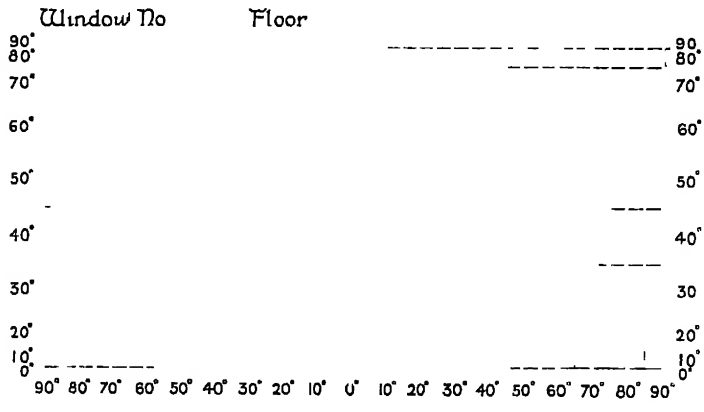


FIG 244 —Waldram's Daylight Factor Diagram.

abscissæ are angles of azimuth ϕ (from any convenient zero direction) and the ordinates are proportional to $(1 - \cos 2\theta)$, where θ is the angle of altitude from the horizontal If P (Fig 245) be an element of sky whose angles of azimuth and altitude are ϕ and θ respectively, the illumination of a horizontal surface at O is $B \sin \theta \, . \, \delta\theta \, . \, \delta\phi \cos \theta$.

Hence the whole illumination is $\frac{1}{2} B \iint \sin 2\theta \,.\, \delta\phi\delta\theta$, which, on putting in the limits, becomes

$$\frac{1}{4} B \,.\, \delta\phi \,.\, (1 - \cos 2\theta)$$

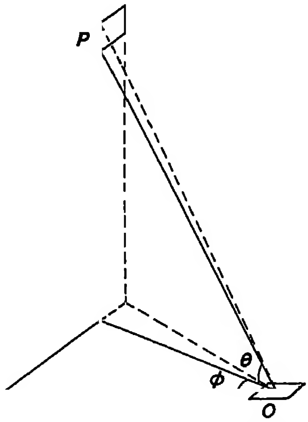


FIG 245 —The Principle of the Graphical Determination of Daylight Factor

Thus the area of the patch of sky as plotted on the diagram is equal to the daylight factor, on the scale to which the area of the whole diagram, $\phi = -90°$ to $+90°$ and $\theta = 0°$ to $90°$, is unity *

If the element of surface at the point considered be vertical instead of horizontal, ϕ must be measured from the normal to the surface, and the lengths of abscissæ and ordinates are proportional respectively to $\left(\phi + \frac{1}{2} \sin 2\phi\right)$ and $\left(\theta + \frac{1}{2} \sin 2\theta\right)$

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* Since $1 - \cos 2\theta = 1 + \sin (2\theta - 90°)$ it is clear that, by a simple renumbering of the scales, the co-ordinate paper suitable for plotting Rousseau diagrams may also be used for finding daylight factors (See note (20), p. 120)

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(47) Illum Eng, 17, *1924*, p 138
C H. Sharp Electrician, 66, *1911*, p 343, and 67, *1911*, p 179 But see also A P. Trotter, *ibid.*, 66, p 552, and 67, p 180
(48) See, *e g*, S W Melsom, Electrician, 69, *1912*, p 6
(49) C H Sharp and P S Millar El World, 60, *1912*, p 266, Assoc Elettrot, Ital, Atti, 16, *1912*, p 907
(50) J S Dow and V. H Mackinney Optical Convention, Proc, *1912*, p 70, Electrician, 69, *1912*, p 691, Phys Soc, Proc, 24, *1912*, p 405.
J S Dow. Brit Patent No 23,931 (*1911*)
(51) The flicker principle is used in the Harrison photometer (see p 352)
(52) See, however, H W Vogel, Wied Ann, 61, *1897*, p 409, J de Phys, 6, *1897*, p 569
(53) Illum. Eng Soc N Y, Trans, 10, *1915*, p 766
(54) See E T Z, 31, *1910*, p 303, L Bloch, *ibid*, pp 382 and 571, Illum Eng, 3, *1910*, p 403, El World, 55, *1910*, p 956, J G W, 53, *1910*, p 684, J Sumec, E T.Z, 31, *1910*, pp 571 and 648
(55) See, *e g*, A. P Trotter, Inst Civil Eng, Proc, 110, *1892*, p 69, Illum Eng, 6, *1913*, p 241
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L Bloch E T Z, 27, *1906*, p 493, Illum Eng (N Y), 1, *1907*, p 580
H T Harrison. Illum Eng, 1, *1908*, p 470, and 3, *1910*, p 307
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S L. Pearce and H A Ratcliff Inst El Eng, J, 50, *1913*, p 596 (and discussion).
See also U Vitry, Acad Nat des Sci nat (Toulouse), Mém, 6, *1850*, p 255.
(56) A H Ford Illum. Eng Soc N. Y, Trans, 8, *1913*, p 155
(57) See, *e g*, W E. Sumpner, *loc cit*, note (38), p 232, and E Mascart, *loc cit*, note (39), p 232.
Elettricità, 32, *1909*, p 171
G Peri. Elettrot, 6, *1919*, p 464
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L F Pello'. Elettrot, 11, *1924*, p 551
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R Edler E u M., 27, *1909*, p 293
H D Austin El World, 56, *1910*, p 1241
J S Codman Illum. Eng Soc N Y, Trans, 5, *1910*, p 638.
F Blacizek E u M, 29, *1911*, p 1025
J Teichmüller E u M, 36, *1918*, pp 261 and 277, J G W, 61, *1918*, p 229 (see also E Liebenthal, J G W, 62, *1919*, p 470), Z f Bel, 26, *1920*, pp 13 and 24, Licht u Lampe, *1919*, pp 108 and 130, and *1920*, pp 142 and 158, E u M, 38, *1920*, p 201, Z f I, 41, *1921*, p 305 (E Liebenthal, "Phys Handw," p 639)
E Kun El World, 81, *1923*, p 801
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(58) See, *e g*, S Meisel, E T Z, 26, *1905*, p 860
W C Clinton. Illum Eng, 7, *1914*, p 189, J G W., 57, *1914*, p 663.
E H Rayner, J W T Walsh, and H Buckley Illum Eng, 15, *1922*, p 107, Licht u Lampe, *1923*, p 7
(59) See, *e g*, H Laurain, C I E, Proc 6, *1924*, p 286, Rev Gén de l'El, 17, *1925*, p 521
J Mariage C I E, Proc, 6, *1924*, p 301, Rev Gén de l'El, 17, *1925*, p 559
(60) J R Cravath and Van R Lansingh Illum Eng Soc N Y, Trans, 3, *1908*, p 518, Illum Eng, 1, *1908*, p 1036
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P Högner E T Z, 31, *1910*, pp 234 and 267 (see also p 584)
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(61) Committee on Nomenclature and Standards, Illum Eng Soc N Y, Trans, 13, *1918*, p 517, C I E, Proc, 6, *1924*, p 171; J. Wetzel, Rev Gén de l'El., 14, *1923*, p 1073

(62) "Illumination, etc.," p 58.

(63) K Norden E T Z, 32, *1911*, p 607, J G W, 62, *1919*, p 416, Z f Bel, 26, *1920*, p 73, and 27, *1921*, p 109, Licht u Lampe, *1923*, p 470

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H H Kimball Monthly Weather Rev, 42, *1914*, p 650

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(65) H. Cohn. Schlesische Gesell f vaterl Cultur, Jahresber, 79, (Abt Ib), *1901*, p 2.

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(66) A P Trotter "Illumination, etc," p 249

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(67) K Kähler Meteorologische Z, 25, *1908*, p 52

(68) P J Waldram. Illum Eng, 1, *1908*, p 811, Soc of Architects, J, 3, *1910*, p 131

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(69) An instrument has been designed for comparing the brightness of different parts of the sky at the same instant See F Herxheimer, Z f I, 32, *1912*, p 55

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H Wild Acad Imp de S Pétersb, Bull, 21, *1876*, col 312, and 23, *1876*, col 290, Les Mondes, 41, *1876*, p 270, Arch des Sci, 56, *1876*, p 366, Z d österr Gesell f Meteorol, 11, *1876*, p 334; Naturforscher, 9, *1876*, p 377, and 10, *1877*, p 157

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See also A Korff-Petersen, Licht u Lampe, *1923*, p 57

(70) W Thorner Hygienische Rundschau, 14, *1904*, p 871, Phys Z, 9, *1908*, p. 855, Illum Eng, 1, *1908*, p 504, Am Gas Light J, 88, *1908*, p. 1108, D R Patent, 159,633 (*1904*), El. World, 56, *1910*, p 216, Z f Schulgesundheitspflege, 22, *1909*, p 174

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K Franz Z f Hygiene, 68, *1911*, p. 477, and 78, *1914*, p 95

L Weber Naturwiss Verein f Schleswig-Holstein, Schriften, 15, *1911*, p 158, Z f Bel, 25, *1919*, p 8, Z f I, 38, *1918*, p 123, Illum Eng, 4, *1911*, p 243

(71) A K Taylor J Sci Instruments, 1, *1924*, p 214

(72) Phys. Z, 3, *1901*, p 137, Naturwiss Rundschau, 16, *1901*, p 589, J G W, 45, *1902*, p 235, Gesell deut Naturforscher u Ærzte, Verh, 73 (Theil II), *1901*, p 37

(73) See also R Mehmke, Z f Math u Phys, 43, *1898*, p 41

(74) F. Pleier. Z f Schulgesundheitspflege, 22, *1909*, p 461, Illum Eng, 7, *1914*, p 359.

Instruments for measuring the angular area of the patch of sky visible from a point have been described by L Weber (Raumwinkelmesser), Z f I, 4, *1884*, pp 343 and 417, Lum El, 22, *1886*, p 314, F Erismann, Arch f Hygiene, 17, *1893*, p 205, F Gotschlich, Klinisches Jahrbuch, 12, *1904*, p 202, M Moritz, *idem*, 14, *1905*, p 95, F Pleier, Z d österr. Ing- u Arch Vereins, 60, *1908*, p 23; K. Franz, Z f Hygine, 68, *1911*, p. 477, and 78, *1914*, p 95

(75) Illum Eng, 16, *1923*, p 90, Architects J, 58, *1923*, p 624 See also H H Higbie, Illum Eng Soc N Y, Trans, 20, *1925*, p 433

(76) In the latest form of the Luxometer the lamp is moved in such a way that, as its distance from the comparison surface is altered, the angle of inclination of the light is changed as well Thus both the inverse square and cosine laws are used simultaneously (Add to (21) *supra*)

CHAPTER XIII

MEASUREMENT OF BRIGHTNESS AND OF REFLECTION AND TRANSMISSION FACTORS

Reflection

The Measurement of Reflection Factors.—It has already been ınted out ın Chapter IV (p 114) that the lıght reflected from a rface varıes in ıts dıstrıbutıon accordıng to (a) the dırectıon of the cıdent light, and (b) the character of the surface The varıatıon reflectıon factor wıth the spectral dıstrıbutıon of the ıncıdent ;ht for all but perfectly whıte or grey surfaces was also consıdered . 111) It thus follows that for a measurement of reflectıon factor be of value ıt must be made with lıght of a certaın colour, and the rectıon of the ıncıdent lıght as well as the dırectıon of the reflected ;ht consıdered must be specıfied Further, ın the case of a surface r whıch any apprecıable part of the lıght reflected ın the dırectıon ›nsıdered ıs specularly reflected, the proportıon of specularly to ffusely reflected lıght wıll depend on the dıstance and brıghtness the source (1)

When knowledge of the dıstrıbutıon of the reflected lıght, or of the flectıon factor ın some partıcular dırectıon, ıs specıally desıred, ıt measured dırectly by an obvıous adaptatıon of ordınary photo- ıetrıc procedure (2) The apparatus may be set up as shown ın lan in Fıg 246. S ıs the surface under ınvestıgatıon, fixed at one

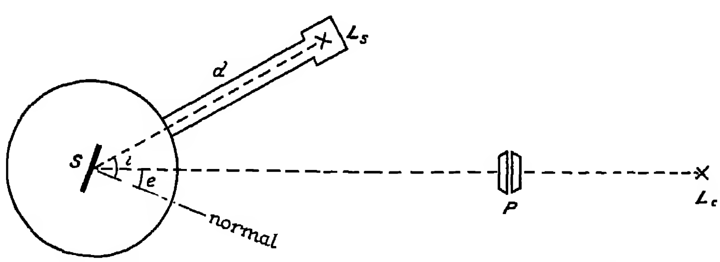

Fig 246 —The Measurement of Reflectıon Factor ın a Sıngle Dırectıon

nd of a photometer bench It ıs ıllumınated by a lamp L_S, whıch s carrıed on a radıal arm capable of rotation through any desıred ngle about a vertıcal axıs through S The brıghtness of S ıs neasured by means of a photometer P and comparıson lamp L_C The surface must be quıte flat and ıts area A, as well as the candle- ›ower of L_S, must be known exactly ıf absolute values of ρ are requıred f i be the angle whıch the ıncıdent lıght makes wıth the normal to he surface, whıle the (fixed) angle of aspect ıs ϵ, the ıllumınatıon ›f S ıs $(I_S \cos i)/d^2$ The brıghtness ın the dırectıon ϵ is therefore $\rho_\epsilon I_S \cos i)/\pi d^2$ candles per unıt area, and its candle-power ın the

direction SP is $A \cos \epsilon\ (\rho_\epsilon I_S \cos i)/\pi d^2$ Since all the other quantities are known, ρ_ϵ can be found It is to be noticed that in all cases, and especially if either ϵ or i be great, I_S must be large, or the candle-power of the surface will be too small and the photometer cannot be kept at a desirable distance from S For instance, if S be 10 cm square, the distance of the photometer from it must be at least 50 cm (for an accuracy of 1 per cent), and the brightness of S should therefore be at least 250 candles per sq. metre L_S may therefore be a 1,000 c p lamp placed at about 1 metre from S.

The above arrangement gives values of ρ_ϵ for different values of i, ϵ being in the plane of the incident light For measurements with i constant and ϵ variable the surface must turn with the radial arm which carries L_S Measurements with i and ϵ in different planes can be made by raising or lowering L_S (3).

A very convenient method of obtaining approximate values of ρ at various angles of aspect, both in the plane of incidence and in other planes, is that in which the reflecting surface is used as the test surface of a portable illumination photometer, such as the lumeter or luxometer (see p 351) (4) Any convenient method may be used for positioning the instrument at known values of ϵ. The relative values thus obtained may be converted to absolute values by comparison with some standard surface for which ρ is known, or alternatively by a measurement of the absolute value of ρ in some *one* convenient position by means of the more accurate method first described

The results obtained in this way for directions in a single plane may be expressed in the form of curves such as those of Fig 218, p 342 For directions in more than one plane a solid diagram is necessary (5) or an iso-candle diagram (see p. 89) may be used

Diffuse Reflection Factor. The Reflectometer.—It frequently happens that the value of ρ for a surface at any single value of ϵ and i is of less importance than the value of the diffuse reflection factor, *i.e*, the ratio of the whole flux reflected by the surface in all directions to that incident at the surface when the illumination is perfectly diffused, *i e*, when the flux density is the same in all directions (6). The advantage of this is that a single figure may be used to give the performance of the surface as regards reflection under conditions of measurement which are perfectly defined Several forms of apparatus have been devised to measure the diffuse reflection factor of a surface, either absolutely or in terms of some standard surface (7), but large errors are liable to occur in the measurements unless the conditions under which the instrument works are such as to give (i.) as perfect diffusion as possible for the incident light, and (ii.) a correct measurement of the total flux reflected by the surface

The most satisfactory form of diffuse reflectometer depends on the principle of the integrating sphere, which may be adapted to the measurement of diffuse reflection factors in several different ways (8). In some of these it is necessary to use some readily reproducible surface of already known reflection factor (such as magnesium carbonate) as a standard of comparison Other methods give the absolute reflection factor directly. The theory of several of these methods has been given by A H. Taylor (9). In the first the sphere

has a circular portion of its surface cut away as shown in Fig. 247, which gives an elevation in two perpendicular planes. A tube T containing a lamp and lens, and fitted with opaque diaphragms, projects an intense patch of light on the sphere at P. B is the observing window of the sphere

The area of the sphere opening in relation to the total area of the sphere must be exactly known. This fraction (a) may conveniently

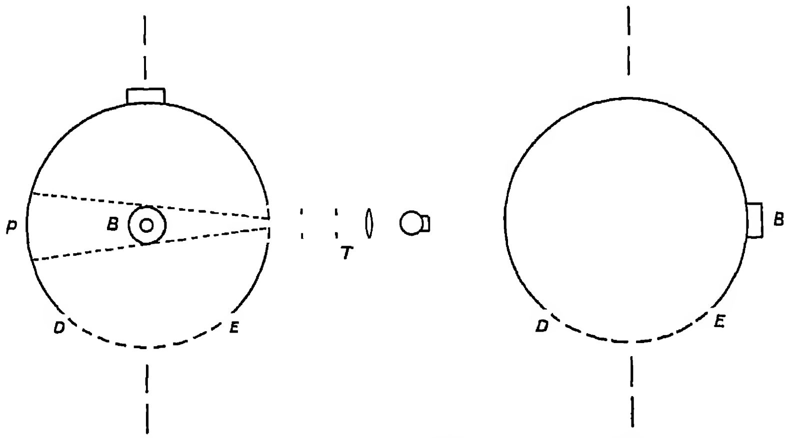

FIG 247 —The Measurement of Diffuse Reflection Factor

be about 0·1 From measurements of the sphere brightness at B when (i) DE is uncovered (b_0), (ii) DE is covered with a surface of the same reflection factor as that of the sphere surface (b), and (iii) DE is covered with the surface under measurement (b_ρ), it is possible to calculate ρ For, in the first case, the flux escaping through DE is made up of two parts · (i.) that directly reflected from the luminous patch at P, and (ii) that reflected from the illuminated walls of the sphere This flux, added to that absorbed (iii) by the walls of the sphere and (iv) by the walls at P, must be equal to the total flux received from the lamp (F), so that

$$a\rho_s F + a'\pi b_0 + \alpha_s(1 - a)\pi b_0/\rho_s + \alpha_s F = F$$

where ρ_s and α_s are respectively the reflection and absorption factors of the sphere surface (so that $\alpha_s + \rho_s = 1$) and a' is the area of the (plane) circle at DE expressed as a fraction of the area of the surface of the sphere.*

This equation gives

$$b_0 = \rho_s{}^2F(1 - a)/\pi\{\rho_s a' + \alpha_s(1 - a)\}.$$

In the third measurement, DE is covered by a surface of reflection factor ρ, so that the flux lost at DE is reduced in the ratio $(1 - \rho)$ 1. Hence, since the brightness is now b_ρ,

$$(1 - \rho)(a\rho_s F + a'\pi b_\rho) + \alpha_s(1 - a)\pi b_\rho/\rho_s + \alpha_s F = F$$

* (i) The total flux from P is $\rho_s F$ and is equally distributed over the surface of the sphere (see p 206) (ii) If the brightness of the sphere is b_0, the flux received by DE is thus πb_0 multiplied by the area of DE relative to the sphere (iii) Since the brightness is b_0 the flux emitted per unit area is πb_0 and the flux received is, therefore, $\pi b_0/\rho_s$.

and

$$b_\rho = \rho_s^2 F\{(1-a) + a\rho\}/\pi\{a'\rho_s(1-\rho) + (1-a)\alpha_s\}$$

Similarly, in the second measurement, when DE is covered by a surface of reflection factor ρ_s,

$$(1-\rho_s)(a\rho_s F + a'\pi b) + \alpha_s(1-a)\pi b/\rho_s + \alpha_s F = F$$

and

$$b = \rho_s^2 F\{(1-a) + a\rho_s\}/\pi\alpha_s\{a'\rho_s + (1-a)\}$$

Thus b/b_0 is a function only of a and a', which are known from dimensions, and of ρ_s, which can therefore be calculated. This done, b_ρ/b_0 gives the required value of ρ

A rather more sensitive method is to project the patch of light on to the surface at DE instead of on to the sphere wall at P. If the value of ρ_s be known by the first method, it is easy to show that the brightness at B when a surface of reflection factor ρ is placed at DE is proportional to $\rho\rho_s/\{(1-a)\alpha_s + a'\rho_s\,\alpha\}$ For the total flux F is absorbed (i) αF directly at DE, (ii) $(1-a)\alpha_s(\pi b/\rho_s)$ on the surface $(1-a)$ of the sphere, which has a brightness b and therefore receives $\pi b/\rho_s$ lumens per unit area, (iii) $a'\alpha\pi b$ at DE, since this surface, of area a', receives πb lumens per unit area by reflection from the sphere of brightness b So that

$$\alpha F + (1-a)\alpha_s\pi b/\rho_s + a'\alpha\pi b = F.$$

Thus $\pi b/F$ is a function of a, a', α_s, ρ_s and α If, then, all except α be known, by placing different surfaces in turn at DE it is possible to find the reflection factor of any of them if one is known A thick piece of depolished opal glass forms a convenient standard surface for use in this way A slight modification of the foregoing methods leads to the simple portable instrument shown in Fig 248 ([10]) The tube T is movable, so that the beam of light from L may be projected on to either the test plate at A or a portion of the sphere wall at B The part of the sphere wall at C is screened from A, so that it cannot receive light directly from the test surface when the latter is illuminated by the beam of light The reflection factor of the test surface is the ratio of the brightness of C when A is illuminated by the beam to the brightness of C when B is thus illuminated, for in the first case, if F be the luminous flux in the beam, the surface acts just like a source of ρF lumens placed in the sphere, and since C is screened from A its illumination is that of a sphere window under these conditions When B is illuminated, however, the conditions are those which exist when a source of F lumens is placed in the sphere, for C, being unscreened from B, is illuminated equally with the remainder of the sphere wall

It will be noticed that in this instrument perfect diffusion is assumed for the sphere walls, but the manner in which the light is reflected from the test surface is immaterial On the other hand, the beam of light is not such as even to approximate conditions of perfectly diffuse illumination, and the values of ρ found may be in error on this account A method in which the illumination is perfectly diffused has been described ([11]), but in this the brightness of the surface is measured in a single direction, so that unless the test surface be a perfect diffuser the values obtained may be in error ([12])

The Measurement of Specular Reflection.—The reflection factor of a mirror may be simply measured by the method indicated on p 199 The candle-power of a source is measured (*a*) directly and (*b*) by reflection in the mirror, the distance of the source from the

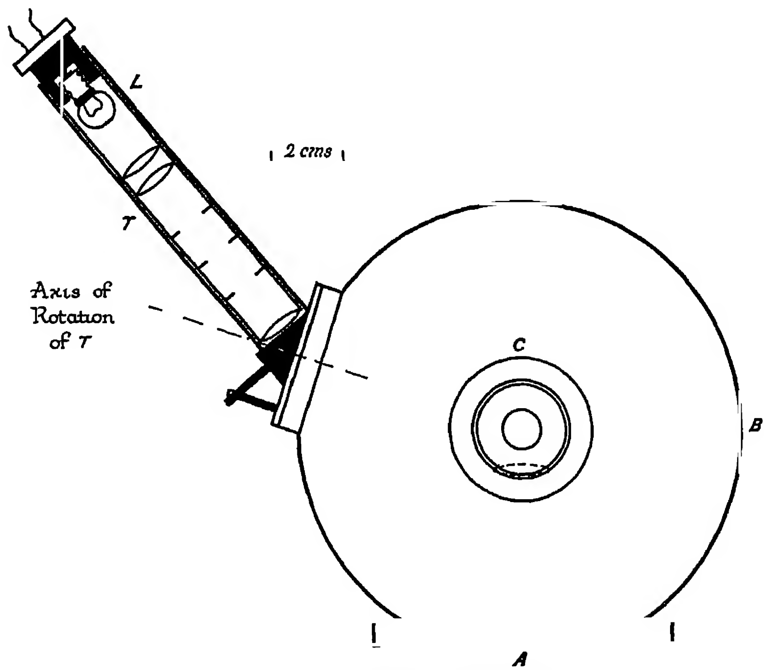

FIG 248 —The Portable Reflectometer

photometer being that actually traversed by the light (13) Since the reflection factor of a good silvered mirror is between 80 and 90 per cent, it is convenient to use several pieces so that the light suffers more than one reflection The reflection factor is then $\sqrt[n]{I_M/I}$, where I_M is the apparent candle-power of the source after n reflections (14) In the case of a glass mirror the value of ρ thus found is that of the mirror as a whole, *i e*, the light reflected from the front surface of the glass is included with that reflected by the metal Since the reflection factor varies slightly with the angle of incidence of the light, this angle should always be stated with a value of reflection factor For an angle of about 45° the arrangement shown in plan in Fig 249 is convenient For most silvered mirrors of good quality the variation of reflection factor with the colour of the light is almost negligible (15)

The Reflection Factor of Semi-Matt Surfaces.—While it is true that the diffuse reflection factor is the most generally useful figure for expressing the reflecting power of matt or semi-matt surfaces, it is nevertheless desirable for certain purposes to obtain a rough measure of the proportion of light regularly reflected within a certain solid angle A rigorous measurement is clearly impossible in this case, since it is found that light is "regularly" reflected in directions appreciably inclined to the direction of specular reflection. The

measurement may, however, be made with an instrument of specified design, such as the "glarimeter" (16), and the values obtained, although arbitrary, are comparable with one another and are useful for such purposes as the control of the calendering process in paper,

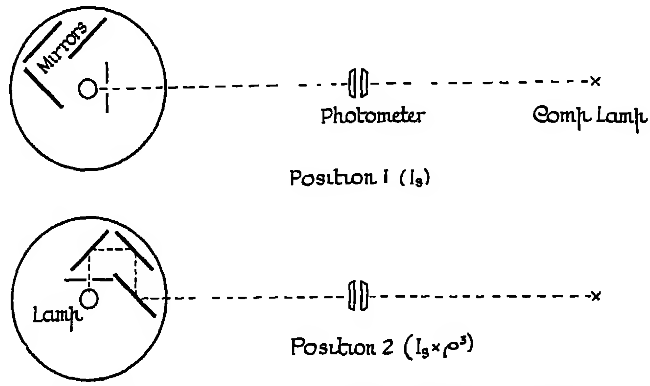

FIG 249 —The Measurement of Specular Reflection Factor

the description of the surface finish in paints or enamels, *etc.* The instrument, which is shown in vertical section in Fig. 250, depends on the experimental fact that the regularly reflected light is almost completely polarised in the plane of incidence, while that diffusely reflected is unpolarised (17) Light from a source subtending a certain small solid angle (actually 0·038 steradian) illuminates the

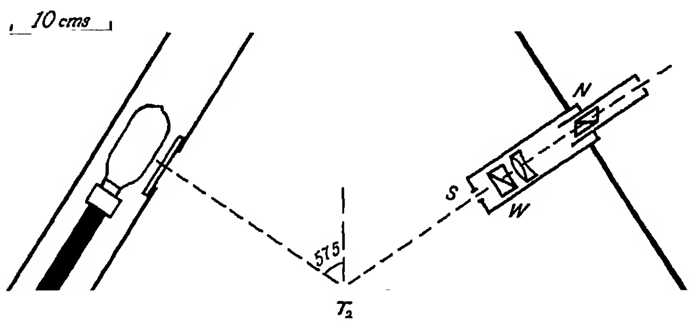

FIG 250 —The Glarimeter.

surface under measurement at T_2, the light being incident at an angle of about 57·5° The Wollaston prism W (see p. 30) gives a double image of the slit S, and is so set that the specularly reflected light is completely extinguished in one image The Nicol N is rotated until the two images are of equal brightness In this case, if A be the angle of rotation of the Nicol, while D and S are the intensities of the diffuse and specular components, $\frac{1}{2}D/(\frac{1}{2}D + S) = \tan^2 A$, or $S/(D + S) = \cos 2A$, i e , the fraction of the whole light which is regularly reflected is equal to $\cos 2A$.

An instrument which does not depend on polarisation is that shown in Fig 251. The lamp L can be moved, by means of a handle

carrying a pointer, between the mirrors M_1, M_2. In the bottom of the box is an aperture under which is placed the specimen to be measured The lower part of the box is divided into two by a thin diaphragm placed parallel to the plane of the paper, and so arranged that half the specimen is illuminated by light from M_1, and the other half by light from M_2 The specimen is viewed either from E_1 or E_2, where are placed small double prisms of the form shown in plan at the bottom of the figure. By this means the two parts of the specimen are seen in juxtaposition The position of L is altered until the brightness (B_s) of the half of the specimen seen by light reflected in the direction of specular reflection is equal to the brightness (B_d) of the half seen from the direction of the incident light The instrument is scaled to give directly, from the position of L, the value of $100\ B_s/(B_d + B_s)$ [18]

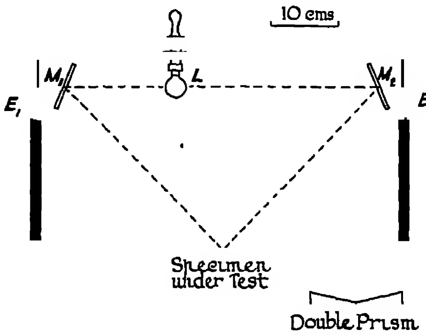

FIG 251 —Trotter's Gloss Tester

Dependence of Reflection Factor on Colour.—It has already been pointed out that the reflection factor of any surface which is not perfectly white depends on the colour of the incident light (see p 111) The determination of the spectral reflection curve of a surface may be carried out by means of a spectrophotometer In the case of specular reflection the apparatus may be that shown diagrammatically in Fig 252 [19]. S is a uniform source of light, and I its

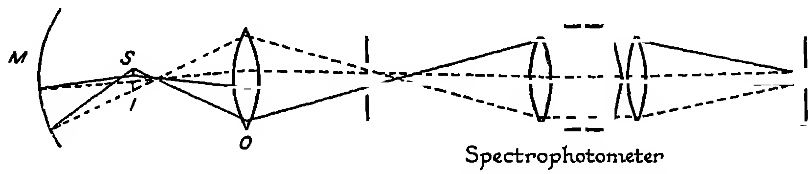

FIG 252 —Spectral Reflection Measurements for a Polished Surface

adjacent image formed by reflection in the concave mirror M, the surface of which is of the material (metal or other polished material) to be investigated O is a lens which forms images of S and I respectively upon the two apertures of a suitable form of spectrophotometer (see p. 281) So long as the whole of the solid angle subtended by the lens O at the object is embraced by the mirror M the ratio of the intensities of the two images formed on the slit is the reflection factor of M for the frequency at which the measurement is made Since in this method the light forming the two images at the spectrophotometer slits comes from opposite sides of S, it is necessary that the latter shall be a perfectly uniform source, *e g*, the two sides of a wide strip of incandescent tungsten Even so it is desirable that two sets of measurements should be made, each side of S facing M in turn.

For spectrophotometric measurements of diffuse reflection factor the apparatus sketched in Fig 253 may be used (20) B is a metal box coated on the inside with a diffusing white paint, and containing a number of lamps sufficient to give a high uniform illumination at the back of the box It is kept cool by exterior water circulation Two small apertures in the front of the box allow light diffusely reflected from the back to illuminate the slits of the spectrophotometer The surface of which the reflection factor is to be measured is placed first at P and then at Q, and the brightness at any part of the spectrum is compared with that of a small block of magnesium carbonate placed first at Q and then at P.

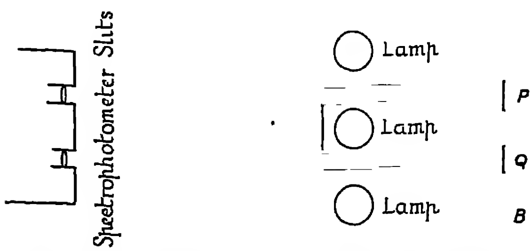

FIG. 253 —Spectral Reflection Measurements for a Diffusing Surface.

It is to be remarked that the colour of the light or the constancy of the illumination do not affect the results, since the comparison surface of magnesium carbonate is equally affected. The reflection factors obtained are all relative to magnesium carbonate, but as the reflection factor for this material is in the neighbourhood of 98 per cent., and is therefore almost independent of colour, a small constant correction factor can be applied throughout the spectrum in order to obtain absolute values (21).

When determining either reflection or transmission factor curves it is sometimes convenient to invert the process described on p 249 for the calculation of the total factor, for if the total factor be known sufficiently accurately by direct measurement by ordinary photometric methods, the *absolute* value of the factor at any frequency can, it is clear, be calculated at once from a curve giving the *relative* values of the factor throughout the spectrum. This avoids the determination of the absolute factor by means of the spectrophotometer, but it cannot be used to give results of great accuracy, since it involves heterochromatic photometry with, probably, a considerable colour difference

A colorimetric determination (on the trichromatic system) of the light reflected by a coloured surface may be made by measuring the reflection factor of the surface with some convenient reflectometer, placing in turn in the eyepiece of the instrument three glasses —red, green and blue—which thus become the instrument primaries (see Chapter X , p 304) (22)

It is clear that the light reflected from any coloured surface illuminated by white light may be measured by any of the methods of colorimetry described in Chapter X The colour of the surface may thus be expressed on the monochromatic or on the trichromatic system The same remark naturally applies to coloured transparent media (23).

Nephelometry.—When light passes through a slightly turbid or cloudy medium, part of it is obstructed by the particles causing the cloudiness. Some of this light is truly absorbed, while the remainder

is scattered in all directions by reflection from the particles It is this scattered light which renders visible the path of a beam of light traversing a dusty atmosphere. If the particles be small and their number not too great, so that the total cross-section of the beam is large compared with the total cross-section of the particles situated in its path, then the absorption factor of a given thickness of the medium bears a linear relation to the concentration of the particles. For a constant concentration the transmission factor varies with the length of path of the beam, according to the ordinary exponential law (see p 116). When the dimensions of the particles are of the same order of magnitude as the wave-length of the light considered, $\tau = e^{-kt}$, where $k \equiv KN\nu^n$, N being the number of particles per unit volume, ν the wave-number (24), and n a constant whose value is 4 when the particles are very small compared with λ (25). When, however, the particles are much larger, this law no longer holds at other than very weak concentrations, and the empirical relation $\tau = 1 - e^{-m}$ has been suggested (26), where $m \equiv B/N^\beta$, B and β being constants whose values depend on the cross-section of the beam as well as on the length of its path in the medium, the frequency of the light, and the nature of the particles, so that the formula can only be regarded as affording a convenient method of interpolation between the readings given by standard suspensions.

When the size of the particles and their concentration are both small, τ is very nearly equal to unity, and an accurate measure of concentration is therefore very difficult to obtain by this method A much more convenient measure in this case is that of the light scattered in a direction making a definite angle (frequently 90°) with the direction of the beam (27) This is the so-called Tyndall beam (28), and several instruments for making this measurement have been described (29) under the name of "Tyndall meters" or "nephelometers." * That of Mecklenburg and Valentiner is shown in Fig 254 The light from the source S is focussed in the plane of B, a diaphragm with a small circular hole, which is completely filled with light. The beam from B is rendered parallel by the lens L_1 and is divided into two parts (Fig 254 (a)), the lower of which, after suffering four total internal reflections in the prism system P (Fig 254 (b)), traverses the lens L_2 and is brought to a focus at F, a point within the liquid under examination which is contained in the parallel-sided glass vessel T. The upper half of the beam from B illuminates a plaster screen G, which is seen through the three Nicols N_1, N_2 and N_3, the total reflection prism P_1, and the Lummer-Brodhun cube L-B. The other face of this cube is illuminated by the light scattered by the liquid at F in the vertical direction (i e , at 90° to the original beam). Thus the brightness of the liquid at F is compared with that of the screen G, and photometric balance is

* The terminology appears to be somewhat confused at the present time The name "nephelometer" has been used to designate an instrument for measuring the transmission factor (C Chéneveau and R Audubert, C R, 170, *1920*, p 728, J de Phys, 2, *1921*, p 19) This is more properly called an "opacimeter" (Lambert, F Vlès and de Watteville, C R, 168, *1919*, p 797), although the name "turbidimeter" is sometimes used when the instrument is employed in connection with the study of suspensions, colloids, etc (J T W Marshall and H W Banks, Am Phil Soc, Proc, 54, *1915*, p 176) Nephelometry proper, the measurement of the Tyndall beam, has also been called "Tyndallimetry" (S. E Sheppard and F A Elliott, Am Chem. Soc, J, 43, *1921*, p 531)

obtained by rotating the Nicol N_2 about a vertical axis. The use of three Nicols avoids the possibility of error due to partial polarisation of the light transmitted at G The vessel T is capable of an accurately measurable movement, both horizontal and vertical. By using the vertical movement it is possible to make measurements

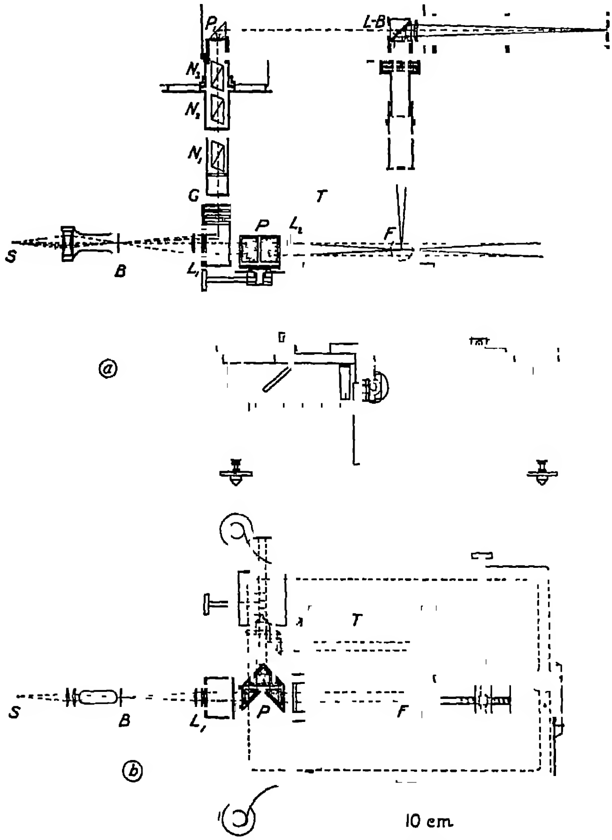

FIG 254 —The Nephelometer of Mecklenburg and Valentiner

when F is at different depths below the surface of the liquid The absorption of the Tyndall beam in the liquid itself can thus be allowed for by extrapolation A small piece of auxiliary apparatus is provided for finding the height of F above the bottom of the vessel T. Measurements are also made with F at different distances from the "entrance" side of the vessel, and the results are again extrapolated to allow for absorption of the incident beam in the liquid

TRANSMISSION

Measurement of Transmission Factor.—A somewhat similar problem to that of measuring the reflection factor of a surface is the determination of the transmission factor of any substance, generally in the form of a plate. The transmission factor of a

transparent plate is easily found by measuring the candle-power of a source with and without the plate between it and the photometer ([30]) The value of τ thus found is that of a plate of the medium regarded as a unit, and not that of a certain thickness of the medium The reflection factor per surface may usually be calculated with sufficient accuracy from the refractive index (see p. 112) If this be ρ, while τ is the actual transmission factor of the medium in a thickness equal to that of the plate, the light transmitted is easily seen to be $(1-\rho)^2\tau/(1-\tau^2\rho^2)$ (see p 117) If $\rho = 0{\cdot}04$, while $\tau = 0\ 5$, it will be seen that this value differs by a quite negligible quantity from $(1-2\rho)\,\tau$ ([31]). It has to be remembered, in applying this method, that slight departures from planeness of the surfaces of the plate may produce large errors in the determination of the transmission factor owing to lens effects, unless the plate be placed quite near to the photometer (see p. 183) Like the reflection factor, the transmission factor of a body naturally depends upon the colour of the incident light unless the body be colourless, *i e*, neutral or grey.

Determination of Spectral Transmission Curves.—The need for spectral transmission measurements of the colour filters used in heterochromatic photometry, and for other purposes, has already been pointed out Similar curves for liquid solutions are of considerable importance in many branches of pure and applied chemistry The modifications necessary in order to make transmission measurements with the spectrophotometric apparatus described in Chapter IX. will generally be obvious In the case of double-slit instruments, or instruments like the Hufner or Nutting, it is clearly only necessary to obtain two parallel beams of light from a single source and place the coloured medium in the path of one of these A brief description, applied to the Nutting-Hilger instrument, has been given on p 284 The application to other similar instruments is immediately apparent A convenient method of producing the two parallel beams is shown in Fig 181, where the prisms *A* and *B* are, in effect, two portions of a single convex lens having its focus in the position of the light source *L* ([32]) Alternatively *A* and *B* may consist of two prisms producing two vertical images of *L*

The substitution method should always be used, so that the transmission of the medium at any given frequency is the ratio of the intensities measured at that frequency for *the same half* of the comparison field (*a*) with, and (*b*) without the medium interposed It is convenient to use a dummy filter of high transmission permanently in that half of the field used for the medium under measurement, partly to avoid any possibility of the intensity on that side being greater than the intensity of the comparison side when the test medium is not inserted, and partly to bring the balance point to a more convenient part of the scale

In the case of an instrument with two collimators, two sources may be used, or a reflecting surface may be placed in front of each of the two slits, both these surfaces being illuminated by the same source ([33]) In either case two comparisons of spectral distribution are made, one with the medium in the path of one of the two beams, and one with the medium removed If the coloured medium be of

appreciable thickness, the increase of illumination due to refraction in the medium must be allowed for (see p 23) A solution has to be contained in a cylindrical glass vessel, which may be of any length up to 100 cm, and the particular instrument chosen for the work must be capable of accommodating such a vessel in the path of one of the incident beams A convenient form of vessel is shown in Fig 255. The ends are plane parallel plates of colourless glass or

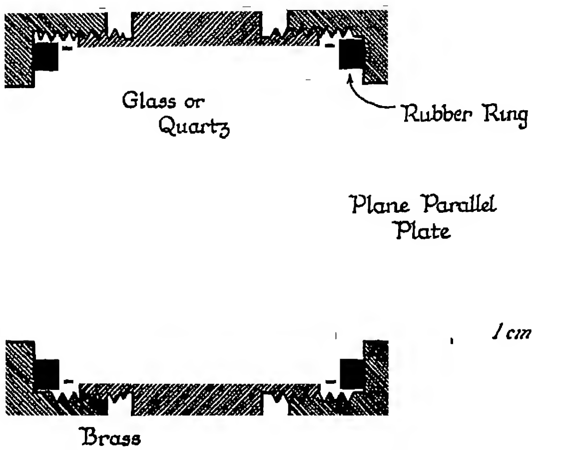

Fig 255 —Cell for Spectral Transmission Measurements on Liquids

quartz pressed hard against a tubular body. Sometimes the whole vessel is immersed in water contained in a larger outer vessel, so that the temperature may be kept constant and accurately measured by means of thermo-couples immersed in the water.

In order to avoid errors due to reflection from the glass ends of the vessel, two methods are commonly adopted In the first a vessel similar to that containing the liquid is placed in the path of the comparison beam of light. This vessel contains some practically colourless liquid, such as water, of which the transmission is either negligible or known It is not sufficient to use an empty vessel, for then there is a double reflection loss of about 4 per cent at each end, whereas in the vessel containing liquid, of which the refraction coefficient may be taken as about 1 3, the loss by reflection at the *inside* surfaces is only $(n - 1)^2/(n + 1)^2$, where $n = 1\ 5/1\ 3$, *i e*, it is about 0 5 per cent [34] In view of the smallness of this loss, it is often sufficient to use a single plate of glass in the comparison beam This plate should, however, be of the same glass as that used in the vessel, and of double the thickness, in order to compensate for any slight selectivity in this glass In the second method of compensation two vessels of different lengths, but otherwise identical, are used. Both vessels are filled with the liquid to be measured and are placed one in each of the beams of light to be compared [35] Alternatively, they may be placed successively in the same beam of light In this case, if τ_1 be the transmission factor for light of a given frequency in the case of a vessel of length l_1, while τ_2 is the transmission factor for a vessel of length l_2, it follows that the

transmission factor of a length of liquid $(l_1 - l_2)$ is equal to τ_1/τ_2 Of the two methods of compensation just described the first is generally more convenient when temperature effects are likely to be important [36].

Instead of using any of the ordinary methods for adjusting to a photometric balance, the length of path of the absorbed beam in the medium to be measured may be arranged so that it can be varied at will by the observer, and this variation may then be used for adjusting the photometric field [37]

For substances which cannot be examined in solution special methods of spectrophotometry must be used [38]

Transmission Factors by means of Physical Photometers.—When there is no colour modification of the light transmitted by the medium, values of transmission factor may conveniently be measured by some form of physical photometer [39]. The most general case of this kind arises in the determination of spectral transmission curves, and for this purpose the photo-electric null method (see p 330) has been employed [40]. The apparatus is shown in Fig 256.

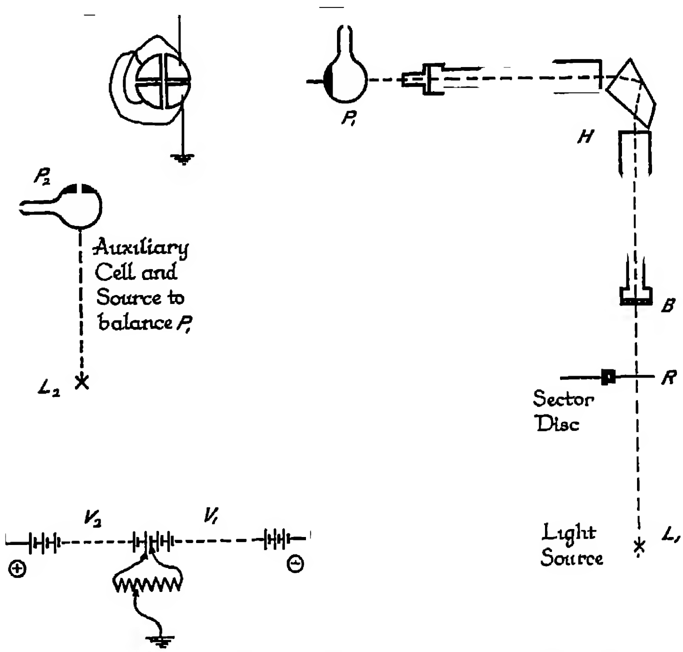

FIG 256 —Spectral Transmission by Photo-electric Photometry

The light from a convenient source L_1 passes through a rotating sector R and the medium B whose transmission is to be measured. H is a constant deviation spectrometer, the monochromatic beam from which illuminates the photo-electric cell P_1 This cell is connected in a bridge arrangement with a second cell P_2, which is illuminated by a lamp L_2 The bridge is first balanced for dark currents by adjusting the voltages V_1 and V_2 P_1 is then illuminated by light of any desired frequency, B being in the beam from L_1.

The bridge is balanced by altering the illumination of P_2. B is then removed from the beam and the opening in R is readjusted until the bridge containing P_1 and P_2 is again balanced. The ratio of the openings in R, with and without B, gives the transmission factor of B for light of the frequency illuminating P_1. Modifications of this method have been employed according to circumstances

Physical Photometers for Transmission Measurements of Neutral Media.—A flicker method which has been used (41) for measuring the transmission of such a medium as a photographic plate in white light is shown diagrammatically in Fig 257. L is a concentrated filament lamp round which a shutter revolves in such a way as to interrupt in turn the two beams of light which respectively traverse M, the medium whose transmission is to be measured, and W, an adjustable wedge. The photo-electric cell P is thus illuminated alternately by these two beams, and W is adjusted until there is no oscillation of the galvanometer. W may be calibrated by substituting for M a standard calibrated wedge.

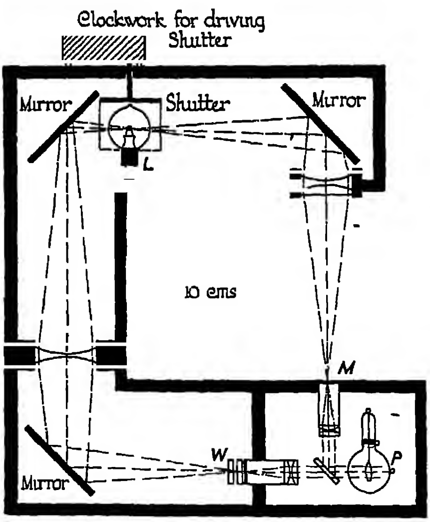

FIG 257 —The Photo-electric Densitometer

Diffuse Transmission Factor.—When the medium itself, or either bounding surface, is even slightly diffusing, the above methods can no longer be applied, and methods similar to those described for the measurement of diffuse reflection factor must be used (see p 377) (42). The apparatus shown in Fig. 246 may, by an obvious transformation, be used for the measurement of transmission factor at different angles of incidence and emergence of the light (43). When the incident light is perfectly diffused, the overall transmission factor may be measured as the ratio of the total flux transmitted to that incident on the surface (44). The globe reflectometer shown in Fig 248, p 381, may be used to give diffuse transmission factors (45), as follows —

(i.) The brightness (b_1) of the sphere at C is measured when B is illuminated and A is uncovered

(ii.) The same measurement is made (b_2) when A is covered with the test material.

(iii.) A sphere similar to the reflectometer sphere is now placed with its aperture against the aperture of the reflectometer, the test material having been removed. The light in the reflectometer (b_3) is now measured

(iv.) The same measurement is made (b_4) with the test material placed between the apertures of the spheres

The transmission factor required is given by $\tau = b_1 b_4 / b_2 b_3$, for b_4/b_3 is the ratio of the flux entering B unimpeded to that entering after transmission through the test material. The correction factor

b_1/b_2 is required because in the first case A is uncovered, while in the second it is covered with the test material, and it will be seen that b_1/b_2 is the ratio of the illumination of C when A is covered with the test material to that when A is uncovered, the flux in the sphere being the same in both cases

The Transmission of Optical Systems.—It has been said above that the transmission factor of transparent objects cannot be measured by ordinary methods if these objects be such as to produce a modification of the spatial flux distribution (lens effect). It is, however, frequently of importance to determine the transmission of a lens or system of lenses in an optical instrument, *e g*, a camera lens or telescope objective ([46]) For this purpose the usual procedure is to use the instrument to form a real image of an extended bright surface, such as a sheet of opal glass forming the front of a whitened cube containing a number of lamps The optical device is placed so as to form an image of this surface on the test surface of a photometer, and a measurement of the illumination, E_1, is made The optical device is then removed, and the direct illumination, E_2, of the test surface by the object surface is measured The ratio E_1/E_2 is the product of the transmission factor of the device, τ, and the lens effect The magnitude of the latter can be calculated from the dimensions of the apparatus, to a first approximation, as follows —

Let S be the area of the object surface and L that of the optical device, and let u and v be the distances between this device and the object surface and photometer surface respectively If S and L be small compared with u and v, the flux reaching the device is BLS/u^2, where B is the brightness of the object surface The transmitted flux $\tau BLS/u^2$ is distributed over an area v^2S/u^2, so that $E_1 = \tau BL/v^2$. Without the device the illumination E_2 is clearly $BS/(u+v)^2$, so that $E_1/E_2 = \tau L(u+v)^2/Sv^2$. When L is not small compared with u and v, the value of E_1 may be calculated for the special case when the optical device is circular and of radius R ([47]). Then

$$E_1 = \{\pi B u^4/(u^2 - v^2)\}\{\log(1 + R^2/v^2) - \log(1 + R^2/u^2) - R^2(u^2 - v^2)/u^2(u^2 + R^2)\},$$

while E_2 is found as described in Chapter IV (p 102). Since u, v, S and L or R are known, τ can be found from the observed value of E_1/E_2 The approximate formula is generally accurate to about 1 per cent so long as R/f does not exceed 0·1, f being the effective focal length of the optical device

Alternatively, the overall transmission factor of a given optical system may be determined by measuring the brightness, B_i, of the image formed by the device when the object is a diffusing surface of known brightness B_o. Since with any optical system of transmission factor unity $B_i = B_o$ ([48]), it follows that in any actual system $\rho = B_i/B_o$ ([49]) B_i and B_o may be measured by one of the methods described later in this chapter (p 401) or by means of a portable photometer, if a large enough field can be obtained. This method is, however, open to the objection that stray light may be included in the brightness B_i, and a modification designed to overcome this difficulty has been described ([50]).

Density of Photographic Images.—A special problem in the measurement of transmission factors is presented in the determination

of the degree of darkening of an exposed photographic plate ([51]) This is of importance not only in general photographic sensitometry, but also in those branches of photometry where photographic methods are employed, such as stellar photometry (see p. 427), the study of line spectra, *etc.* ([52])

Photographic sensitometry consists, briefly, in exposing an area of a photographic plate to a known illumination for certain fixed periods of time and then developing in a specified manner, the temperature being controlled throughout ([53]) The opacity ([54]) is then measured by placing the exposed area of the plate between a source of light and a photometer ([55]) When the area to be measured is small, a modification of the Martens photometer (see p. 159) may be used ([56]). These methods, however, are open to the objection that the exposed film acts to a greater or less extent as a diffuser ([57]). Consequently the only certain measure is that of diffuse transmission factor (*vide supra*). Various instruments have been devised for the purpose. In most of these a sheet of opal glass is placed behind the negative so as to diffuse the incident light ([58]). The candle-power of a given area of this opal glass is measured with and without the photographic plate in front of it, and the transmission factor is taken as the ratio of the figures thus found A convenient form of the apparatus is that illustrated in Fig 258 ([59]), which gives a develop-

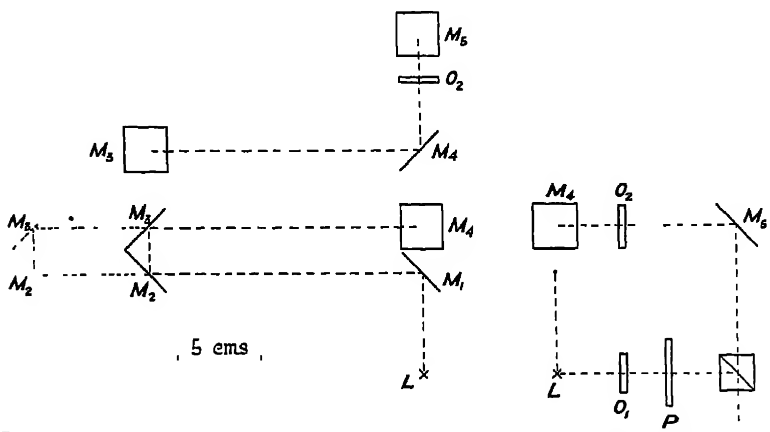

FIG 258—The Measurement of Diffuse Transmission of a Photographic Plate.

ment of the light path. The grid filament electric lamp L is enclosed in a screen with two apertures. From the first of these the light passes upwards through the opal glass O_1, and the plate P whose density is to be measured, to a horizontal Lummer-Brodhun cube From the other aperture the light proceeds horizontally, and is reflected at the vertical mirror M_1 to the pair of movable mirrors M_2, M_3. From these it is reflected back to the 45° mirror M_4, whence it proceeds vertically through the opal glass O_2 to the mirror M_5. It then passes through the Lummer-Brodhun cube The photometric balance is obtained by sliding M_2, M_3 along a bench in the direction shown, as in the Martens illumination photometer (see note (14), p 372)

Alternatively, the diffuse transmissometer described above (p. 390) may be used in the ordinary way, the photographic plate being inserted between the two spheres ([60]).

For the measurement of very low densities the thalofide cell (see p 325) has been found useful on account of its high sensitivity ([61]) For very high densities an instrument giving a very high illumination must be used ([62])

The selenium bridge can also be used for transmission measurements, two beams of light originating from the same source being thrown in turn upon the bridge. One of these beams passes through the plate to be measured, and the other through a calibrated neutral wedge The position of the wedge is altered until the resistance of the bridge is unaffected by passing from one beam to the other ([63])

Microphotometry.—In such work as stellar photometry or the study of line spectra by photographic means it is necessary to measure the density at any point of an image on an exposed plate ([64]) Since the variation of density from point to point is frequently very rapid, the problem really reduces to that of measuring the transmission factor of an exceedingly small area of a photographic plate The instrument used for this purpose includes a microscope as an essential part, and this special branch of photometry has received the name "microphotometry" Microphotometry also provides a method of testing the performance of any type of photographic emulsion as regards sharpness of image ([65]) The first microphotometer was that of Hartmann ([66]), which is shown diagrammatically in vertical section in Fig. 259. *P* is the plate under examination, supported on an ebonite table *L*, over the surface of which it can be moved at will so as to bring any desired portion of *P* into the centre of the field of the microscope *ABG*. This microscope has a common eyepiece *A* with a second horizontal microscope *ABD* At *B* is a glass cube, constructed after the manner of a Lummer-Brodhun cube, as shown in detail above. The image of part of the plate, isolated by means of a diaphragm ([67]) in order to avoid errors due to light reflected from the objective of the microscope, is brought to a focus by means of *H* at the centre of the side *ab* of this cube, and is viewed at *A* by total reflection from the silvered part *gh* The comparison surface *O* is focussed, by means of *E*, at the same point, and is viewed by transmission through *ag* and *hb*. The silvered area *gh* may conveniently be elliptical in form so as to appear circular from *A*. Its apparent diameter must depend on the size of the area of *P* to be included, and on the magnification produced by *G* The latter may conveniently be as high as 12.

The comparison surface at *O* is a neutral wedge, which may conveniently be a strip of photographic plate exposed in such a manner as to produce a gradually increasing density from one end to the other Methods of preparing such wedges are described on p 180 This strip may be calibrated by inserting filters of known transmission factor (determined as described in Chapter VI, p 182) in place of *P* ([68]) The use of a substitution method such as this avoids the necessity for correcting for the difference between the two parts of the microscope and between the illuminations behind the two surfaces *P* and *O*. In order to secure that these illuminations shall bear a constant ratio to each other, the system *SRT* is employed.

This consists of a matt translucent glass plate R, illuminated from outside the tube either by daylight or by an artificial source, and two silvered mirrors S and T, which reflect the light from R to P and O respectively One of the objections to the Hartmann instrument

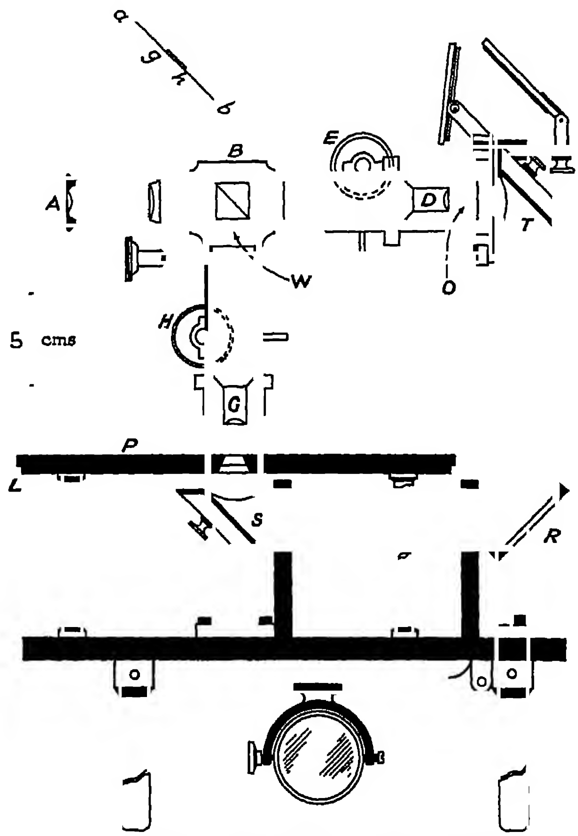

FIG 259 —The Hartmann Microphotometer

is that, owing to the considerable magnification necessary, the grains of silver in the image may become separately visible This difficulty has been overcome in two later instruments.

The first is simply a modification of the Hartmann instrument (69) by the insertion at W of a diaphragm with a very small aperture, upon which is focussed the part of P under examination. The silvered part of the prism B is then diffusely illuminated by light from that region of P whose image covers the aperture in W.

A still later instrument, in which the Maxwellian method of view is employed (70), is shown diagrammatically in vertical section in Fig. 260 The light from the illuminated aperture A is rendered parallel by the lens O, and one-half of the resulting beam is brought to a focus by the lens L_2 at the point of the plate P which is being measured. A second image of A is similarly formed by O and L_1 at a point of the neutral wedge W The two beams of light from these images are brought to a common focus at the eyepiece E,

one by reflection, and the other by transmission in the Lummer-Brodhun cube L-B Since the aperture in E is sufficiently small to act as a stop to both beams, the brightness of each of the two comparison surfaces in L-B appears uniform In making a measurement

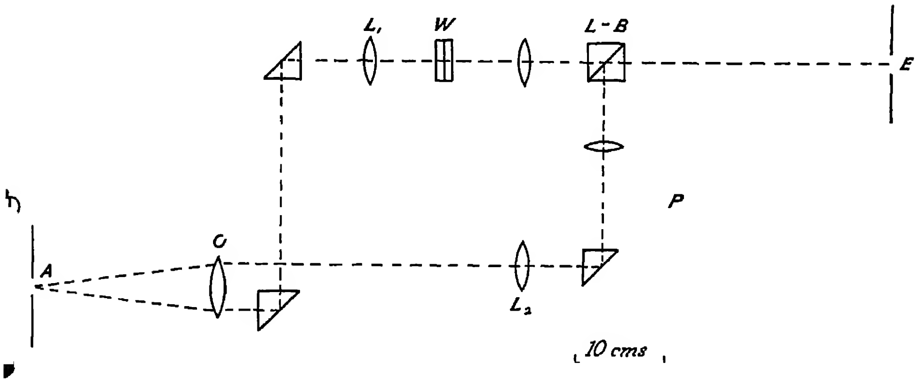

FIG 260 —The Fabry-Buisson Microphotometer.

the transmission of W is altered until a photometric balance is obtained in L-B An instrument has also been described in which a polarisation method of obtaining the photometric balance is employed ([71])

It is evident that, unless the image in the photographic plate be truly neutral, the transmission factor found will depend on the colour of the light used for the measurement. The same is true if the wedge used as a standard of comparison be not neutral

An instrument has been designed which depends on measuring the brightness of a strip of incandescent tungsten viewed through the photographic plate ([72]). The measurement is made by altering the current through the filament of a small comparison lamp contained in the eyepiece of a microscope until it disappears on the image of the strip, when both are viewed through a red pyrometer glass.

As would naturally be expected, it is found that different values are obtained for the density of a photographic plate according as it is illuminated by a parallel beam or by diffused light

The principle of the Hartmann microphotometer has been used in conjunction with photo-electric cells in order to obtain a continuous record of the variation of transmission factor along any given line on a photographic plate, as, for example, along the centre of a spectrum band The apparatus used for this purpose by Koch ([73]) is shown diagrammatically in Fig 261 N is a small source of light of high surface brightness, such as the filament of a gas-filled lamp The lens A forms an unmagnified image of N at that part of the plate P which is being measured The microscope objective B forms a second image within the aperture of a diaphragm W, and the light which passes through W illuminates the photo-electric cell Z, which may be of the form described on p 326 An auxiliary glass plate and microscope enable the position of P to be correctly adjusted on the table before the measurements are begun The current (from the 180-volt battery L) which traverses Z passes

through a safety resistance to the string electrometer E. The artificial leak from E to earth is formed by the system of photo-

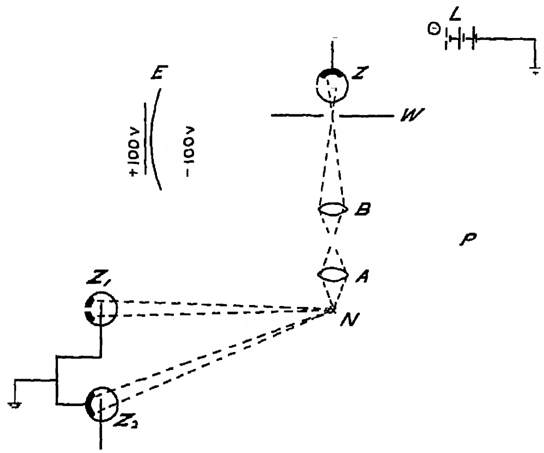

FIG 261 —The Photo-electric Microphotometer

electric cells Z_1 and Z_2, both of which are also illuminated by N. By this means small variations in N have no effect on the results of the measurements The table which carries the plate P is moved across the image of N by a clockwork motor, which also drives a photographic film on which is obtained a continuous record of the deflection of E Since the deflection is proportional to the illumination, it follows that this record is a graph of the transmission factor of P along the line of travel

A registering microphotometer in which a thermopile is used instead of the photo-electric cell has been described ([74]) The instrument, which is shown diagrammatically in Fig 262, consists of a lens C, which projects an image of a lamp filament on the slit S_1. The microscope objective O_1 forms an image of S_1 on the plate P, and O_2, a similar objective, throws an enlargement of this image on S_2 T is a sensitive thermopile connected to a d'Arsonval galvanometer of suitable type With this instrument it is possible to obtain a record of the transmission curve of an Abney

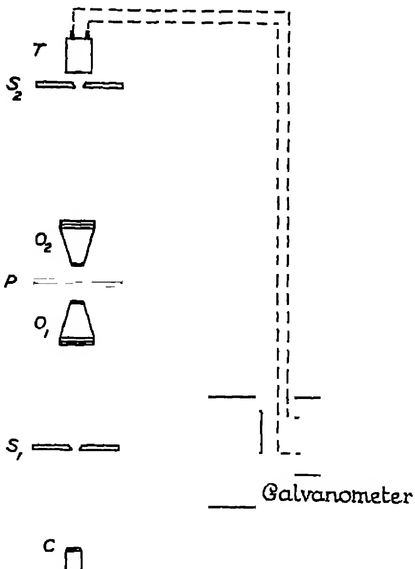

FIG 262 —The Thermopile Microphotometer

test plate (lines 0 02 mm wide and 0·02 mm apart), and the relative photographic intensities of the different elements of a line spectrum may readily be obtained.

A microphotometer which can be used to measure either the transmission or reflection factors of very small surfaces has been described by Nutting ([75])

It should be noticed that in the microphotometry of very fine detail there is an inherent source of error, analogous to the error in spectrophotometry arising from the use of a finite slit width (see p. 287) This will be clear from Fig 263 If curve *A* represent the true transmission curve of a line on the plate, the effective "slit width" of the microphotometer being 10 units, then the density assigned to the point *A* will be the mean ordinate within the region *BC*, so that the values obtained on a convex portion of the curve are too low, and those on a concave portion are too high. The maximum density of a line, therefore, is always reduced, unless the size of the microphotometer field be very small compared with the breadth of the line.

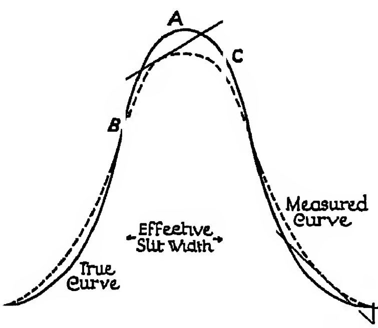

FIG 263 —The "Slit-width" Error in Microphotometry

ABSORPTION

The Measurement of Absorption Factors.—For all practical purposes it may be said that there is no method of measuring the absorption factor of a substance directly, since there is no means of evaluating the light unless it can be caused to illuminate a photometric test surface. It follows, then, that absorption can only be measured as a difference, using the relation $\rho + \alpha + \tau = 1$ In the case of an opaque substance $\tau = 0$ and $\alpha = 1 - \rho$, so that all that is necessary is a measurement of reflection factor Since, in general, the reflection factor of a surface depends on the direction and colour of the incident light, so also must the absorption factor similarly depend on these variables. In the case of a transparent or translucent body, both ρ and τ must be measured in order that α may be found For a truly transparent body $\rho = 0$, except at the surface, so that $\alpha = 1 - \tau$ when surface effects have been allowed for in the measurement of τ Although α is thus found indirectly from a measurement of τ, it is for many purposes a far more convenient constant to use, since for transparent bodies it is connected with the thickness by the simple exponential relationship given in Chapter IV. (p 116).

Atmospheric Absorption.—When objects have to be illuminated from considerable distances, or when a luminous object has to be viewed from afar, the absorption of light by the atmosphere, in reality a combination of true absorption and scatter, becomes very important. It has, too, to be taken into account when making photometric measurements over considerable ranges in the open

(see p 418, Chapter XIV) It is therefore necessary to have sor means of measuring the transmission factor of a certain length the atmosphere at any given time. One of the chief difficulties the measurement lies in the fact that when the absorption is mo important it is most liable to rapid and extensive fluctuations (⁷' In its simplest form the apparatus used for this measurement consis of (i) an evenly illuminated surface set up at a convenient distan (of the order of half a mile) from the observing station, and lar enough to fill completely the field of view of (ii) an instrume capable of measuring brightness. If the size of the surface is n to be prohibitive (*e g* , more than 10 ft square) an ordinary portab illumination photometer, such as the lumeter (see p. 353), cannot l used for the brightness measurement (see p 401) at distances more than about 200 yards, except by the use of an undesirab small aperture In general, therefore, a telescope is used A image of the illuminated surface is formed in the focal plane of tl eyepiece (see Fig 264), and in this plane is an opaque white diaphrag illuminated by a compariso lamp The aperture in th diaphragm is slightly small than the image of the illu minated surface, so that th eye sees a continuous fiel A photometric balance obtained by varying the illu mination of the diaphrag surface If a telescope wit an objective of 3 ft foc length be used, a 10-ft scree at a distance of half a mil gives an image which will fi an aperture of $\frac{1}{8}$ inch dia meter This instrument i called a " telephotometer "

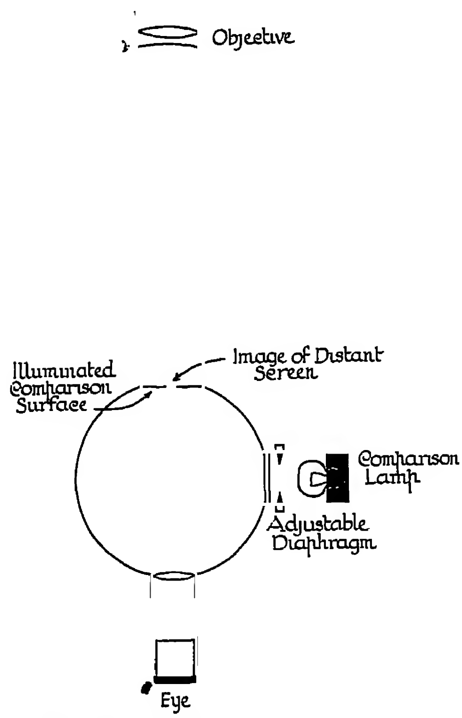

FIG 264 —Telephotometer (diagrammatic)

A different form has bee devised with the object c avoiding the necessity for large screen ([77]) This i shown diagrammatically i Fig 265 L_1 is a lamp o known candle-power I_1 place at the distant station O_1 i a telescope objective, whic forms an image of L_1 at th position of the observer' eye E L-B is a Lummer-Brodhun cube, which enables a comparison to be made between the brightness of O_1 and th brightness of O_2, another objective, which forms at E an imag of L_2, a local comparison lamp The brightness of O_2 is varie by means of a pair of Nicol prisms (not shown) placed between L and O_2. The illumination of O_1 is $I_1/O_1L_1{}^2$ Now in the Max wellian view the brightness of the lens surface is $\tau E v^2$, where E is the illumination at the lens and τ its transmission factor, whil

v is the distance of the eye from the lens (see p. 109) It follows that the brightness of O_1 is $\tau I_1 v^2/O_1L_1^2$. But $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$, so that the brightness becomes $\tau I_1 f^2/(u-f)^2$. This is independent of the size of the lens, but it is necessary to ensure that the image of each of the sources is smaller than the pupil of the eye, so that the latter does not act as a stop. The instrument may be calibrated by measuring L_1 on a clear day at a distance of a few hundred yards (u_1), so that atmospheric absorption is negligible If the angle of rotation of the movable Nicol be θ_1 in this case, then

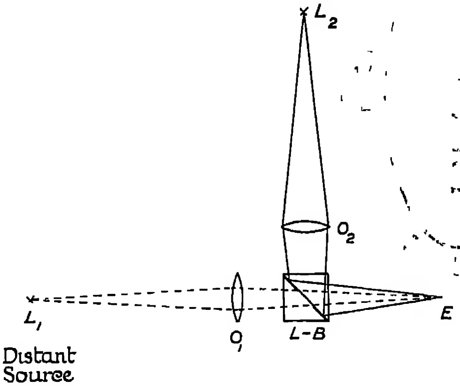

FIG 265 —The Telephotometer with Maxwellian View

$$I_1 f^2/(u_1-f)^2 = K\cos^2\theta_1,$$

where K is the instrument constant If now the lamp be measured at a distance u_2, the atmospheric absorption over the distance (u_2-u_1) is given by

$$\sqrt{1-\alpha}\,(u_1-f)/(u_2-f) = \cos\theta_2/\cos\theta_1$$

Instruments have been designed in which a mirror placed at the distant station reflects to a photometer a beam of light from a powerful source near the photometer The reflected beam is then compared with a direct beam from the source (78) If this method be used, the mirror at the distant station must be optically worked to an exact plane, or the results will be quite valueless owing to the introduction of lens effects.

The chief disadvantage of the ordinary telephotometer is that the light scattered towards the photometer by the particles between it and the comparison surface causes the brightness of the latter to be increased In other words, the error is that which would be introduced into the measurement by ordinary methods of the transmission factor of a vessel containing a slightly turbid liquid This defect is overcome in an instrument which measures the contrast between the comparison surface and its background (79) The lens O (Fig 266) forms an image of the comparison surface (which must have a sharp edge and a non-luminous background) on the translucent screen S, behind which is placed a black disc of such a size and shape that the image *exactly* fits it The screen is illuminated from behind by the comparison lamp L. The photometric balance is obtained by moving L or (as shown) by using a variable diaphragm D over O If B represent the brightness of the image of the comparison surface, it follows that $B\rho = B'$, where ρ is the reflection factor of the screen S and B' is its brightness due to transmitted light It is assumed that the "haze" due to scattered light is sufficiently uniform to produce an equal increase of brightness on

both sides of the edge of the image, so that it does not affect the photometric balance If measurements at different parts of the spectrum be desired, colour filters may be placed in front of the eye.

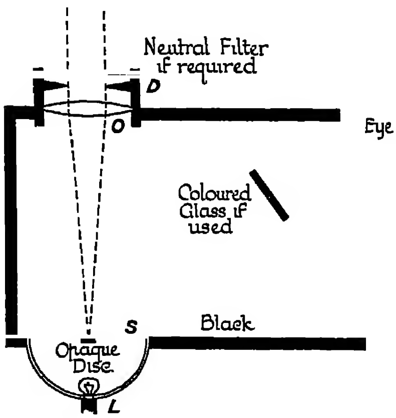

FIG 266—Richardson's Absorption Photometer

Variation of Absorption with Colour.—The absorption factor of a misty atmosphere is different for light of different frequencies ([80]) The factor at any spectral region may be found by using a suitable colour screen in the eyepiece of either of the instruments described in the last section Alternatively, a spectrophotometric method may be used over short ranges (of the order of one-half mile). The light from a powerful source is collimated by a suitable lens and projected to a mirror placed at the distant station The reflected beam is then returned to the near station and compared with the original light from the source by means of a spectrophotometer ([81]). As mentioned above, the mirror used at the distant station must be optically flat, or a lens effect is introduced into the measurement Further, the mirror must be of such a size, relative to the collimating lens, that it does not act as a stop in the optical system

BRIGHTNESS

It was pointed out at the beginning of Chapter VI. that the whole science of visual photometry is based on a comparison by the eye of the brightnesses of two surfaces in juxtaposition The ultimate object in view, however, is generally a measurement of the illumination of those surfaces, or of the candle-power of the sources producing that illumination Since the photometric unit is one of luminous intensity, while brightness is luminous intensity per unit area, it follows that the most direct method of measuring the brightness of a surface (supposed uniform) is to measure its candle-power in a given direction and divide this by the projected area of the surface in that direction In order to avoid "edge effects," it is often convenient to place an opaque diaphragm with an aperture of accurately known area (a) close to the surface The combination then acts as a source of candle-power Ba, situated in the position of the *diaphragm* (see p 108) If this candle-power be measured, B is readily found ([82]) It is assumed here that the area of the surface is not so great as to render it impossible to apply the inverse square law in the candle-power measurement This proviso severely limits the application of the method, for it has been shown that if the accuracy of measurement is to exceed 0 5 per cent the radius of a disc source should not exceed 0 07d, where d is the distance from the photometer head Hence if the brightness of the disc be B, the illumination of the photometer surface is 0 005πB, so that for a minimum illumination of 10 metre-candles B must exceed

650 candles per square metre This brightness is equal to that of a matt white surface having an illumination of 2,000 metre-candles, and is much above the brightness of ordinary illuminated surfaces The method can therefore generally be applied only to self-luminous bodies whose area can readily be determined

All other cases fall into two classes, *viz*, those in which the brightness to be measured is below the limit above mentioned, and those in which the area of the surface is small and not readily measurable. In the former class, the most direct method is to use the surface whose brightness is to be measured as one comparison surface in a photometer, and to obtain a photometric balance with a second comparison surface, of which the brightness is variable at will and determinable in absolute value This involves a knowledge of the reflection factor of the second comparison surface in the direction of view, and the provision of a variable illumination ([83])

As an example of this method, the use of the Lummer-Brodhun head for the measurement of brightness may be described. A piece of silvered glass mirror of known reflection factor (see p. 381) is placed over one surface of the plaster screen in the photometer so that, when this faces the surface to be measured, the image of this surface in the mirror is seen through the Lummer-Brodhun cube, and so becomes one of the comparison surfaces in the photometer. The brightness of the other comparison surface is equal to its illumination divided by π and multiplied by the reflection factor of the plaster screen The latter must be determined for normally incident light and angle of view 45° by the method described on p 377 Alternatively, if a surface of known and suitable brightness be available, a substitution method may be used so that the errors due to lack of symmetry in the head are avoided

For purposes where an accuracy of 2 to 3 per cent is sufficient a portable photometer with detached test plate may conveniently be employed ([84]) The reflection factor of the test plate, ρ_T, must be known from previous laboratory measurements by more accurate methods. If, now, the photometer be directed towards the surface to be measured and a photometric balance be obtained in the usual way, using this surface instead of the ordinary test surface, the brightness required is equal to the reading given by the photometer multiplied by the factor ρ_T/π, for the photometer is calibrated to show the illumination of the test plate so that a reading E corresponds to a brightness $\rho_T E/\pi$.

A coloured surface, or a white surface illuminated by coloured light, naturally introduces all the uncertainties of heterochromatic photometry It is desirable to avoid these as far as possible by the use of colour filters. This is especially necessary when the brightness to be measured is below the limit at which the Purkyně effect is active Since by no optical means is it possible to increase the brightness of a surface, it is impossible to avoid this source of error otherwise than by using colour filters whose transmission factors can be determined at normal intensities A problem of this kind is often presented in the measurement of the brightness of substances which are self-luminous owing to either photo- or electro-luminescence ([85]), cathode bombardment ([86]), chemi-luminescence ([87]), radioactive bombardment ([88]), X-ray radiation ([89]), *etc*

Brightness of very small Objects.—When the extent of the surface to be measured is too small for either of the methods described in the last section to be employed directly, a different arrangement of apparatus must be used. An important case is that of a glowing filament, of which it is often difficult to measure the dimensions accurately. If the brightness be not too great, the filament may be placed in front of an extended surface of adjustable brightness, such as a piece of matt glass illuminated from behind. The brightness of this surface is altered until the filament just disappears on its background, and is then measured by one of the means described in the last sections If, however, the brightness of the filament be greater than about 10 candles per square inch (brightness of a gas-filled lamp with diffusing bulb), this method is not suitable, and an alternative in which the background is formed by the image of a tungsten strip or a Nernst filament may be used instead ([90]). The apparatus is shown in Fig. 267. *N* is the standard surface, of which

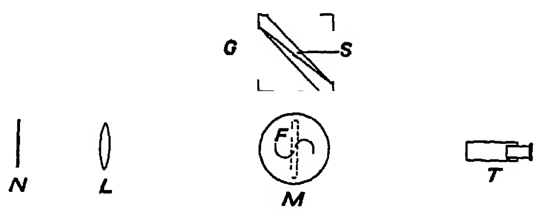

FIG 267 —The Measurement of Brightness of a Glowing Wire

an enlarged image is formed at *M* by means of the lens *L* *F* is the filament to be measured, placed in the same plane as, and superposed on, *M*. *T* is the observation telescope. The variation of brightness of *M* is achieved either (*a*) by altering the current through *N* when very bright filaments are measured, or (*b*) by inserting a pair of Nicol prisms between *L* and *M*.

If a tungsten strip be used at *N* the front surface of its enclosing bulb should be uniform in thickness and as nearly plane as possible, to avoid distortion in *M*. If a Nernst glower be used it should be enclosed in a box to avoid draughts. In either case the apparatus may be calibrated most conveniently by placing at *F* a tungsten filament lamp of standard pattern for which the brightness variation with change of current or voltage may be accurately found on the photometer bench. The absolute filament brightness of this lamp at a low efficiency may be found by causing its filament to "disappear" in front of a "black-body" radiator at a known temperature

The brightness of a flame source may be measured with this apparatus by substituting for *F* the device shown at the top of the figure, *G*. This consists of a skeleton box containing a diagonal double wedge of clear glass in the centre of which is a very narrow strip of silver, *S*. The flame is placed so that a thin line of it, reflected in *S*, is seen by the eye at *T* to be superposed on the image *M*.

Another method of measuring the brightness of a small area of a lumineus object is to form an enlarged real image of the object by means of a convex lens, as shown in Fig 268 (*a*), and to measure

the illumination at the corresponding part of the image by means of any ordinary photometer with a small comparison surface (91) If s be the area of the lens aperture, E the illumination of the photometer surface, and d the distance between this surface and the second focal point of the lens L_1, then the required brightness $B = Ed^2/s\tau$

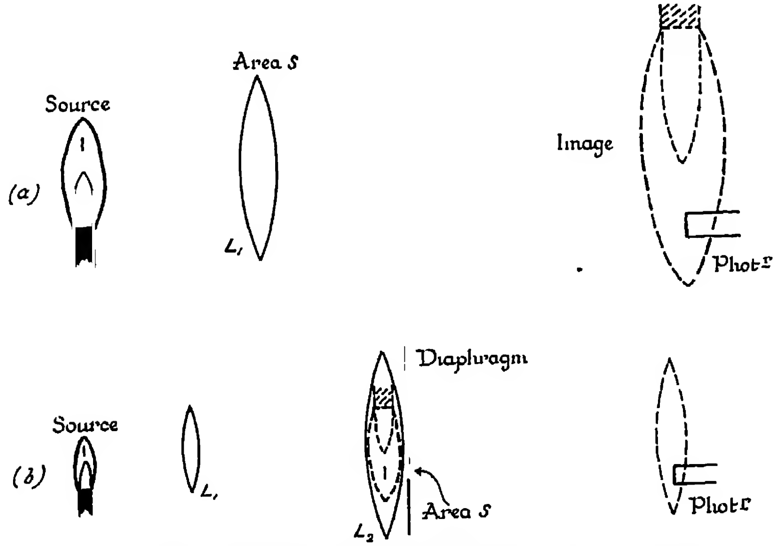

FIG 268 —The Measurement of Brightness of a Luminous Object

(see p. 109) An alternative scheme is shown in Fig 268 (*b*) L_2 is placed so as to coincide with the real image of the luminous object formed by L_1, and a diaphragm with an adjustable opening is placed close to L_2 If a photometer screen be placed in the position of the image of L_1 which is formed by L_2, then the illumination at the photometer gives the brightness of the luminous object by the same formula as before, but the area s is now the area of the opening in the diaphragm. Blondel's "nitometer" is designed on this principle (92)

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CHAPTER XIV

PHOTOMETRY OF PROJECTION APPARATUS

Of all the problems of practical photometry, the one which, it is probably true to say, presents the most difficulty, and is not infrequently carried out under conditions which render the results liable to considerable error, is the photometry of projectors All candle-power photometry depends on the measurement of the illumination produced at a standard surface by a source placed at a measured distance from it (see Chapter VI). In order to determine the candle-power of the source it is necessary to assume that all the light illuminating the surface proceeds from a restricted area which, to the degree of accuracy aimed at in the measurements, may be regarded as a mathematical point. In other words, the source must, for the purpose of the measurement, be such that it can legitimately be regarded as a point source situated at a definite distance from the photometer.

It is frequently impossible to make this assumption in the case of light emitted from an optical device (1), for in many cases this device redistributes the light in such a way that it appears to proceed from a source at infinity, while in other cases different parts of the optical system produce separate images of the primary light source, so that the resulting beam is in reality composed of a number of primary beams, each of which appears to proceed from a different point in space The simplest problem in projector photometry is that in which the optical device produces a single image of the source at some definite position. The image produced by a lens or spherical mirror is of this kind, provided (*a*) the aperture of the optical device be small compared with its distance from the source, and (*b*) the source be not at the focus of the lens or mirror. Examples of projectors of this type are the magic lantern and kinema projector.

In a problem of this kind the only departure from ordinary photometric procedure is that the measurements involved in the application of the inverse square law must be made from the position of the image, and not from the source or the optical device (2) This is only true so long as the whole of the image is visible from every point of the photometer screen It is clear that the inverse square law cannot be applied at all when the optical device is delimiting the image, so that the fraction of the whole which is visible in any particular direction depends upon the distance of the photometer from the device When the image is so much larger than the device that the surface of the latter appears bright all over, the device itself may be regarded as the source, if the image be of uniform brightness, for the edge of the optical device then acts in the same manner as a diaphragm placed in front of a bright surface (see p 108) * (3)

* The above conclusion applies equally if the image be real, and lie between the device and the photometer.

It must be remembered that in the case of a magnified image, even if the optical device be not acting as a stop, it may be necessary to use the photometer at a greater distance than would be necessary in the case of the original source For example, in the case of a line source of length $2l$, placed along the axis of a lens of focal length f, with its centre at a distance u from the lens, by means of the formula given on p. 22 it is easy to show that the length of the image formed by the lens is $2f^2l/(f+u-l)(f+u+l)$ If, then, $f = -6$ inches, $l = 0.5$ inch, and $u = 3.5$ inches, the length of the image is 6 inches, and the minimum permissible distance of the photometer from the image has to be six times that which would be allowable in measuring the original source

Parallel Beam Projectors.—Probably the most important case of projector photometry is that in which the source is at the focus of the optical device, so that the image is at infinity This is the problem presented in the photometry of searchlight projectors, where the source is placed at the focus of a parabolic mirror, or of lighthouse lanterns, where the source is at the focus of a plano-convex lens These two cases will be considered separately The parabolic mirror has been very fully dealt with by F A Benford (4). In the case of a uniform point source the rays proceeding from the mirror are all parallel, so that the illumination is constant at all distances from the mirror, and the apparent candle-power is therefore entirely dependent on the distance of the photometer The

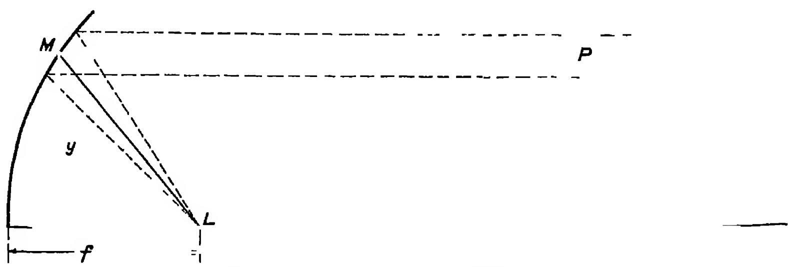

FIG 269 —Projector with a Point Source

illumination at any point in the beam is easily obtained. For (Fig 269) if the candle-power of the source be I, the flux density at M is I/LM^2, since the flux per unit solid angle from L is I lumens If the focal length of the parabola be f, then LM is given by $LM^2 = y^2 + \left(f - \frac{y^2}{4f}\right)^2$, where y is the distance of M from the axis

Hence the illumination at a point in the beam P, distant y from the axis of the mirror, is $\rho I / \left(f + \frac{y^2}{4f}\right)^2$, where ρ is the reflection factor of the mirror The distributions of illumination for mirrors of various focal lengths can readily be obtained from Fig 270.

The case of a source of finite size is somewhat different If this source have a definite brightness B, which is the same in all directions, then from a distant point P on the axis of the beam the whole surface

of the mirror appears to have the brightness $B\rho$, for (Fig. 271), considering an element M of the mirror surface, since the angles of the incident and reflected beams are equal, the angular density

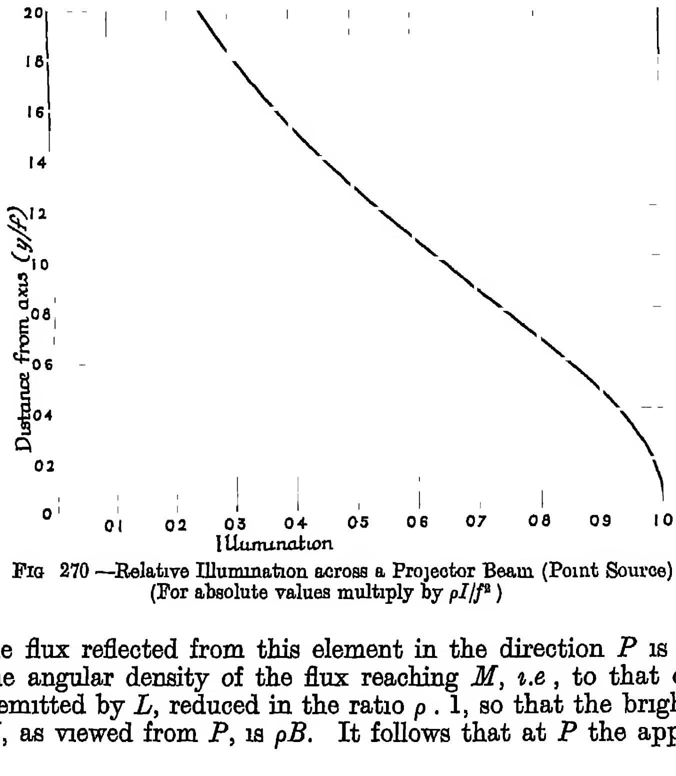

FIG 270 —Relative Illumination across a Projector Beam (Point Source) (For absolute values multiply by $\rho I/f^2$)

of the flux reflected from this element in the direction P is equal to the angular density of the flux reaching M, i.e., to that of the flux emitted by L, reduced in the ratio ρ . 1, so that the brightness of M, as viewed from P, is ρB. It follows that at P the apparent

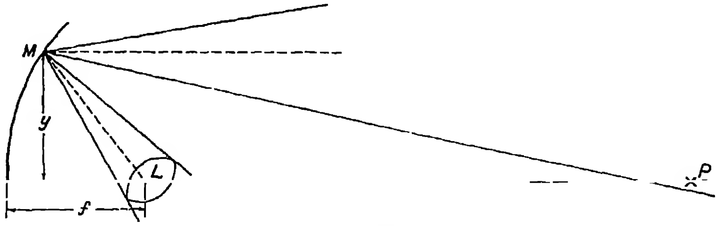

FIG 271 —Projector with a Finite Source

candle-power of the whole mirror is $\pi\rho R^2 B$, where R is the radius of the mirror This holds whatever be the shape of the source, provided its brightness be uniform in all directions In the case of a disc source, it must be remembered that the candle-power is zero at angles of emission greater than 90°, so that a mirror embracing a total angle of more than 180° is of no advantage in this case

In the above discussion of the apparent candle-power of a mirror it is assumed that P is situated so far from the mirror that the latter appears bright or " flashes " all over In other words, it is assumed that the elementary beams due to two opposite points at the extreme

edge of the mirror cross at some point between P and the mirror (see Fig. 272), so that P receives light from every part of the mirror surface. No point, such as P' or Q, for which this is not the case can be assumed to have an illumination based on the formula for the apparent candle-power given above, and within the crossing point on the axis the inverse square law must not be assumed to hold

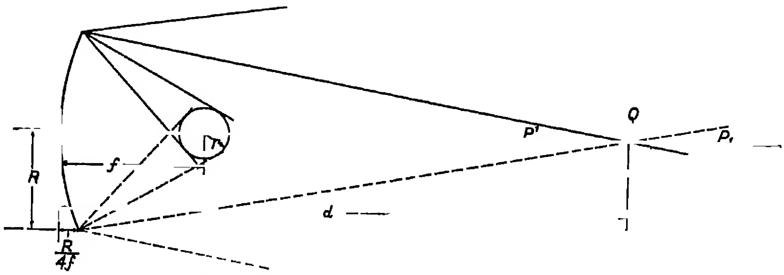

FIG 272 —Projector with a Spherical Source

For a spherical source of radius r (Fig. 272) ([5]) the limiting distance d for the application of the inverse square law along the axis of the mirror is given by

$$R/\left\{d - \frac{R^2}{4f}\right\} = r/\left(f + \frac{R^2}{4f}\right),$$

so that

$$d = \frac{R^2}{4f} + \frac{R}{r}\left\{f + \frac{R^2}{4f}\right\}.$$

For a mirror of 60 cm aperture and 30 cm focus, with a source of 1 cm radius, d is 11·3 metres.

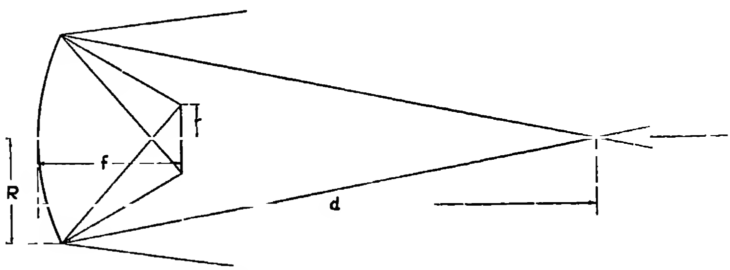

FIG 273 —Projector with a Disc Source

For a disc source (see Fig. 273), instead of r in the above expression, it is necessary to write

$$r(f - R^2/4f)/(f + R^2/4f),$$

so that ([6])

$$d = R^2/4f + R\left(f + \frac{R^2}{4f}\right)^2 \Big/ r\left(f - \frac{R^2}{4f}\right)$$

It is to be noted that this expression becomes infinite when $R = 2f$, *i.e.*, when the mirror embraces a total angle of 180° with a disc source In the case of the dimensions used in the above example with a spherical source, d is now equal to 18·8 metres

The case of prism reflection may be treated similarly, for in this case, too (see Fig 274), the angular density of the incident flux is equal to that of the reflected flux, provided the prism be isosceles. This is the case of the outer catoptric elements of a lighthouse lens.

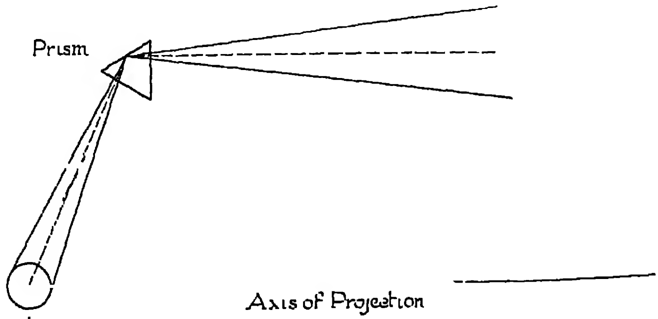

FIG 274 —Prismatic Reflection

The inner elements, however, depend on refraction If in Fig 275 the light from a source situated at L be refracted by the prism K so as to emerge in an approximately horizontal direction, it is easy to show that, in the special case when the first refracting face of the prism is vertical, if n be the refractive index of the glass (see

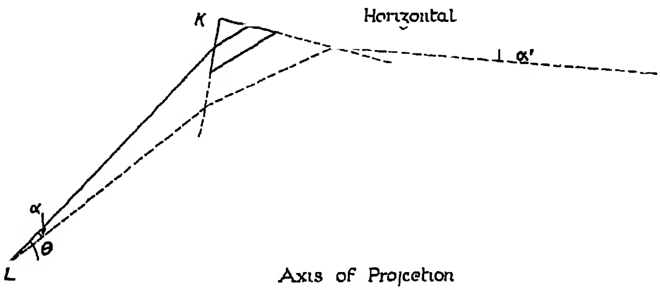

FIG 275 —Projection by Refraction

p. 20), incident rays inclined at an angle α will be refracted and emerge at an angle α', where $\alpha'/\alpha = \cos\theta\ (n^2 - \sqrt{n^2 - \sin^2\theta})/\{n^2 - \sin^2\theta - \sqrt{n^2 - \sin^2\theta}\}$ The value of this expression will be found to *increase* from 1 as θ increases from zero, so that for a constant value of α the elementary beams given by the more extreme elements of the lens will be more divergent than elementary beams from the intermediate elements If f be the focal length of the system and the source a sphere of radius r, an element at an angular distance θ from the axis will give a beam having a semi-angular depth of α', found by putting $\alpha = (r/f)\cos\theta$ in the above expression Since the distance d at which this beam meets the axis is equal to $y \cot\alpha'$ or y/α', *i.e*, to $(f/\alpha')\tan\theta$, it follows that the minimum distance at which the inverse square law can be applied may be found in a manner similar to that described for the reflector The problem of the disc source may be treated similarly (7)

In all the above cases it will be noticed that, to a first approximation, d varies as R/r, as might, indeed, be expected *a priori* [8]

In the above discussion of the minimum distance at which it is safe to apply the inverse square law for the calculation of apparent candle-power there are three important considerations which have not so far been mentioned These are as follows —

(i) It has been tacitly assumed that the size of the optical device is so small compared with d that no appreciable error would be introduced in applying the inverse square law to a uniformly bright disc of the same radius (R) (see p 102).

(ii) Throughout the work d has been measured from the centre of the optical device itself, it must not be assumed that this point is the effective source from which distances are to be measured. This is not so, for in the case of the parabolic reflector, since the whole mirror surface appears to have a uniform brightness, *viz.*, ρB, it may be replaced by a disc of this brightness and of the same area as the aperture of the mirror placed in the plane of the front edge of the mirror [9] In the case of the lens projector the brightness is not quite uniform, but the error introduced by assuming the effective source to be in the plane of the lens is quite small

(iii) All the above work has referred to candle-power measured in the direction of the axis of the beam While this is the most important direction in the majority of problems, it is frequently necessary to determine the candle-power distribution across the beam, and the method of finding the value of d for various directions inclined to the axis will therefore be described in the case of the parabola The dioptric lens may be treated similarly, but the calculations are more lengthy [10].

Variation of Apparent Candle-Power across the Beam.—From Fig 276, if L be a disc, and d the distance from the mirror at which

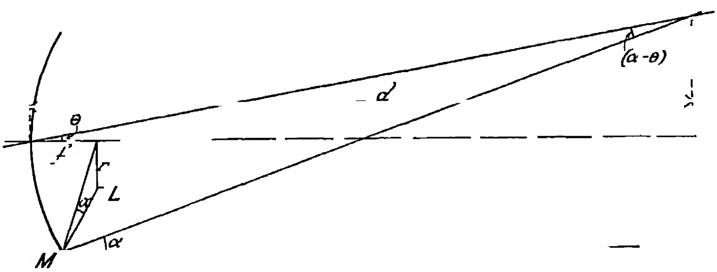

FIG 276 —Beam Measurements at Oblique Angles

the extreme ray of the elementary beam proceeding from M crosses the line passing through the centre of the mirror and inclined to the axis at an angle θ, then, to a first approximation, since α and θ are both small, $R/d = \alpha - \theta$, and $\alpha = r(f - R^2/4f)/(f + R^2/4f)^2$, so that

$$d = \frac{R}{r(f - R^2/4f)/(f + R^2/4f)^2 - \theta}.$$

d becomes infinite when $\theta = r(f - R^2/4f)/(f + R^2/4f)^2$ With the dimensions given in the example used above this value of θ is 0·016, or about 0 9°. When θ is 0 015, however, d = 300 metres. It follows

that in this case, where the maximum beam divergence is 0·033 (semi-angle), the inverse square law may be applied to measurements of illumination made at distances of 300 metres or more over an angular breadth from the axis of the beam of only 0·45 times the total breadth. While this is theoretically the case, in practice the variation of the illumination with distance departs but little from this law for wider angles, since the elementary beams which cross the observation line between the 300-metre point and infinity contribute but little to the total illumination. For example, with the projector system already calculated, when $\theta = 0{\cdot}03$, *i e*, only 10 minutes of arc from the extreme edge of the beam, the percentage error introduced by measurements at 300 metres instead of infinity is less than 1·6 per cent. It thus appears that measurements of such a projector system may be made at a range equal to about 1,000 times the radius of the projector when the semi-divergence of the beam is not less than 2° ([11]). For approximate work one-half, or even one-quarter, of this range may be sufficient, but in practice it has to be remembered that the above discussion is based on the assumption of a perfect mirror. The imperfections and irregularities of form met with in practical apparatus make it desirable to use an even longer range than that indicated above if really accurate results are required.

Photometry of Large Projectors. Searchlights.—For large projectors there is a very serious practical difficulty in the use of long ranges, and this may well counterbalance the extra accuracy theoretically obtainable at greater distances. Searchlight projectors of double the size used in the above examples are now common, so that ranges up to at least half a mile are necessary, with the result that measurements have to be made in the open, or at least the beam from the projector has to traverse half a mile or more of atmosphere at a distance, generally, of a few feet above the ground. The result is that absorption of light by a very slight amount of mist or other suspended matter in the air causes a reduction in the beam intensity at the end of the range, which may be as much as 20 to 30 per cent. before the presence of the mist, *etc.*, becomes noticeable to the eye ([12]). If atmospheric absorption be present to any marked degree it is impossible to obtain any satisfactory photometric measurements, for the rapidity with which the transmission factor varies, both from time to time and from place to place, makes it a very uncertain method to attempt to allow for the variations by subsidiary measurements such as those to be described later. The interference due to ground mist may generally be much reduced if the beam be projected across a valley, the projector and the photometer being on the opposite slopes of the hills on either side.

In determinations of the apparent candle-power in various parts of the beam it is desirable to move the projector in altitude and azimuth, since at a range of half a mile a beam of 4° total divergence has a linear diameter of over 60 yards, and an angular movement of the projector is usually much easier to arrange than a lateral movement of the photometer over such a long distance ([13]). In the case of vertical distribution, tilting the projector is clearly the only course practically possible. Angles may be accurately determined by means of large scales attached to the projector or, more accurately,

by means of a small telescope rigidly fixed to the projector and reading on bold horizontal and vertical scales placed at a distance of 50 to 100 feet in front of it. The projector and photometer stations should be in constant communication, preferably by telephone

The actual photometric apparatus may take many different forms A very convenient arrangement consists simply of a standard surface supported vertically, or so as to be normal to the light from the projector. The illumination at this surface is then measured by means of a portable illumination photometer, and the candle-power is calculated by means of the inverse square law. Alternatively, a Lummer-Brodhun or other photometer head may be mounted on a small photometer bench whose axis is in the direction of the beam A small comparison lamp on the side of the photometer away from the projector enables candle-power measurements to be made in the ordinary way A difficulty in this method is the very wide range to be covered, the apparent candle-power in the centre of the beam being often 100 times that near the edge

Nearly all outdoor projector photometry, except that of very high candle-power searchlights at close range, has to be carried out at night If, however, the illumination to be measured exceed about 1 metre-candle, approximate measurements may be carried out in the daytime by suitable screening arrangements, such as those shown in Fig 277 The area of the aperture in the front screen

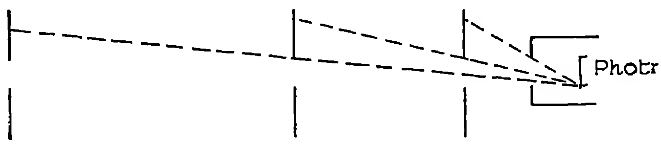

FIG 277 —Screening for Daylight Photometry of Projectors

should be the minimum permissible, *i e*, the same as that of the photometer standard surface, so that if the distance between this screen and the photometer be, say, forty times the diameter of the aperture (*e.g*, 2 metres for a 5 cm photometer disc), the "daylight factor" of the card is about 0 02 per cent It follows that on a day in which the general illumination is 10,000 metre-candles, if the background of the projector have a reflection factor of about 10 per cent, the illumination of the photometer surface due to stray light will be $1{,}000 \times 0\ 0002$, *i.e*, 0 2 metre-candles This illumination can be determined to the necessary accuracy by means of blank measurements before and after the experiment, and the amount found can then be subtracted from the measured illumination in the beam

In all photometry of projection apparatus it is essential that the source shall be maintained accurately at the focus of the optical device This is not always easy to arrange when the source is an electric arc in which the carbons are gradually being consumed. The provision of a lens and scale device, termed a "focus-scope," at the side of the barrel is a great help in this adjustment. Since the arc is frequently difficult to control for long periods, the value of

telephonic communication between the projector station and the photometer station can hardly be over-estimated.

Arc-light photometry with ordinary comparison sources has the great disadvantage of introducing a large colour difference into the photometer field. Since the accuracy aimed at (or, indeed, attainable) is generally not higher than 2 to 5 per cent, the use of coloured glasses to produce an approximate colour match is almost universal in searchlight photometry

Atmospheric Absorption.—Various methods have been used for measuring and allowing for atmospheric absorption when this is not more than 10 to 20 per cent These methods may be referred to as (i.) direct measurement, (ii.) the standard-beam method, and (iii.) the double-range method.

The first of these methods consists in a subsidiary determination of the absorption factor of the atmosphere by means of a telephotometer or other special apparatus (see p 398)

The second method really relies on the same principle as the first. A known constant source having a high surface brightness, such as a tungsten arc lamp, is placed in a standard projector at one end of the base, and the axial beam candle-power is measured at the other end in exactly the same way as for the test projectors. Allowance for atmospheric absorption is then made as before.

The third method consists in having two photometer stations instead of one and making measurements of the illumination at a given part of the beam at both stations. If these are nearly in a straight line with the projector, only a slight movement of the beam is necessary, and the two measurements can be made very quickly one after the other If the distances of the stations be d_1 and d_2, then when the transmission factor of the atmosphere is τ per unit distance the illuminations will be respectively $E_1 = I\tau^{d_1}/d_1^2$ and $E_2 = I\tau^{d_2}/d_2^2$, or, eliminating τ, $d_2 \log (d_1^2E_1/I) = d_1 \log (d_2^2E_2/I)$. The calculations are much simplified if $d_2 = 2d_1$, so that $I = d_1^2E_1^2/4E_2$ This method depends on the assumption that τ is constant over the whole range d_2, and consequently very contradictory results are sometimes obtained, especially over land, where slight ground mists, which are the chief source of trouble, are very variable in density from place to place (14).

Lighthouse Projectors.—The lens system used for lighthouses consists of two parts, *viz*, a central convex lens, divided into a number of steps known as "Fresnel" or "dioptric" elements, and an outer zone of total reflection prisms. Both of these systems are so arranged as to give a horizontal beam of light of the smallest divergence possible with the size of source used. The source is often a large incandescent mantle and may, to a first approximation, be regarded as spherical. The whole optical system may be as much as 10 feet in diameter. From the theoretical treatment given on p 414 it will be seen that the measurement of apparent candle-power of a projector of this kind may be made in exactly the same way as for a parabolic mirror of equal size (15). The total reflection prisms, in fact, act in exactly the same way as a mirror, giving elementary beams whose divergence is equal to that of the beams they receive. The outer parts of the central lens give beams of

increased divergence, so that errors calculated on the same basis as for a mirror cannot at any rate be exceeded by the lens. It follows that for a projector of 3 metres diameter and 150 cm focal length, used with a spherical source of radius 7·5 cm , accurate measurements of apparent candle-power may be made in the axis of the beam at any distance over 37·9 metres

The range required for measurements of apparent candle-power in directions inclined to the axis may be found as in the case of the mirror projector The beam of the projector above quoted as an example has a semi-divergence of rather more than 0 05 radian The error made in a measurement of apparent candle-power at a range of 1,000 metres in a direction which makes an angle of 0 04 radian with the axis is less than 2·8 per cent

Sometimes the method is adopted of measuring the apparent candle-power of separate portions of the whole optical system by blocking out the remainder with opaque screens The candle-power of the whole system is then obtained by adding the candle-powers of the separate portions ([16]). In this way it is possible to make the measurements at shorter ranges, since the range may be reduced more than in proportion to the aperture of the system exposed The position of the photometer must be arranged separately for each element measured, so that the line joining that element with the photometer makes the same angle in every case with the axis of the main beam Thus if a projector is being measured in a direction making an angle α with the axis, the photometer must be placed on the line L_1P_1 (Fig. 278) when the element L_1 is being measured, and on the line L_2P_2 when L_2 is being measured, otherwise the candle-powers for the two elements measured are in directions inclined to each other at the angle L_1L_2/L_1P_1, which is equal to 1·7° if L_1P_1 is 100 metres when L_1L_2 is 300 cm

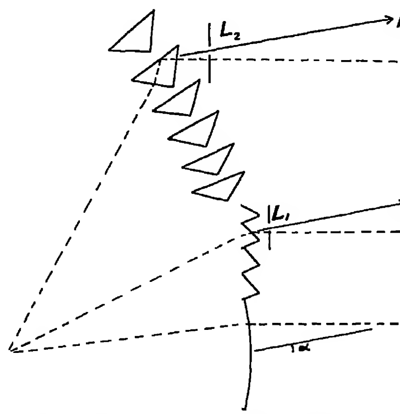

FIG 278.—The Step-by-Step Method of Projector Photometry

Photometry of Small Projection Apparatus.—The same principles as regards the range at which measurements may be made apply in the case of such apparatus as automobile or locomotive headlights as for searchlights, if all the dimensions be reduced in the same proportion For example, if a headlight of 20 cm diameter used with a source of 3 3 mm radius be measured at a range of 100 metres, the measurements will be subject to the same errors as those discussed on p. 416 above. The measurements are simplified, however, by the fact that in a photometric laboratory provided with a suitable long photometer room it is possible to make the measurements indoors

The projector may conveniently be mounted on a turn-table provided with a degree scale, while the turn-table itself is carried

on a kind of cradle such as that shown in Fig. 279 ([17]) A movement of $\pm$ 15° to 20° in altitude is generally sufficient

The distance used in calculating the apparent candle-powers should be that from the photometer surface to the front face of the

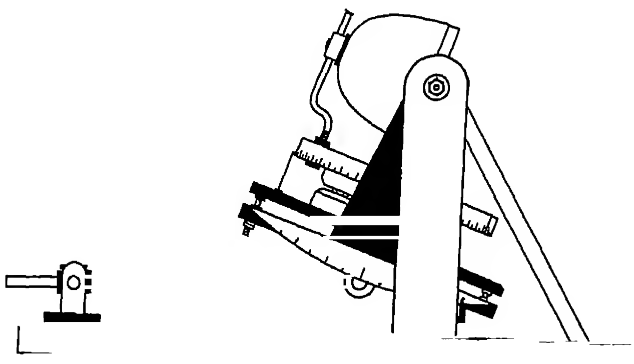

FIG 279 —Tilting Frame for Small Projectors

projector (see p 415) This distance should always be quoted with the candle-power values so that an estimate may be formed of the magnitude of the errors to which the measurements are liable

Heterogeneous Beams.—In some classes of projection apparatus, as, for example, ships' navigation light lenses, the application of the inverse square law to illumination measurements is complicated by the fact that different parts of the optical system form different effective images of the source, and these images may be separated so widely from one another that it is extremely difficult to locate their photometric centre of gravity or to determine the minimum distance at which this centre of gravity may be assumed to remain sufficiently constant for the application of the inverse square law.

In this connection it will be useful to obtain an expression for the centre of gravity of two sources of candle-powers I_1 and I_2, placed on the axis of a photometer bench at a distance apart equal to a

The illumination produced at the photometer screen, distant x, will be the same as that given by a source of candle-power $(I_1 + I_2)$ placed between the original sources and at a distance z from I_2, if $(I_1 + I_2)/x^2 = I_1/(x - z + a)^2 + I_2/(x - z)^2$ Neglecting powers of $\left(\frac{1}{x}\right)$ above the first, this becomes—

$$(I_1 + I_2) = I_1\left(1 + \frac{2(z-a)}{x}\right) + I_2\left(1 + \frac{2z}{x}\right),$$

i e, $(a - z)I_1 = zI_2$ or $z = aI_1/(I_1 + I_2)$, and the inverse square law may be assumed to hold if applied from the position of the centre of gravity so long as the second order terms may be neglected. The error in the illumination at distance x, owing to the omission of these terms, is $\left\{3I_1\frac{(z-a)^2}{x^4} + 3I_2\frac{z^2}{x^4}\right\}$, which, expressed as a fraction

of the true illumination $(I_1+I_2)/x^2$, is $3\{I_1(z-a)^2+I_2z^2\}/(I_1+I_2)x^2$. Putting $z = aI_1/(I_1 + I_2)$, this becomes $3a^2I_1I_2/x^2(I_1 + I_2)^2$, so that the error involved in assuming the inverse square law to hold from the centre of gravity is less than 1 per cent so long as x exceeds $10a\sqrt{3I_1I_2}/(I_1 + I_2)$ If $I_1 = I_2$ this becomes $5a\sqrt{3}$ As the ratio I_2/I_1 departs from unity the accuracy is improved. For instance, if $I_2 = 9I_1$, x may be reduced to $3a\sqrt{3}$.

In projection apparatus where different parts of the optical system produce separate images, the position of the centre of gravity of the images is obtained by measuring the light from each image separately, calculating its position, and then finding the centre of gravity as a result of these measurements, using the ordinary formula $\bar{x} = \Sigma I_1x_1/\Sigma I_1$ In each set of measurements the whole of the optical system, except that being measured, is covered with an opaque screen, so that the light reaching the photometer is derived only from the particular part of the optical system under investigation. It sometimes happens that the position of the centre of gravity of the images, or the "effective light centre" as it is called, is immaterial under the conditions of use In that case the sum of the separate apparent candle-powers of the images produced by the different parts of the optical system is the required apparent candle-power of the whole system, but in basing calculations of illumination on the figure of candle-power thus obtained, it must be remembered that the distance at which the inverse square law may be assumed to hold must be large in comparison with the maximum separation of the various images contributing to the total illumination.

Total Flux Measurements.—For many purposes a measurement of the total luminous flux in the beam given by a projector may be of value quite apart from the details of distribution contained in a curve of effective candle-power This measurement can often be made quickly and conveniently by means of some form of photometric integrator (see Chapter VII) A small projector may be placed inside a sphere or cube so long as the beam is directed towards a suitable part of the sphere wall (see p 214) and the projection apparatus is whitened outside (see p 216) Alternatively, the projector may be placed outside an opening in the sphere so that the beam is projected on to the opposite wall In this case allowance must be made for the effect of the opening, as explained on p 222 If the projector be large it may be convenient to move the source of light away from the focus in such a way as to produce a real image of suitable dimensions, as shown in Fig 280 Allowance must then be made for the alteration in the amount of flux from the source which reaches the effective aperture of the projector For example, a searchlight projector consisting of a mirror of radius R and focal length f, with an arc crater of radius r, receives an amount of flux which may be calculated from the formula given in Chapter IV (p 102), putting d equal to $f - (R^2/4f)$. If the crater be moved a distance 0 1f away from the mirror, a real image of radius 10r is formed at a distance 11f from the mirror, while the flux reaching the mirror from the crater is now less by a calculable amount, so that the readings obtained must be increased in a determinable ratio.

An alternative method depends on the use of the integrating hemisphere described on p 227. The projector may be placed at one end of a long blackened room and the beam projected on to the open hemisphere placed at the other end The total flux contained

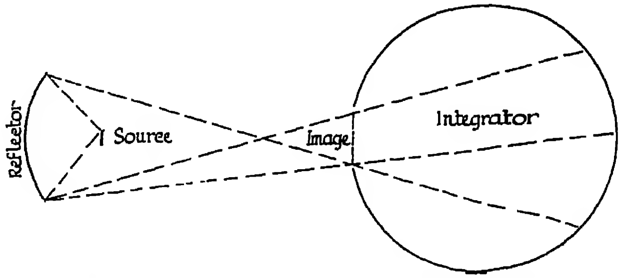

FIG 280 —The Measurement of Total Flux from a Projector

in different zones of the beam may be measured by providing a series of diaphragms to fit over the face of the hemisphere. Owing to the imperfect diffusion of the light by the surface of the hemisphere, especially at large angles of incidence, it is necessary either to use a compensating screen which has been specially designed to equalise the effect of flux reaching any part of the hemisphere when the light is *parallel to the axis*, or, alternatively, a series of slender radial wedges may be placed in the plane of the opening so as to weight the flux reaching the central parts of the hemisphere ([18]) A similar method might be used with a flat surface in place of a hemisphere

Instead of a diffusing surface, a paraboloidal mirror may be used to concentrate the flux on to a small whitened disc placed at or near the focus of the mirror, and the candle-power of this disc may then be measured by means of a photometer This candle-power is proportional to the total flux received by the mirror Apparatus of this kind must be calibrated by means of a steady beam, the total flux in which can be measured by a step-by-step method ([19])

Polished Shades and Reflectors.—An important practical case for the application of the principles described in this chapter arises in the photometry of lighting fittings, such as street lighting units, which consist partly of a polished reflector or a band of refracting elements ([20]) In such cases it is generally possible to obtain some idea of the proportion of light which is "projected" either by refraction or by specular reflection, and to adjust the distance at which photometric measurements are made so that the permissible error is not exceeded in the measurement of the combined candle-power when the inverse square law is applied.

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CHAPTER XV

STELLAR PHOTOMETRY

ONE of the earliest applications of photometric measurement was to the determination of the relative luminous intensities of the heavenly bodies (1). The light-giving power of stars has, since the time of Ptolemy (2), been expressed in terms of an arbitrary scale of "magnitudes," the magnitude of any particular star being obtained by comparison with one or more reference stars in its immediate neighbourhood (3) The scale of magnitudes has always been approximately, and is now strictly, logarithmic, being of the form $\log_{10} E = 0\ 4(1 - m)$, where m is the magnitude of a star, and E its intensity in terms of that of a first magnitude star The ratio of intensity of successive magnitudes is thus $\sqrt[5]{100} = 2\ 512$ (4).

The estimation of magnitude was at first made purely by eye (5), and later by bringing telescopic images of the two stars to be compared into as close juxtaposition as possible, generally by placing inclined mirrors or reflecting prisms in front of the objective (6). Various methods of altering the brightness of one image to equality with the other were used by different workers (7). A favourite method was the reduction of the effective aperture of one objective by means of a variable diaphragm (8), or by a wire gauze screen or a rotating sector (9) In some cases an artificial "star" was used, and the various real stars were compared with this in turn instead of with each other

The Zollner Photometer.—There are three types of photometer chiefly in use for visual measurement at the present time. The first of these is a polarisation instrument with an artificial comparison star, designed by J C F Zollner (10) This instrument has undergone a number of modifications of form at various times (11), but the general principle remains the same, and will be clear from the section shown in Fig 281. An artificial star is formed by a small aperture in the diaphragm A, behind which is placed a source of light of known brightness This source may conveniently be an electric lamp (12) The light from A passes through the Nicol prisms N_1, N_2 and N_3, the quartz plate Q, and the lens L, and is then reflected from the front surface of the glass plate M The intensity and colour of the reflected image are varied by rotating the different parts of the optical train, the colour by rotating N_1 relative to N_2 and Q (13), and the intensity by rotating the whole system N_1QN_2, relative to N_3, by means of the handle F N_3 is fixed in relation to the tube BC, which forms part of the eyepiece of a telescope through which the star is viewed. The telescope is so directed that the image of the star under observation, and the reflected image of A, are sufficiently close to each other for accurate adjustment to equality By an obvious change of design (see Fig 282) the real star may be seen by reflection, and the artificial star by transmitted light (14).

Apparatus have been designed for automatically registering the readings made at night with this instrument [15].

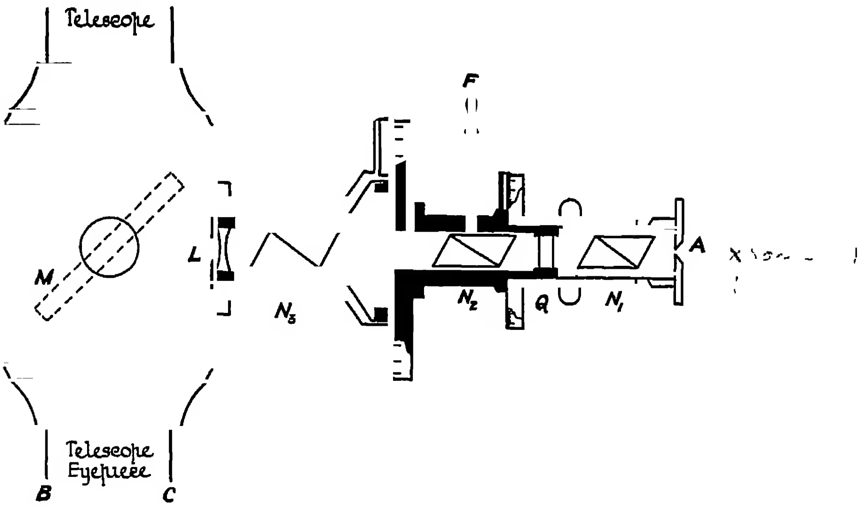

FIG 281 —The Zöllner Photometer

Photometers in which a neutral wedge is used in front of an artificial star to reduce its brightness to equality with that of the image of a real star have been extensively used [16], while an artificial star variable by means of a diaphragm, or otherwise, has also been proposed [17]

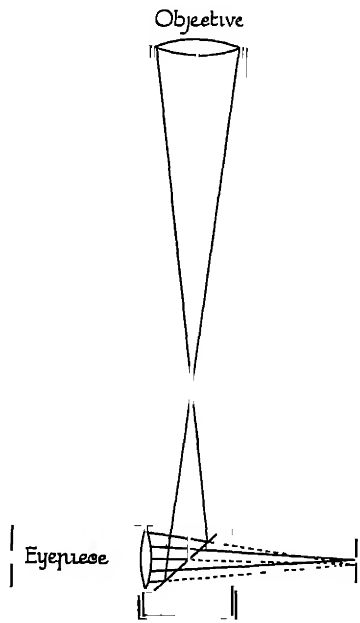

FIG 282 —Modification of the Zöllner Photometer

The instrument described on pp 398-9, for measuring the absorption of the atmosphere, may clearly be used for stellar photometry It has the great advantage that by the employment of the Maxwellian view the photometric comparison is made between extended surfaces instead of point images.

The Meridian Photometer. — A polarisation photometer essentially different from the Zollner instrument is the meridian photometer of E C Pickering, shown in Fig 283 [18] This consists essentially of a telescope with a double objective and a single eyepiece The telescope is placed horizontally in the east-west direction, and each objective is furnished with a mirror or 45° reflecting prism which can be rotated about the instrument axis so that the images of any two stars on or near the meridian can be compared. In practice the scale of

magnitude is based on the mean values assigned to a large number of circumpolar stars, so that periodic variations are eliminated [19] In the first form of the instrument the beams of light from the two objectives O, O' both pass through the same double-image prism K, which, consisting of a wedge of quartz or of Iceland spar cemented to a glass wedge, is approximately achromatic

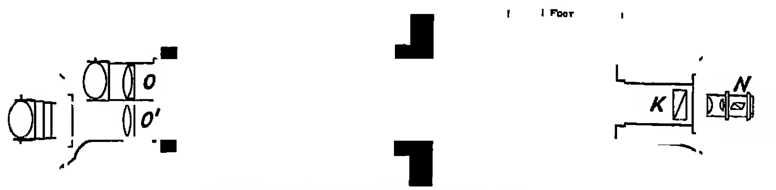

FIG 283 —The Meridian Photometer

for a mean value of separation of the images The position of this prism in the telescope is adjusted for each experiment, and is such that the extraordinary image of one of the stars to be compared is brought close to the "ordinary" image of the other, the two remaining images being stopped off In a later form of the instrument [20] each beam of light passes through a separate achromatic prism, and the positions of these two prisms are so adjusted for each experiment that the two required images in the double-image prism are side by side when the latter is kept fixed at a point just within the common focus of the objectives In both instruments a rotatable Nicol N in front of the eyepiece gives, by the angle of rotation necessary to produce equality, the relative intensities of the images (see p 30) Each observation must be repeated with the position of the star images reversed in the field This is necessary, owing to the fact that the retina is not equally sensitive all over, so that there is generally a tendency to over-estimate the magnitude of the lower stars in any field This error may amount to several tenths of a magnitude [21], but when it has been eliminated by reversal the visual method, using a good instrument, is capable of an accuracy of at least 0 1 magnitude for neighbouring stars Varying atmospheric conditions prevent a similar accuracy when the stars to be compared are widely separated Sometimes an artificial star is used for the comparison in this case [22].

The Wedge Photometer.—The third type of stellar photometer depending on visual methods is the wedge photometer, described by de Maistre and extensively employed by Pritchard [23] The principle of extinction had been used in conjunction with variable diaphragms or sector discs used over the objective [24], but the accuracy attainable in this way was not great, partly owing to errors due to diffraction when the aperture was small [25] Pritchard used a wedge of neutral glass, which was placed in the path of the rays, forming a star image in a telescope. The wedge was moved across until the image just disappeared, and the ratio of the magnitudes of any two stars could readily be found from the distance between the positions of the wedge at which disappearance took place For if θ be the angle of the wedge, and d the distance between the positions for stars whose intensities are I_1 and I_2, then $I_1/I_2 = e^{-\alpha d \tan\theta}$, where α is

the absorption factor of the material of the wedge (see p 116) Hence $m_1 - m_2 = \mu\alpha d \tan\theta$, where μ is the modulus of common logarithms. A movable double wedge of the form shown in Fig 106 (p 179) is preferable to the single wedge [26]. The wedge may be calibrated directly by comparing the results obtained by its use with those obtained by an objective diaphragm or any other device for producing a diminution of light intensity in a star image [27].

It is clear that the readings obtained on any single occasion with this photometer, as with every photometer based on visual acuity, will depend to a certain extent on the state of the observer's eye and its recent history, as well as on other circumstances of the experiment, even assuming the use of an artificial pupil The effect of removing the eye from the instrument in order to read the scale on the wedge is appreciable, and this has been avoided by the provision of a device for automatically recording the position of the wedge by the depression of a lever, the eye remaining in position at the eyepiece [28].

The Purkyně effect (see p 65) still remains, however, and since stars compared may differ considerably in colour, it is clear that an error of considerable magnitude may be introduced by this effect, and this has been found actually to be the case [29]

Photographic Methods.—Although a vast amount of useful work in stellar photometry has been done by visual methods [30], it will be clear from the brief account given above that there are special features in this particular problem of photometry which make physical methods more promising for accurate work if these are carefully developed with due regard to the necessity for bringing the results obtained by purely physical means into agreement with those which would be obtained visually

Of the physical methods available, radiometric methods seem to be the least promising, on account of the exceedingly small amount of radiant energy available [31] Selenium bridges have been used to a small extent [32], but the principal work in this branch of stellar photometry has been by methods depending on photography and on the use of photo-electric cells, and these will be described briefly in what follows

It has already been pointed out in Chapter XI (p 332) that a photographic plate possesses the great advantage of being able to perform a time integration of the light reaching it Unfortunately, however, length of exposure and increase of intensity are not exactly equivalent in producing blackening of a photographic plate, and the connection is found to be different for different types of plates (see note [76], p 338) The relation may, however, be determined for any given plate by subsidiary experiment [33] Two methods are in general use for stellar photometry by photographic methods In the first of these an image of the star to be measured is focussed on the plate. The magnitude may then be determined from a measurement of either the intensity or the diameter of the image, for, owing to slight imperfections in the optical system of the telescope, the image of a star is not a mathematical point, but a disc, which more or less rapidly decreases in intensity from the centre outwards It follows that increase in either the brightness of the star or the time of exposure will cause an enlargement of the area, over which perceptible darkening takes place. From what has been said concerning

the effect of exposure on blackening, it naturally follows that the law connecting diameter with exposure is dependent on the particular plate used. Various empirical formulæ have been proposed at different times ([34]), the most used being, probably, the Greenwich formula $m = a - n\sqrt{d}$ where a and n are constants ([35]). The formula, $m = B/(d + C)$, B and C being obtained independently for every plate and exposure by comparison with visual magnitudes ([36]), is useful as a means of comparison of stars photographed at the same time on any one plate. The magnitude of an undetermined star may thus be found by comparison with reference stars photographed *on the same plate* ([37]), the magnitudes of these reference stars having previously been determined visually ([38])

It is to be noticed that the values of the constants in any formula which may be employed are liable to vary with the colour of the light from the star photographed, since it has been found that the photographic plate shows an effect which is analogous to the visual Purkynĕ effect ([39]).

The Grating Method.—A scale of magnitudes may be formed from one star at a single exposure by placing over the objective of the telescope a coarse diffraction grating (see p 25) formed of wires spaced at equal intervals The effect of such a grating is to convert each star image into a line of separate images of gradually diminishing brightness If the thickness of the wires be t and the breadth of the spaces between them s, the distance apart of the images is $f/\nu(t + s)$, where f is the focal length of the telescope objective and ν the wave-number of the light Further, it can be shown ([40]) that the ratio of the brightness of the nth diffraction image to the brightness of the central undiffracted image is $\{ns\pi/(s + t)\}^2/\sin^2\{ns\pi/(s + t)\}$, and thus, from a knowledge of s and t, the relative magnitudes of the series of diffraction images is accurately known If, then, two stars be photographed side by side on the same plate in this manner, their magnitudes may readily be compared by determining between which pair of diffraction images of the brighter star the central image of the lesser star appears to fall in order of density.

There are several considerations which limit the application of this method in any particular case. It is clearly necessary that the diffraction images should be distinct from one another, and, as the formula for their separation shows, an upper limit is thus set to $(t + s)$ for any given value of f and ν On the other hand, it is seen from the same formula that, since the light forming the image is not monochromatic, each diffraction image will be spread out into a spectrum to an extent depending on (i) the frequency range of the light, and (ii) the value of $f/(t + s)$. Hence it is desirable that $(t + s)$ should approach as closely as possible to the permissible maximum The value of $(s + t)$ may therefore be regarded as fixed within a very narrow range The respective values of s and t must be determined by a compromise, for, on the one hand, the absolute brightness of the images naturally increases with s, so that t must not be too great if the loss of light is not to be excessive, while, on the other hand, the formula for the brightness ratio shows that increase of s produces a rapid increase of ratio, so that the scale becomes too coarse and the number of images too small ([41]).

Comparison of Densities.—The relative magnitudes of two stars

may be estimated by comparing the *densities* of their photographic images instead of the *diameters* of these images For this purpose the image of one star may be compared with a series of images of another star obtained with gradually increasing exposures. The comparison may conveniently be carried out with a duplex microscope in which, by means of two widely-separated objectives with a common eyepiece, the images of two objects may be viewed side by side for purposes of comparison (42).

More accurate results can be obtained, when the light available is sufficient, by using images which are considerably out of focus, so that they no longer vary appreciably in size, but only in density (43). The density measurement is readily made by some form of microphotometer (see pp 393 *et seq.*) or otherwise (44). One of the chief sources of error in this method is sky fog, the effect of which is naturally greatest for the lesser magnitude stars (45) A visual method depending on the use of an out-of-focus image has also been described (46)

It will be obvious that the results obtained by any photographic method will not agree with those found visually if the stars being compared differ in colour (47) To overcome this difficulty the so-called "photo-visual" method is used, in which a colour filter is interposed in the path of the light, the transmission curve of this filter being such that, for the particular kind of photographic plate used (isochromatic) the sensitivity curve of the combination approximates to that of the eye (48). A comparison of the results obtained by this method with those obtained with an ordinary plate gives useful information concerning the colours of the fainter stars (49).

The Photo-electric Method.—The most recent development in stellar photometry has been the increasing use of the photo-electric cell (see p 325) Since the cell measures the total energy incident on it, and not the illumination, an out-of-focus image may be used, and its size is without influence on the result, so that the perfection of the optical performance of the telescope is unimportant A convenient arrangement for the apparatus is shown diagrammatically in Fig. 284 (50) *AA* is the ocular end of the telescope, the image

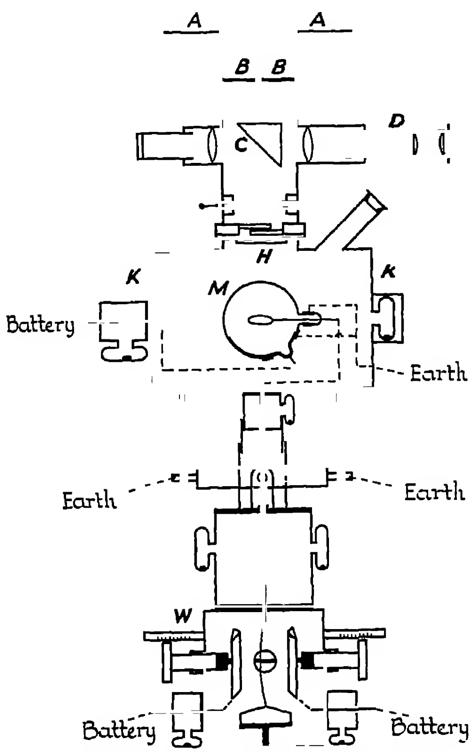

FIG 284 —Photo-electric Stellar Photometry

of the star under observation being formed within a small aperture in the diaphragm BB, so that the energy from this star alone reaches the cell. The reflecting prism C and auxiliary telescope D are provided for the purpose of positioning the image at B. C is swung out to the side when a measurement is made, and the light passes on to the photo-electric cell M, which is enclosed in a light-tight box KK open only at H, where a glass filter may be placed The electrometer W, which is used for the measurements, is attached to K by a Cardan suspension, which allows it always to remain vertical as the telescope moves round, or if a string galvanometer be used it may preferably be attached rigidly to the case containing the photo-electric cell (51). The cell may be of any ordinary type, such as those described in Chapter XI. of this book. It is desirable to use a colour filter in order to bring the sensitivity curve of the cell into approximate coincidence with that of the eye With due precautions an accuracy approaching 0 01 mag can be obtained (52), as compared with 0·04 mag. with a polarising photometer (53), or by the photographic method (54)

A particularly valuable application of the photo-electric cell to stellar photometry is in the study of variable stars (55)

Other Problems.—In addition to the determination of stellar magnitudes, there are various problems of celestial photometry which it is impossible to do more than mention here. Such are the measurement of brightness of different parts of the sun's disc (56) and of the corona (57), the brightness of nebulæ (58), the general brightness of the night sky (59), and the brightness of the sky in the neighbourhood of the sun (60)

The spectrophotometry of celestial bodies is also a subject of great and growing importance in astrophysics (61) A simple measure of the colours of stars may be made by finding their relative magnitudes at different parts of the spectrum, using a visual method and inserting coloured media in the eyepiece of the telescope (62).

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CHAPTER XVI

THE PHOTOMETRIC LABORATORY

THE object of the present chapter is to give a brief description of the accommodation required for a small photometric laboratory, and of the more important auxiliary apparatus needed for use in the different branches of photometric measurement dealt with in the foregoing chapters It will be assumed that the laboratory is to be equipped for the following work · (i.) The measurement of candle-power in a single direction, and of the total flux given by a source of light of any kind ordinarily met with in practice , (ii) the determination of candle-power distribution in the horizontal and vertical planes for any source or for any lighting fitting, shade, reflector, *etc* , (iii) the measurement of illumination and the calibration of illumination photometers , (iv) the spectrophotometry of light sources and the measurement of spectral transmission or reflection curves of coloured media , (v) physical photometry , (vi) the life-testing of electric incandescent lamps or of gas burners and mantles , (vii) the determination of candle-power distribution from projection apparatus In many laboratories, no doubt, one or more of the branches of work enumerated above need not be considered, and it will frequently be found impossible to provide the full accommodation or the whole of the apparatus mentioned below. The following description is intended mainly to serve as a guide by which the actual requirements in any particular laboratory can be approximately estimated with due regard to the special circumstances of each individual case (1).

In considering the general design of the building it should be remembered that it is generally undesirable, if not impossible, to carry on two sets of photometric measurements simultaneously in one room owing to the general necessity in photometric work for avoiding the presence of any lights in the room other than those actually being compared. It follows, therefore, that a number of small rooms must be provided if much time is not to be lost A very convenient size for a small photometer room is about 20 by 24 feet, as such a room will comfortably accommodate a 5-metre bench in addition to the necessary tables, desk, *etc* At the same time it is very desirable to have one room *at least* 100 feet long for the measurement of sources of high candle-power and polar curve determinations

Power Supply.—Before the different rooms are described in further detail, the general electrical supply for the laboratory must be considered, since this affects with almost equal importance every branch of photometric work

For work of precision, such as candle-power measurement to an accuracy within 1 per cent , the supply voltage must be absolutely steady, and nothing but a storage battery or batteries can be used. Other things being equal, two batteries of 100 amp.-hrs. capacity

each are more convenient than one of 200 amp -hrs , unless lamps taking over 5 amps. are likely to be worked with frequently. The highest potential available should be at least 300 volts, since 240-volt lamps are common and it is often necessary to measure these at an excess voltage of 10 to 15 per cent The method of splitting up the battery into sections is very important. Since two workers using the same battery must necessarily tend to affect each other, the voltage in one room rising, perhaps, 0·1 per cent when the circuit in another room is switched off, it is a convenience to have the battery so split that, if only 100 to 120 volts are required in each of two rooms, the circuits in these rooms can be run off two separate portions of the battery. In addition to this, it is very convenient in measuring low-voltage lamps taking a large current to be able to use a few cells from the battery instead of having to absorb a large amount of energy in resistances. For these reasons it is generally found convenient to divide a 300-volt battery into, say, five sections of thirty cells each, and then to subdivide the two end sections still further. If a 100-volt supply be used for charging the battery, the 60-volt sections can be connected in parallel for charging purposes. A convenient scheme of sub-division is that indicated in Fig 285, which also shows the method of distribution to different rooms in the laboratory. It will be seen that, by means of the flexible connectors joining the battery bus-bars to the plugs connected with the terminal boards in the different rooms, any desired potential may be obtained in any room in the building.

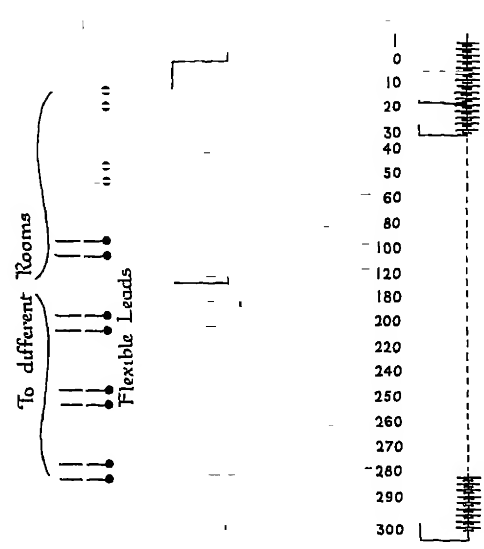

FIG 285 —The Battery Distribution Board

A special room, isolated as completely as possible from the rest of the laboratory, should be provided for the storage batteries, while generators should be located as far as possible from rooms in which it may be necessary to use sensitive galvanometers.

It may be mentioned in passing that, in order to avoid interference between two circuits run off the same battery, when either or both have to be switched on and off frequently (as in measuring a number of lamps in succession), it is customary to use a " dummy " or " balance " circuit method, that is to say, when a circuit is switched off, an artificial load, consisting of one or more electric lamps in parallel, is put on the battery at the same instant by means of a two-way switch, as shown in Fig 286 If the current taken by the balance circuit is nearly equal to that taken in the test circuit, no disturbance of the battery voltage is caused

The Photometer Room.—As all photometric work must depend ultimately on the measurement of candle-power by means of comparison with sub-standards on the photometer bench, the room or rooms in which this work is carried out may be regarded as the basis of the photometric laboratory The bench room, as it may be called, is conveniently of the size mentioned above It contains a bench mounted on rigid supports, so that the eyepiece of the photometer

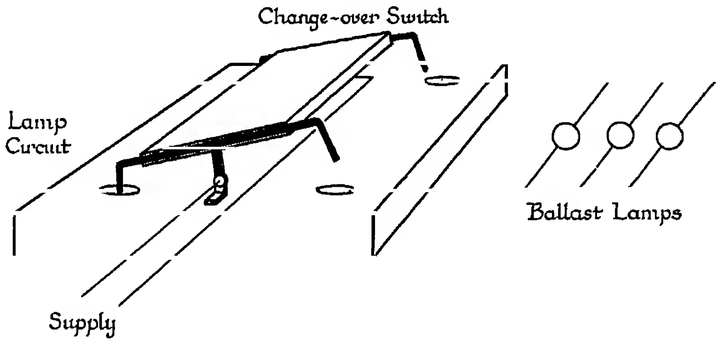

FIG 286 —The Balance Circuit

head is at a convenient height for an observer when seated. It also contains (i.) a table, upon which may be placed the lamps and fittings to be tested, small pieces of apparatus in current use, *etc* , (ii) a desk for the use of the observers, (iii) special cupboards for storing the sub-standards, working standards and comparison lamps, (iv) an ordinary cupboard for storing small apparatus, (v) a table or bench to accommodate the potentiometer and other apparatus for the precise control of the electric lamps, and (vi) a table for the auxiliary apparatus required in the photometry of gas lamps The bench room should be provided with a set of fixed sector discs accurately calibrated as described in Chapter VI (see p 179)

The main lighting of the room should be controlled from a point near the bench as well as from the door The electrical measuring instruments and the scale on the bench (Fig. 76, p 149) should have separate individual lighting by low candle-power lamps completely shaded from the observer's eyes (2) When the general lighting is not in use no surface in the room should have a greater brightness than the photometer field It has been said already that the larger part of the errors made in photometry are due to stray light reaching the photometer head Although the screening system described on p. 170 should be sufficient to avoid any possible error from this cause, the custom of painting the walls and ceiling of a bench room a dead black is one to be recommended (3) The room should, for hygienic reasons, have an adequate number of windows, and these should be provided with internal shutters blackened on the inside so that the daylight can be completely excluded when measurements are in progress It is convenient to have a vestibule to all doors leading to rooms in which much photometric work is carried out (4).

The Electrical Equipment.—Much of the work in an ordinary photometric laboratory is concerned with electric lamps, and, as

already stated in Chapter VI, the most convenient sub-standards and comparison lamps used in modern photometry are electrical. It follows that two very important parts of the installation in the bench-room are the means provided for the ready connection of lamps on the bench to a source of electric supply, and the apparatus used for the adjustment and accurate measurement of potential and current

Since for a tungsten lamp the candle-power varies at about 3·7 times the rate of the potential, while the potential varies nearly twice as much as the current owing to the high positive temperature coefficient of resistance of tungsten (5), it follows that the electrical measurements should be to an accuracy of at least 0 1 per cent. in ordinary work, and 0 02 per cent. in standardisation. The best form of indicating instrument is sufficient in the former case, but in the latter a potentiometer method of measurement must be used (6) Either potential or current may be used as the basis for photometric work The latter possesses the advantage that the current measured must necessarily be that actually passing through the lamp filament, so that potential drop at the lamp terminals or in the leads may be ignored On the other hand, potential measurement is, as stated above, nearly twice as sensitive, and if care be taken to avoid bad contact and, by means of a separate pair of leads, to measure as close as possible to the lamp terminals, the errors due to voltage drop can be eliminated

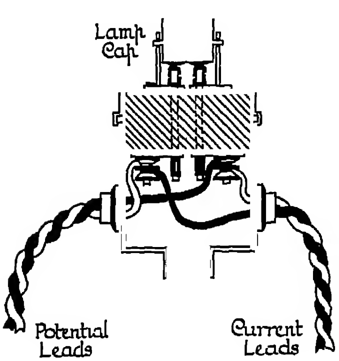

FIG 287.—The Standard Lamp Socket

A convenient form of bayonet lamp holder, in which provision is made for measuring potential at the lamp contacts, is that shown in section in Fig 287, which is self-explanatory Similar holders for screw-capped lamps are also required (7). If the two pairs of leads be each terminated in a small ebonite holder such as that shown in Fig 288, connection to the terminal board fixed on the photometer bench (*T* in Fig 75) is quick and easy (8) In the case of sub-standards mounted specially as described on p 137, the potential is measured across the ends of permanent leads which are soldered to the lamp contacts The leads are then connected to either pair of terminals on the terminal board, and the corresponding members of each pair on the board are connected by means of short horizontal copper strips

The method of control by potential will be described here, but it will be seen that the same circuit, with the omission of the auxiliary pair of potential measuring leads, is equally suitable when current control is employed (9) The system of electrical connections is shown in Fig. 289. Two or more pairs of leads are brought from the battery distribution board to a terminal board in the bench room. One of these circuits is connected through a regulating resistance R_1

and a switch S_1 to the pair of terminals marked "LAMP" on the small ebonite terminal board fixed to the left-hand end of the bench (see Fig 75) From this pair of terminals the lamp on the left of the photometer is fed directly The potential across its contacts is measured by means of the auxiliary leads, which, by way of the other pair of terminals, marked "VOLTS," are

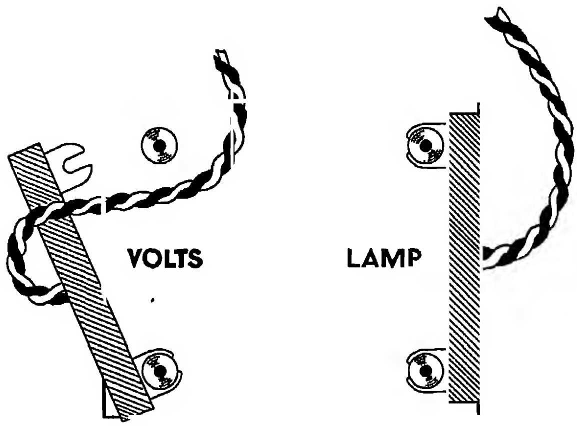

FIG 288 —The Terminal Board on the Photometer Bench

connected through a change-over switch S_2 to the ends of a constant high resistance R_3, generally 10,000 ohms This resistance has tapping-off points at 100 and 1,000 ohms, so that either one-hundredth or one-tenth of the potential across the lamp contacts may be measured by means of the potentiometer P, using a Weston cell W as the standard of potential, and a galvanometer G For the purpose of current measurement the small resistance R_2 is included in the circuit supplying the lamp The value of this resistance is 1 or 0 1 ohm, according to the magnitude of the current to be measured, and its value should be accurate to at least 1 part in 10,000 for precision work By measuring the potential E_2 across this resistance the current in the lamp circuit is known, being E_2/R_2 Allowance must, however, be made for the small current which is flowing in the potential circuit, for this circuit contains the resistance R_3, so that from the measured current must be subtracted the current through R_3, *viz* , E_1/R_3, where E_1 is the measured potential across the lamp terminals

The electrical connections to the lamp L_2 on the right of the photometer may be made in an exactly similar way, but it is generally more convenient to use the same potentiometer for both lamps, and this is achieved by using a double-pole change-over switch at S_2 A separate battery or other source of supply is preferable, but the same source as that supplying L_1 may be used

In the substitution method of photometry, which is the one most frequently employed in accurate work (see p 161) the lamp L_2 has to be maintained at a constant candle-power, and therefore at a constant potential, throughout a series of photometric measurements. Its potential is adjusted at the beginning of the series and checked at intervals during the course of the work. If, however, the source of

supply is at all liable to variation owing to either the fall of potenti of a freshly-charged battery, variation in external load, or even th variation of current due to change of the lamp L_1 when the sam battery is being used for L_1 and L_2, then it is unsafe to assum

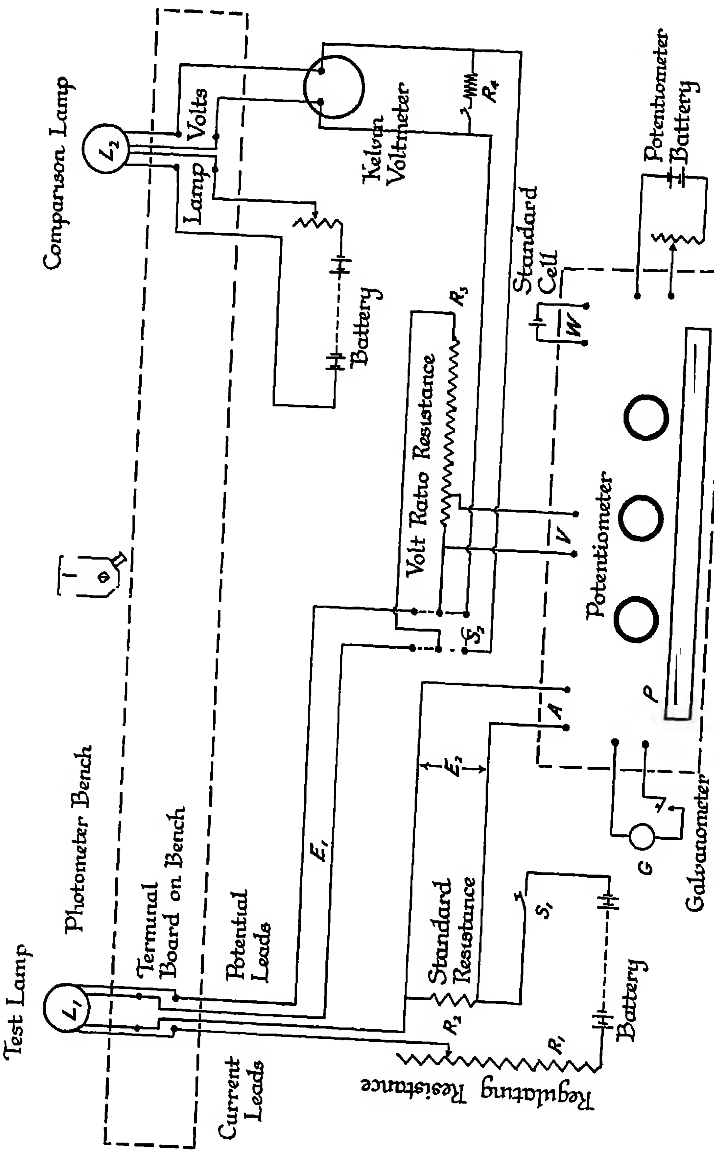

FIG 289 —The Electrical Connections to the Photometer Bench

constancy of potential on L_2, even over short periods of time, and an electrostatic voltmeter is used for keeping a watch on the potential in the intervals between the check measurements with the potentiometer. It is unsafe to rely on this instrument for very long periods, however, owing to creep in the suspension, so that check

measurements should be made at intervals of not more than twenty minutes to half an hour.

The voltmeter should be shunted with a high balancing resistance R_4, which is equal to R_3, and which can be open-circuited when the check measurements are being made. By this means the current in the potential circuit of L_2 is maintained at the same value whether the potentiometer be connected to it or not The Kelvin electrostatic voltmeter may conveniently have a circular scale of about 10 feet radius, on which 1 volt is represented by about 2 inches in the 100-volt region (10)

It will be clear that the difficulties met with in making the electrical measurements necessary in photometry are, in the main, those met with in all kinds of precision electrical work Faulty insulation, often due to surface leakage over exposed ebonite parts, or over wood, may be a source of considerable trouble and can only be cured by thorough cleaning, although such expedients as immersion in paraffin wax may prove efficacious Loose contacts manifest themselves by unsteadiness of indication on the measuring instruments It may, perhaps, be mentioned here that some electric lamps develop an inherent unsteadiness due to uncertainty of contact between the filament and the leading-in wires (see p 138) This may be detected by gently tapping the lamp while alight, the galvanometer key on the potentiometer being held down continuously. If the lamp be unsteady the spot will be jerked to one side or the other at every tap

In laboratories where the accuracy aimed at is not so high, or where it is impossible for some reason or another to adopt the methods of electrical supply and measurement described above, considerable simplifications may be made in the arrangements of the photometer room Indicating instruments of the laboratory standard type with large scales may, with suitable precautions, be used in place of the potentiometer and Kelvin voltmeter (11), but even in the most approximate photometry the electricity supply from the public service mains is too unsteady for use in the ordinary way If, however, two electric lamps *of the same type* are being compared, so that the candle-power/voltage characteristics are the same on both sides of the photometer, the two lamps may be put in parallel on the outside supply (12), and if the voltages be adjusted so that both are correct at any instant, the photometric comparison will remain valid to a reasonable degree of accuracy in spite of fluctuations of voltage during the course of the observations (13) Tungsten vacuum lamps may be compared in this way quite satisfactorily, and tungsten vacuum lamps may be compared with gas-filled lamps if the voltage changes do not exceed 1 or 2 per cent. Tungsten filament and carbon filament lamps, however, cannot be so compared, since the exponent n in the voltage-c p relationship $I = aV^n$ is approximately 3·6 for tungsten and 5 to 6 for carbon lamps (see App. X., p. 481).

When flame sources are being measured the above procedure cannot be followed, and either a flame standard such as the pentane or Hefner lamp must be used (see pp 127, 129), or some automatic device for regulating the voltage on an electric sub-standard may be employed. Such a device is that shown in Fig. 290. It consists

of an unbalanced Wheatstone bridge, of which the opposite a
AC, *BD* are of tungsten lamps in parallel, while *BC* and *AD*
constant resistances. A rise of voltage across *AB* produces
increase in the current flowing through the lamp arms of the bri
and this, owing to the positive temperature coefficient of resista
of tungsten, causes an increase in the resistance of *AC* and
which, if the relative resistances of the arms be properly proportior

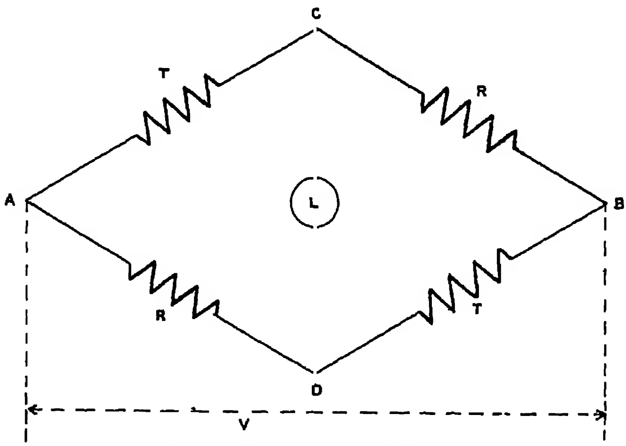

FIG 290 —The Automatic Voltage Regulator

is sufficient to maintain the potential across *CD* at its original val
It follows that a sub-standard lamp may be placed in circuit acr
CD as shown, and will not be affected by fluctuations of the sup
voltage across *AB* It has been shown [14] that with 240 vo
across *AB* the resistances of the arms *AC* and *BD* should be 90 ohr
and of *BC* and *DA* 120 ohms, in order to maintain a constant potent
of 68 volts across *CD* when the current in this circuit is 0 2 am
The regulation is then within 0 1 per cent voltage for a variati
across *AB* of 5 per cent.

When flame standards are used for the measurement of flai
sources, it is frequently assumed that the effect of changes
barometric pressure and humidity are the same for the lamp bei
measured as for the standard. Though the corrections for the
conditions are in the same direction for all flames, and they a
generally of the same order of magnitude (not more than twice
great) for an open gas flame as for a pentane lamp [15], the sar
assumption cannot always be made in the case of mantle flames,
which the conditions governing the luminous efficiency are altogeth
different [16]. For this reason a mantle lamp is often used as
comparison lamp in gas photometry (see p 140)

If indicating instruments be used in place of the potentiomete
they may be connected in either of the two ways shown in Fig 29
In the first arrangement the voltmeter *V* is placed in parallel wi
the lamp, and the current taken by it must therefore be deducte
from the current indicated by the ammeter *A* in order to obtain th
true lamp current In the second arrangement the voltmet
measures the potential drop across both ammeter and lamp, so th

the potential drop in the ammeter must be subtracted from the indication of V in order to obtain the true voltage applied at the lamp terminals (17) If the switch changing from the test lamp to the balance lamps (see p. 436) be on the lamp side of the indicating instruments, the latter are always used with the coils heated to the same temperature.

It should be noted that all electrical indicating instruments require checking at frequent intervals by means of a potentiometer.

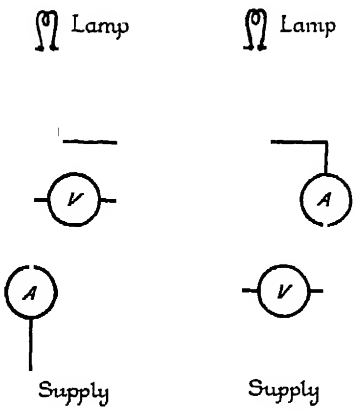

FIG 291—Alternative Arrangements of Measuring Instruments

The Gas Equipment.—For the purpose of testing gas lamps the bench room should be provided with an adequate gas supply by means of pipes of large bore (2½ inches at least) running round three sides of the room and having a large number of outlets, so that the necessity for using long flexible connection pipes may be avoided. It is a great convenience, and a necessity if accurate work is to be undertaken, to have a second supply system fed from a compensated standard gas holder of, say, 5 cubic feet capacity, situated in the neighbourhood of the bench room. This may be filled before a test is commenced, and its use will ensure that the quality of the gas used does not vary during a set of measurements. Further, it enables pressures in excess of the normal supply pressure to be obtained when required For work on high-pressure gas a small gas compressor is required.

The measuring apparatus required in gas photometry includes (*a*) a consumption meter, (*b*) a pressure regulator, (*c*) a pressure gauge, (*d*) a calorimeter, and (*e*) apparatus for determining the atmospheric conditions, *viz*, pressure, temperature and humidity (18).

The consumption meter used may be of any standard form Two meters are generally needed, one for single burners, where the consumption lies between 3 and 7 cubic feet per hour, the other for the higher consumptions which have to be measured when cluster burners are tested A Gas Referees' $\frac{1}{12}$ cubic foot measure (19) or a 5-foot meter prover should also be available for checking the consumption meters periodically If high-pressure gas be used, a special meter designed to give accurate readings of consumption at the pressure in question must also be provided

The pressure of the gas should be regulated both before and after it passes through the meter * Any ordinary form of regulator may be used for this purpose The pressure gauge may consist of a simple U-tube containing water or, if high-pressure gas is being used, mercury. It should be placed between the second pressure regulator and the burner, so that the pressure measured is that actually provided at the burner inlet This is important, since in

* This does not apply in measuring calorific value

mantle lamps the change of candle-power is about 2 per cent per tenth of an inch of water, assuming a normal pressure of 2·5 inches ([20]) In the case of high-pressure burners the variation of candle-power is about $1\frac{1}{2}$ per cent for every inch of pressure variation from an assumed mean of 50 inches of water ([21])

A very important factor in the photometry of incandescent gas burners is the calorific value of the gas used, for it has been found that the candle-power of a mantle increases by about 1 per cent. for every 1 to 2 per cent increase in calorific value ([22]) It is therefore necessary to measure the gas with a calorimeter every time photometric measurements are made, since the calorific value may vary by several per cent from time to time The type of calorimeter generally used is the Junkers ([23]) or that designed by C V Boys ([24]) The latter is described in the Notification of the Metropolitan Gas Referees for 1923, and is shown in section in Fig. 292 It consists of a base holding a double burner, at which the gas to be measured is burnt at the rate of 4 to 5 cubic feet per hour A steady flow of water passes through the coiled copper pipe P, P, and the inlet and outlet temperatures T_1 and T_2 are measured when a steady state has been reached If the quantities of water and gas entering the calorimeter in a given time be respectively W litres and G cubic feet, the gross calorific value of the gas ([25]) is $1{,}000(T_2 - T_1) W/G$ calories per cubic foot, T_1 and T_2 being measured in degrees Centigrade. This figure is converted to British Thermal Units per cubic foot by multiplying by the constant 0·003968. Small correction factors are applied for (*a*) difference between the room temperature and the temperature of the air and products of combustion leaving the calorimeter, and (*b*) the barometric pressure and room temperature The method of making these corrections and the procedure to be followed in carrying out a test are described in the Notification of the Metropolitan Gas Referees for 1906.

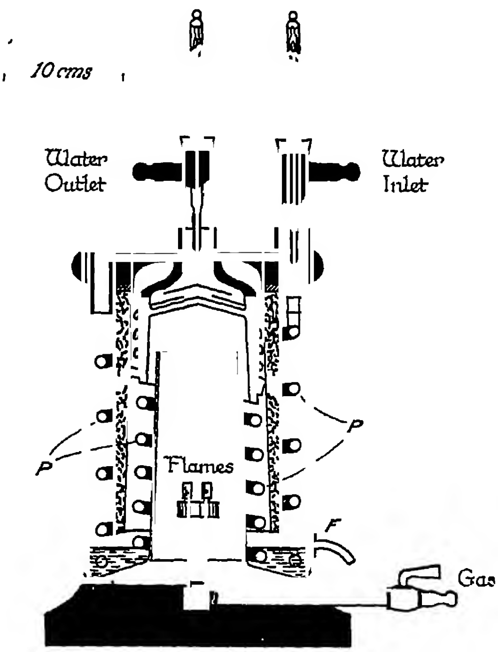

FIG 292 —The Boys' Calorimeter

The calorimeter may be calibrated from time to time by using hydrogen instead of coal gas. The gross calorific value of hydrogen is 344 B.Th.U per cubic foot. To obtain the net calorific value, the heat evolved in the condensation of the water vapour formed in combustion must be subtracted from the total heat calculated as

above The water condensed in any given time (t_1) is collected at the tap F, and its weight in grams (*i e.*, volume in c c) multiplied

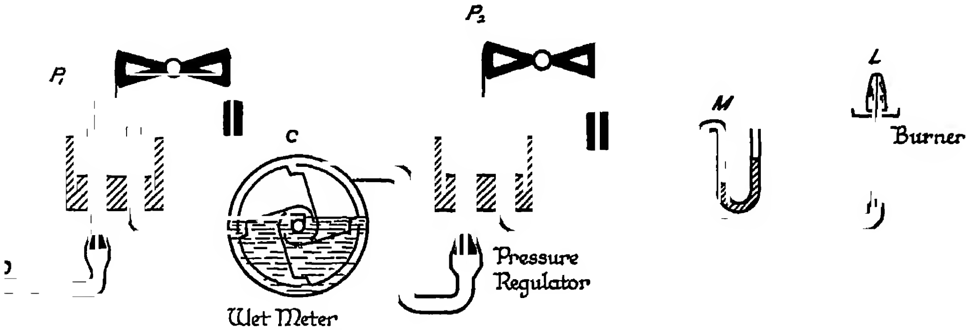

FIG 293 —The Control Apparatus in Gas Testing

by 620 (the approximate heat of condensation from vapour at 100° to liquid at room temperature) gives the amount of heat (C) evolved in condensation during that time If the time of flow used in the measurement of the gross calorific value be t_2, the net value is given by

$$\{1{,}000(T_2 - T_1)W - t_2C/t_1\}/G$$

calories per cubic foot

The general arrangement of the auxiliary apparatus may be as shown diagrammatically in Fig 293 O is the gas outlet from the feed pipes, C is the consumption meter, P_1 and P_2 the pressure regulators, and M the manometer or pressure gauge, L being the burner A barometer and thermometer for recording the air pressure and temperature are required, as well as a hygrometer for measuring the humidity The type generally employed for this work is some form of ventilated instrument, such as the Assmann type, shown in section in Fig 294 ([26]). By means of the clockwork-driven fan F, air is drawn at a constant rate over the wet-bulb thermometer T_1 The readings of both thermometers are taken and the humidity is then found from tables given in the Smithsonian Meteorological Tables and elsewhere ([27]).

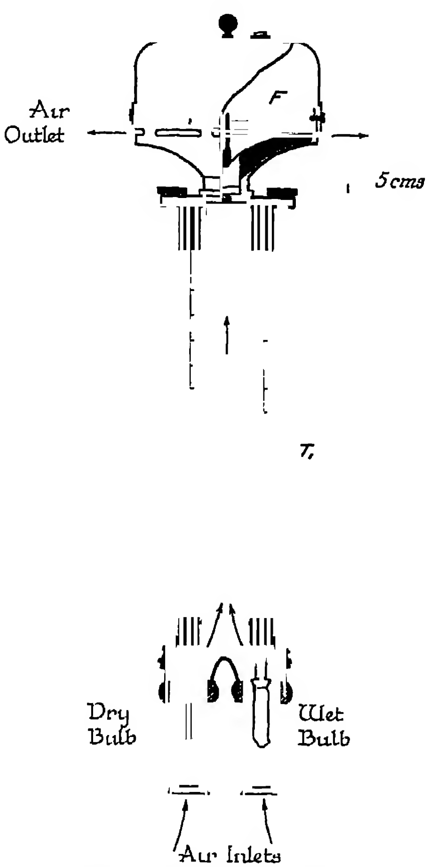

FIG 294 —The Assmann Hygrometer

Special Room for Photometry of Flame Sources.—If much work on the candle-power of flame sources be contemplated, the laboratory should contain a special bench room, in which adequate ventilation can be secured ([28]) in combination with an equable temperature, freedom from draughts and, as far as possible, constant humidity (see p. 130) The size recommended above for a bench room should be regarded as the minimum admissible for work on flame sources, the height of the ceiling being at least 12 feet. The room should be enclosed in a second "jacketing" room from which a steady supply of pure and warm air can be obtained by means of suitable light-tight openings ([29]) These openings should have a total area of not less than 20 square feet, distributed among at least 50 openings more or less evenly spaced around the room

Sub-standards.—The standards of candle-power used in the bench room and elsewhere in the laboratory should be lamps of special construction, as described on p 137, whose candle-powers have been determined at a standardising laboratory They should be handled with great care and should be stored in the bench-room in a cupboard specially fitted with shelves as shown in Fig 295. The upper surface of the shelf at *A*, where the cap of the lamp rests, should have a washer of rubber or felt to prevent jarring when the lamp is put away. The lamps should always be stored in their normal burning position, *i.e*, upright in the case of the type of sub-standard described in Chapter V. The candle-power value assigned to each lamp is generally accurate to about one-quarter of 1 per cent It should be marked on a small label tied to the flexible lead attached to the lamp ([30]). This label should also show (i) the correct voltage of the lamp, (ii) the distance from the photometer screen in millimetres at which the lamp gives an illumination of exactly 10 metre-candles, (iii) the current taken by the lamp, and (iv.) the date on which the lamp was last standardised

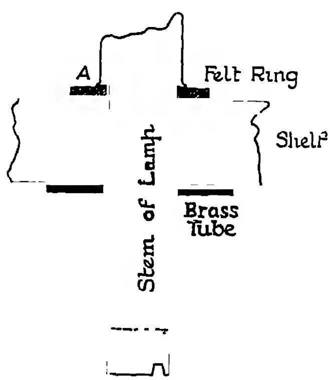

FIG 295—Section of Shelf for Storing Standard Lamps

It may be assumed that a tungsten filament vacuum standard lamp will fall in candle-power at a rate not exceeding 5 per cent per 100 hours of actual burning, *i.e*, if used regularly for five minutes per day, its candle-power will decrease at a rate of about 1 per cent per annum The frequency with which it is sent back to the standardising laboratory for restandardisation must depend on the number of times it is used and the period of use on each occasion, five minutes is probably an under-estimate of the time taken for a standardisation by two observers (see p 165) Where much photometric work has to be done it is frequently found convenient to have several sub-standards and to reserve one or more of these for checking the others at intervals of two or three months Further, other lamps may be standardised at the laboratory by comparison with the sub-standards issued by the standardising laboratory, and

these may be used for all photometric work in which the highest accuracy is not required [31] These lamps may be re-checked at frequent intervals so that changes in their candle-power values may at once be detected.*

The current taken by a sub-standard should be measured on every occasion on which it is used for standardisation, and any difference greater than about 1 part in 2,000 from the value marked on the label should be regarded as an indication of some variation in the lamp itself, or else of some defect in the measuring apparatus involving, possibly, a slight error in the voltage applied to the lamp The following table shows the results actually obtained over a period of ten years in the re-standardisation of a satisfactory tungsten filament sub-standard lamp —

TYPICAL PERFORMANCE OF STANDARD LAMP AT 100 VOLTS

Date	C P	Amps
February, 1913	$14\ 2_1$	0 2337
July, 1916	$14\ 1_9$	0 2338
May, 1919	$14\ 1_0$	0 2338
September, 1920	$14{\cdot}0_2$	0 2337
July, 1922	$14\ 0_0$	0 2336

It is hardly necessary to repeat that the candle-power value of a lamp may be altered seriously by even a momentary excess voltage of more than about 5 per cent [32] Most standards are somewhat under-run, so that a very slight excess voltage applied for a brief period generally produces no change of candle-power. It is always necessary, however, after any such over-run to measure carefully the current taken by the lamp If this be unchanged, the lamp may safely be assumed to have received no harm If a change of current is found, or if the over-running has been at all severe, the lamp should be re-standardised before being used again

Sub-standards of mean spherical candle-power, for use in photometric integrators, may conveniently be kept in the sphere room. They should be treated in exactly the same way as other sub-standards, but if normally used in the pendent position when in the sphere, they should be stored in this position

Heterochromatic Photometry.—If work involving considerable colour differences has to be provided for, one or more of the methods of heterochromatic photometry described in Chapter VIII should be chosen as suitable for the particular kind of colour difference to be dealt with. If only sources giving black-body distributions at various temperatures have to be measured, the provision of a set of sub-standards at various efficiencies is probably the most direct method. Alternatively, one or two sub-standards may be

* In some laboratories the sub-standards are adjusted, in use, to a definite value of watts instead of a definite voltage This reduces the effective rate of deterioration of candle-power

standardised at a number of different efficiencies. This method, however, while economical in lamps, has the great disadvantage that the breakage or accidental over-running of a single lamp renders useless a number of valuable standardisations.

When other than black-body distributions have to be compared, recourse must be had either to colour filters or to a flicker photometer. When gelatine filters are used, they should be stored away from daylight, since some of them are bleached by continued exposure to strong light The transmission factors of colour filters should always be determined at a standardising laboratory, and each filter should be marked indelibly so that there is no risk of confusing it with a similar filter which may have a slightly different transmission factor. The box containing a filter should show (i) the distinguishing mark of the filter, (ii) its colour, (iii) its transmission factor, (iv.) the source and, in the case of an electric lamp, efficiency of the light with which the filter was standardised, (v) the date of standardisation

Illumination Photometers.—The bench room may conveniently serve as the permanent home of the illumination photometers used in the laboratory Each photometer should be kept in a separate case with its own standard surface, the latter being clearly marked with a reference to the particular instrument with which it is used The containing case should bear a sheet giving (i.) the name and distinguishing mark of the photometer, (ii) the correct voltage or current for the lamp, (iii) a list of correction factors or a reference to the instrument calibration curve, (iv) the transmission factors of any neutral or coloured filters with which the instrument is fitted, (v.) the date on which the lamp in the instrument was first taken into use, (vi) the date when the instrument was last checked on the photometer bench

The Sphere Photometer Room.—It is convenient to have a special room for measurements of total flux when these are carried out in an integrating photometer A separate bench can then be allotted to the work, and the bench and sphere can be kept fixed in relation to each other Further, the wiring of the sphere can be made permanent. It is, however, desirable to arrange the wiring so that if the sphere be not in use the bench can be used for other work. The structural features, such as rails, overhead tackle, *etc* , necessary for enabling the sphere to be moved, or to be opened for periodical repainting, must be designed to suit the special circumstances of the case [33] The sphere should, if possible, be so arranged that (*a*) the height of the translucent window, which governs the height of the photometer head, is convenient for an observer when seated, and (*b*) the lamp in the sphere can be easily and quickly changed without any necessity for disturbing the observer at the photometer head Devices to achieve this have been described already in Chapter VII , p. 219.

Physical Photometry.—The rooms for spectrophotometry and for physical photometry require no special mention In the latter, accommodation must be provided for one or more sensitive galvanometers, and the site should be so chosen that there is no trouble due to vibration from generators or to stray fields from heavy currents carried by cables passing near the room. If photo-electric cells be

used, one or more 100-volt batteries of small capacity will be required for use in this room alone.

Spectrophotometry.—For spectrophotometer work a sub-standard of spectral distribution is necessary. This may be either a "black body" of the form described in Chapter V, p. 132, or a sub-standard electric glow lamp, of which the spectral energy distribution curve at some known voltage and current has been determined by comparison with a black body at the standardising laboratory

For calibrating a spectrometer or spectrophotometer it is necessary to have various discharge tubes and a good induction coil (see p. 274). Suitable colour filters for use with the discharge tubes may also be required.

Projection Photometry.—For the photometry of projectors a long room is necessary Many pieces of projection apparatus, such as automobile headlights, should be tested at distances of 50 feet or more, so that 100 feet by 20 feet is the minimum size for the projector room. This should be provided with a white screen at one end, so that the distribution of light in the beam of a projector may be examined visually It is most important that either the walls, ceiling and floor of this room should be kept as dark as possible, or, preferably, large portable screens should be set up at intervals along the room so that the standard surface of the photometer cannot receive light from anywhere but a small region surrounding the source of light under test. These screens should be dimensioned and placed so that they act in the same way as the screens used on a photometer bench (see p. 170).

Unless in frequent use for the photometry of projection apparatus, the projector room may conveniently be used also for the measurement of the polar distribution curves of large sources and fittings by means of the mirror apparatus described on p. 197. The candle-powers to be measured often cover a very wide range of magnitude, even with a single source, so that either a very long bench must be employed or, preferably, a shorter (*e.g* 3-metre) bench may be mounted on a table provided with rollers, so that it can be moved bodily along rails laid in the floor for the whole length of the room. The table must be provided with a pointer which can be set at any one of a number of fixed marks on the rails These marks are generally at intervals of a metre, so that the bench can be placed in such a position that the photometer head, when clamped at the zero end, is at such a distance from the source that the comparison lamp takes up a convenient position on the bench, *e g*, between 1,500 and 2,000 millimetres.

In this work, again, careful attention must be given to the arrangement of screens to give adequate protection from stray light. Portable screens are required, too, for (*a*) covering the mirrors and (*b*) cutting off the direct light from the lamp during the test (see p 199). The electrical measurements, both in the case of projection photometry and in the determination of candle-power distribution curves, may be made with precision indicating instruments, since in neither case can an accuracy better than about 1 per cent. be obtained without very elaborate precautions

Life Tests of Gas-lamps.—An important branch of the work of a photometric laboratory is, frequently, the life testing of lamps,

i e , the measurement of their candle-power at intervals during a long period of operation under controlled conditions [34] In the case of gas lamps, these may be set up in rows, a series of lengths of 2½-inch pipe being arranged side by side, with outlets at convenient intervals. The pipes should on no account be placed one above another, but should be in a horizontal plane, preferably about 4 feet from the floor, with a gangway at least 4 feet wide between pairs, so as to facilitate removal of the lamps to the bench room for measurement The outlets for upright burners may be placed at intervals of about 8 or 9 inches along the pipes In the case of inverted burners, however, the interval should be at least 12 to 15 inches. It is convenient to have each burner connected with the pipe by means of a well-fitting cone joint, so that lamps can be removed with as little mechanical shock as possible. The room used for the life tests should be as close as possible to the bench room in which the photometric measurements are made. Mica chimneys may be used during a life-test run, but naturally the photometric measurements should be made with the chimneys normally used with the lamps Each chimney should be cleaned before each measurement, and, unless the mean horizontal candle-power be measured, care should be taken to ensure that the same part of the chimney and the same side of the mantle face the photometer head on the occasion of each measurement Burners should be attended to during a life-test run just as they would be in normal use under good conditions, i e., dust should be blown out and the air and gas regulation adjusted at specified intervals. Candle-power measurements are often made initially (i e , about half an hour after burning off), and at 25, 100, 300 and every subsequent 200 hours Frequently no photometric measurements are made after 300 hours [35], as the fall in candle-power is generally exceedingly slow after this period of burning, and the chief interest of the further test is to determine how long the mantle will remain intact.

The pressure of the gas supplied to the life-test lamps should be carefully regulated, and a recording pressure gauge should be attached to the supply at some convenient point as near the burner pipes as possible. The calorific value of the gas should be measured at least once a day during the test.

Arrangements for the Rapid Measurement of Life-Test Lamps.—When the lamps have to be moved to the bench room for making the photometric measurements, it is necessary to allow at least twenty minutes to elapse after a mantle has been lighted before the candle-power is measured (see p. 169). The considerable delay thus caused may be avoided by making the measurements in the life-test room. The apparatus is shown in plan in Fig 296 T is a table carrying a 3-metre photometer bench and moving along the room on rails R, R L_C is a comparison lamp, and P a Lummer-Brodhun photometer head The gas burners B_1, B_2, B_3, *etc* , are arranged in a line parallel to, and at a fixed distance (say, 1,500 mm) from, the line of travel of the bench zero. L_S is a sub-standard lamp in the same straight line with B_1, B_2, *etc* During the life run the burners are supplied from the pipe D, but by means of a three-way tap each one in turn can be transferred to pipe E, so that while its candle-power is being determined its consumption can also be measured

by means of the meter M. The photometric procedure will be clear from the figure. The fixed distance for L_C is first determined in the ordinary way, with the bench opposite L_S. Several sub-standards are used for this determination, as described in Chapter VI. The

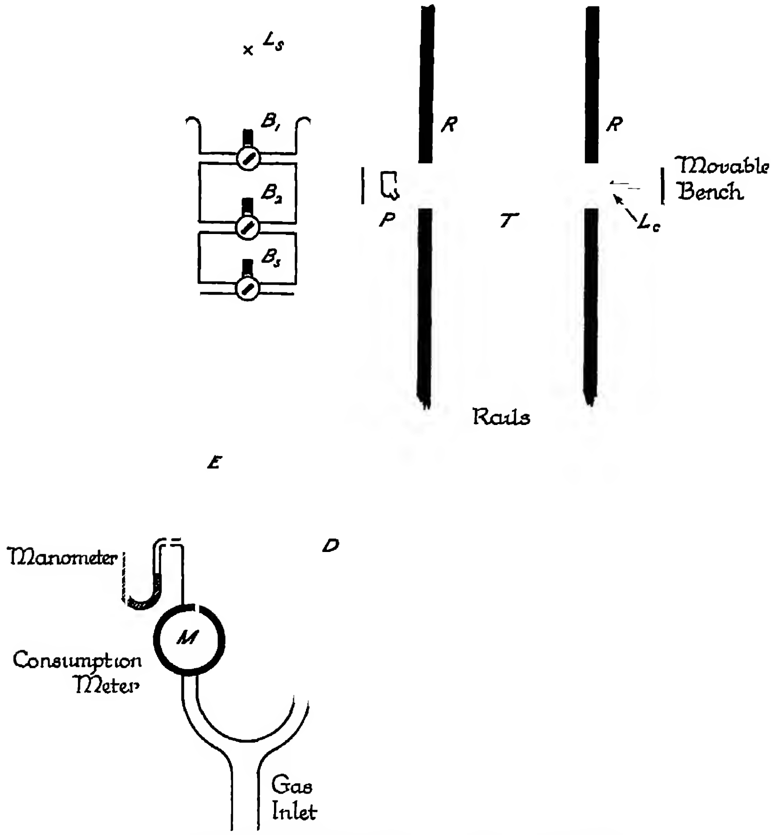

FIG 296 —Life Test Arrangements for Gas Mantles

bench is then moved until it is opposite burner B_1, say, this burner is, by means of its three-way tap, transferred to pipe E, and the photometric measurement is made as usual Similarly, B_2, B_3, *etc*, are measured in rapid succession It will be seen that, owing to the fact that all the burners are alight, special attention must be paid to the screening of the photometer head on both sides.

The particular arrangement above described may clearly be modified in many ways to suit local conditions (36), but the essential feature is that by making the measurements on the mantle *in situ* not only is delay avoided in making of the measurements, but, further, there is no risk of damage to the mantle while the burner is being transferred from the life-test room to the bench room.

Life Test of Electric Glow Lamps.—Electric lamps have to be tested for life on a supply of which the mean voltage is carefully regulated to an accuracy of about 0·1 per cent, momentary fluctuations not exceeding 1 per cent. (37). They may be run either at a

The resistance in series with a lamp is, to the nearest tenth of an ohm, of the correct value required to give the necessary drop of

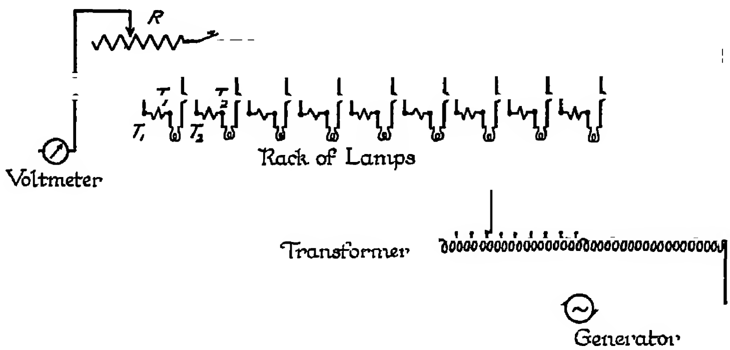

FIG 298—Arrangements for Life Test of Electric Lamps

potential between the rack leads and the lamp terminals. For example, if the highest voltage of all the lamps on the rack be E_1, the rack potential is adjusted to E_1 by means of R, and a lamp requiring E_2 volts, and taking a current I, has a resistance of

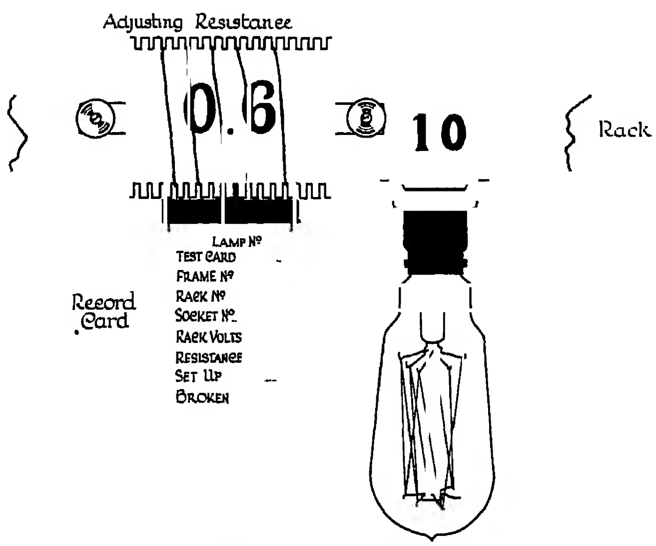

FIG 299—Set-up for a Life Test Lamp

$(E_1 - E_2)/I$ placed in series with it The general arrangement for a single lamp is shown in Fig 299.

The supply must be from a storage battery if a test on direct current be required. For a test on alternating current ([43]) the most convenient arrangement is that in which a generator, governed by a form of automatic voltage regulator, such as the Tirrill or Brown-Boveri ([44]), supplies an autotransformer with a number of tappings in convenient voltage steps. A continuous record of the voltage on

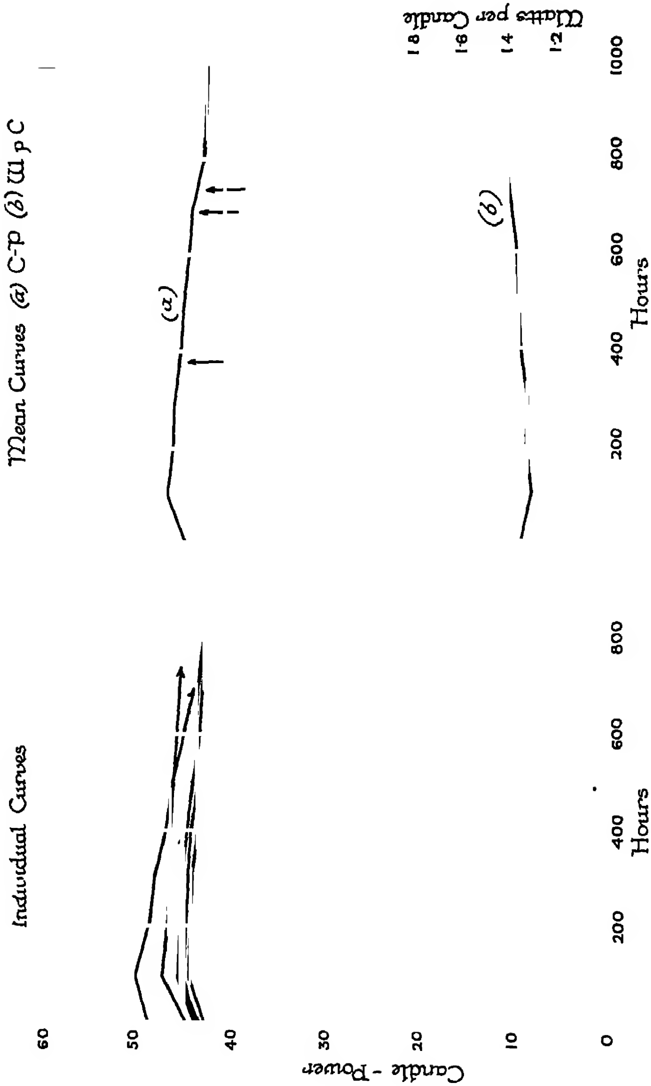

FIG 300 —Typical Life-Test Performance Curves

the supply should be kept by means of a recording voltmeter. The racks are inspected at convenient time intervals, and lamp failures are noted ([45]). It is generally sufficient if the period of burning of each lamp be known to the nearest ten or twelve hours. Alternatively, some automatic device may be used for recording the lamp failures ([46]).

The candle-power and current measurements on a lamp are usually made at rated voltage for the sake of uniformity. In a test at normal efficiency (1,000 hours) they are often made at the expiration of 100, 200, 300, 500, 750 and 1,000 hours from the commencement of the test ([47]) No attempt is made to readjust the voltage, as the efficiency of the lamp changes during the life If the filament of a lamp fracture during actual burning it is removed from the test and noted as broken, even though the loose limb of the filament may fall across another portion and so cause the lamp to continue burning. If a filament fracture during handling, the lamp is considered to be accidentally broken, and the results on it are not included in the life-test figures ([48]).

In order to avoid the long delay caused by a full 1,000-hour test and the cost of the power consumed in such a test, especially in the case of high wattage lamps, it is often desirable to carry out a "forced" life test, i e, a test with the lamps running at a voltage at which they have an efficiency which is some definite fraction, 5 or 10 per cent., above the normal efficiency A correction factor is then applied to the results of such a test in order to arrive at the result which would probably have been obtained by means of a normal life test on the same lamps ([49]) The intervals at which candle-power measurements are made are generally shorter in the case of a forced life test than in a normal test, and the number of lamps chosen to represent a given batch is usually larger

Life tests may be carried out on the basis of mean horizontal candle-power or total flux While the latter is generally used for rating, and therefore for determining the efficiency at which the lamp is run, the reduction factor is usually very constant throughout the life of a *vacuum* lamp, so that the mean horizontal candle-power can be used for the life-test readings on such lamps if it is found more convenient to make measurements of this quantity This remark does not apply in general to *gas-filled* lamps.

Life-Test Curves.—The results of life tests, whatever the source tested, are usually exhibited in the form of a performance curve ([50]), of which the abscissæ represent hours of actual burning from the beginning of the test, and the ordinates (*a*) candle-power and (*b*) efficiency. A typical set of performance curves for a set of electric lamps is shown in Fig. 300, where the left-hand curves refer to the individual lamps, while the right-hand curves refer (*a*) to the mean candle-power and (*b*) the mean efficiency of all the lamps tested

Shock Tests.—Tests of mechanical strength are sometimes included as part of a life test, and in such cases the photometric laboratory must be equipped for carrying out such tests at intervals during the life run. The apparatus used is not in any way standardised, and any description of it does not fall within the scope of this book ([51])

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(2) Illum. Eng, 2, *1909*, p 153
(3) F J Rogers Phys Rev, 16, *1903*, p 166
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(4) W. M Stine "Photometrical Measurements," p. 199
(5) In a carbon filament lamp the potential and current changes are almost equal, while the candle-power change is five to six times as great See Appendix X, p. 481.
(6) For the principle and use of the potentiometer see any book of practical electricity, such as.
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(7) See, *e g*, G Lebaupin, C I.E, Proc, 6, *1924*, p 132, Rev Gén de l'El., 16, *1924*, p 418
(8) For a description of apparatus designed to facilitate the rapid changing of lamps on the photometer bench see, *e g.*, El World, 33, *1899*, p 216
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(11) See, in this connection, E J Edwards, Am I E E, Trans, 31, *1912*, p 1517
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(13) A slightly different mode of procedure is described by C H Sharp, "Lectures on Illum Engineering" (Johns Hopkins Univ Press, *1911*), p 453
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(16) See references in note (54), p 190
(17) See, *e g*, S Nowacki, Elektrot Anzeiger, 26, *1909*, p. 737
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(19) Notification of the Gas Referees for 1923 (H M Stationery Office), p 31
(20) N. Macbeth. Illum Eng Soc N. Y, Trans, 5, *1910*, p 685
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(22) R. S. McBride, W A Dunkley, E C Crittenden and A H. Taylor. Bureau of Standards, Technol Paper No 110, *1918*
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(25) This value includes the heat evolved by the condensation of the steam formed to water at room temperature
(26) R Assmann Das Wetter, 4, *1887*, p 265, and 5, *1888*, p 1, Deut Phys Gesell, Verh, *1889*, p 105, Meteorolog Z, 6, *1889*, p. 278, Z f. I, 12, *1892*, p. 1
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(27) "Aspirations Psychrometer-Tafeln" (Vieweg, Brunswick, *1908*).

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(29) Report of Committee of Am Gas Inst, Proc., 2, *1907*, p 454, J of Gas Lighting, 100, *1907*, p 486, and 101, *1908*, p. 236, Illum. Eng, 2, *1909*, p 203

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(30) Needless to say, the lamp itself should be indelibly marked with a reference number, and the four items of information above enumerated should be entered in a book under this number, in case the label described should be lost

(31) R W Shenton, El Rev and Western Elect, 62, *1913*, p 154

(32) On the effect of over-running in carbon lamps, see J S. Dow, Electrician, 57, *1906*, p 855

(33) See, *e g*, E T Z, 29, *1908*, p 10, Z f Bel, 14, *1908*, p 37, Illum. Eng, 1, *1908*, p 228

(34) See, *e g*, R H Maurer, Illum Eng Soc N Y, Trans, 16, *1921*, p 13

(35) See, *e.g*, the regulations of the Deut Verein von Gas- u Wasserfachmännern, J G W, 41, *1898*, pp 828 and 842, 44, *1901*, pp 697 and 737

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See also C Carpenter, Brit Patent No 1,239 (*1914*)

(37) British Engineering Standards Association, Spec No 161, *1924*, Clause 18 (a)

(38) See also J F Skogland, Bureau of Standards, Bull, 12, *1916*, p 269 Apparatus has been devised for obtaining, by means of a single observation, the voltage at which a lamp has a certain specified efficiency See R P Wilson and G G M Hardingham, Brit Patent No 24,325 (*1902*), E P Hyde and H B Brooks, Bureau of Standards, Bull, 2, *1906*, p 145, El World, 46, *1905*, p 942, E T Z, 27, *1906*, p 450, Electrician, 59, *1907*, p 427, Ecl El, 46, *1906*, p 436, and 53, *1907*, p 32 C Paulus, E T Z, 29, *1908*, p 166, Lum El, 1, *1908*, p 413 H E Ives, Bureau of Standards, Bull, 5, *1909*, p 543, El World, 53, *1909*, p 732, and 59, *1912*, p 1268, Nela Bull. 1, *1913*, p. 108, Lighting J., 1, *1913*, p 178 W T Birdsall, El. World, 60, *1912*, p 157 E P Hyde, El Rev and W Elect, 62, *1913*, p 248, Nela Bull. 1, *1917*, p 203 H W B Gardiner, J Sci Insts, 1, *1923*, p 90 B S Willis, Illum Eng, Soc N. Y, Trans, 18, *1923*, p 62

(39) See note (49), *infra*

(40) The method adopted in the selection of lamps for life tests and the interpretation of the results obtained are matters which fall outside the scope of this book. They have been dealt with at some length in various published papers See, *e g*, W H Preece, Electrician, 37, *1896*, p 733, P S Millar and L J Lewinson, Illum Eng Soc N Y, Trans, 6, *1911*, p 744, and El Rev (Chicago), 68, *1916*, p 1062 Brit Eng Stds Assn Specification for Electric Lamps, No 161 (*1924*), Bureau of Standards, Circular No 13 (10th edit), *1923*

(41) This is clearly indicated in the B E S A Specification for Electric Lamps, No 161 (*1924*), Table IV, and in Circular No 13 (10th edit) of the Bureau of Standards, where it will be seen that the tolerance allowed in the life performance increases rapidly as the number of lamps tested for life diminishes See, too, L. J. Lewinson, Illum Eng Soc N Y, Trans, 11, *1916*, p 1093, C C Paterson and E H Rayner, Electrician, 65, *1910*, p 893, N P L, Coll Res, 7, *1911*, p 203, and C H Sharp and P S Millar, El World, 56, *1910*, p 665

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A somewhat different method of voltage adjustment is used at the Electrical Testing Labs, N Y See El World, 72, *1918*, p 206, Rev Gén de l'El, 4, *1918*, p 594

(43) It appears that, for tungsten lamps, whether vacuum or gas-filled, the fall of candle-power when the lamps are run on direct current is slightly more rapid than when they are run on alternating current See J W Lieb, Illum Eng Soc N Y, Trans, 18, *1923*, p 5, Am I.E E, J, 42, *1923*, p 544, Licht u Lampe, *1923*, p 163 But see also C H Sharp, Am I E E, Trans, 25, *1906*, p 815, G Mongini, Assoc Elettrot Ital, Atti, 17, *1913*, p 690, M G Lloyd, Phys Rev, 32, *1911*, p 256

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(46) See, *e g*, E T Z, 22, *1901*, pp 67 and 184

An automatic arrangement might very easily be devised to take a photograph of each life-test rack every hour

(47) B E S A. Specification No. 161, *1924*, Clause 18 (b)

(48) *Idem*, Clause 17 (c).

(49) It may be assumed that for vacuum lamps the life and efficiency are connected by the relation $l \propto (w/c)^n$ where n is 5 83 for carbon lamps, and varies for tungsten from 6 for small lamps to about 7·4 for the larger sizes For gas-filled lamps no reliable value of n can be given

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(51) See, *e g.*, J. H Coste and W E F Powney, Soc Chem Ind, J, 30, *1911*, p 65, Chem News, 103, *1911*, p 21 (Gas mantles)

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APPENDIX I

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(6) *L'unité d'intensité lumineuse est la* **Bougie Internationale** telle qu'elle résulte des accords intervenus entre les trois laboratoires nationaux d'étalonnage de France, de Grande-Bretagne et des Etats-Unis en 1909.* Cette unité a été conservée depuis lors au moyen de lampes à incandescence électriques, dans ces laboratoires qui restent chargés de sa conservation

* Ces laboratoires sont. le Laboratoire Central d'Electricité à Paris, le National Physical Laboratory à Teddington, et le Bureau of Standards à Washington

The following translation of the above definitions has been officially adopted by the National Illumination Committees of Great Britain and of the United States of America † —

(1) **Luminous Flux** is the rate of passage of radiant energy evaluated by reference to the luminous sensation produced by it

Although luminous flux should be regarded, strictly, as the rate of passage of radiant energy as just defined, it can, nevertheless, be accepted as an entity for the purposes of practical photometry, since the velocity may be regarded as being constant under those conditions.

(2) *The Unit of Luminous Flux is the* **Lumen.** It is equal to the flux emitted in unit solid angle by a uniform point source of one international candle

(3) **Illumination.** The illumination at a point of a surface is the density of the luminous flux at that point, or the quotient of the flux by the area of the surface when the latter is uniformly illuminated.

(4) *The Practical Unit of Illumination is the* **Lux.** It is the illumination of a surface 1 square metre in area, receiving a uniformly distributed flux of 1 lumen, or the illumination produced at the surface of a sphere having a radius of 1 metre by a uniform point source of 1 international candle situated at its centre

In view of certain recognised usages, illumination may also be expressed in terms of the following units —

Taking the centimetre as the unit of length, the unit of illumination is the lumen per square centimetre, it is known as the "**Phot.**" Taking the foot as the unit of length, the unit of illumination is the lumen per square foot; it is known as the "**Foot-Candle.**"

1 **Foot-Candle** = 10 764 **Lux** = 1 0764 **milli-phot.**

(5) **Luminous Intensity (Candle-power).** The luminous intensity (candle-power) of a point source in any direction is the luminous flux per unit solid angle emitted by that source in that direction (The flux emanating from a source whose dimensions are negligible in comparison with the distance from which it is observed may be considered as coming from a point)

(6) *The Unit of Luminous Intensity (Candle-power) is the* **International Candle,** such as resulted from agreements effected between the three National Standardising Laboratories of France, Great Britain and the United States, in 1909. ‡ This unit has been maintained since then by means of incandescent electric lamps in these laboratories, which continue to be entrusted with its maintenance

‡ These Laboratories are the Laboratoire Central d'Electricité in Paris, the National Physical Laboratory in Teddington, and the Bureau of Standards in Washington

The following definitions were officially adopted at the 1924 meeting of the International Commission on Illumination § —

(7) **Brillance.** La brillance dans une direction donnée d'une surface émettant de la lumière est le quotient de l'intensité lumineuse mesurée dans cette direction par l'aire projetée de cette surface sur un plan perpendiculaire à la direction considerée

† Report of National Illum Com. of Great Britain, *1922* Illum Eng, 16, *1923*, p 52, Gas J, 162, *1923*, p 165, Inst. Gas Eng., Trans, *1922-23*, p 78, Illum Eng. Soc N Y., Trans, 20, *1925*, p 629

§ C I.E., Proc. 6, *1924*, p 68, etc

(8) *L'unité de brillance est la bougie par unité de surface*

(9) **Facteur de transmission d'un corps.** C'est le rapport du flux transmis par le corps au flux incident qu'il reçoit

(10) **Facteur d'absorption d'un corps.** C'est le rapport du flux absorbé par le corps au flux incident qu'il reçoit

(11) **Facteur de réflexion d'un corps.** C'est le rapport du flux réfléchi par le corps au flux incident qu'il reçoit

Le flux réfléchi selon les lois de la réflexion régulière est appelé flux régulièrement réfléchi, et le facteur de réflexion correspondant prend le nom de *facteur de réflexion régulière* Le flux diffusé, c'est-à-dire envoyé dans autres directions que celle de la réflexion régulière, donne le *facteur de réflexion diffuse* Lorsqu'on considère l'ensemble du flux renvoyé par le corps, on obtient le *facteur total* de réflexion

(12) **Flux total d'une source.** C'est l'ensemble du flux émis par cette source

(13) **Flux hémisphérique supérieur (super-horizontal).** C'est le flux émis par la source au-dessus du plan horizontal passant par son centre

(14) **Flux hémisphérique inférieur (sub-horizontal).** C'est le flux émis par la source au-dessous du plan horizontal passant par son centre

(15) **Intensité moyenne sphérique d'une source.** C'est la moyenne des valeurs de l'intensité de la source dans toutes les directions de l'espace

(16) **Intensité moyenne hémisphérique supérieure.** C'est la moyenne des valeurs de l'intensité de la source dans toutes les directions au-dessus du plan horizontal passant par son centre

(17) **Intensité moyenne hémisphérique inférieure.** C'est la moyenne des valeurs de l'intensité de la source dans toutes les directions au-dessous du plan horizontal passant par son centre

(18) **Intensité horizontale moyenne.** C'est la moyenne des valeurs de l'intensité de la source dans toutes les directions du plan horizontal passant par son centre

(19) **Facteur de réduction** de l'intensité moyenne sphérique d'une source. C'est le rapport de l'intensité moyenne sphérique à l'intensité moyenne horizontale

(20) **Facteur d'efficacité d'une source.** C'est le rapport du flux lumineux total à la puissance totale consommée Dans le cas d'une lampe électrique, il est exprimé en lumens par watt, dans le cas d'une source utilisant la combustion, on peut l'exprimer en lumens par unité de temps et par unité thermique.

(21) **Facteur de visibilité** pour une radiation monochromatique C'est le rapport du flux lumineux au débit du flux d'énergie correspondant Le facteur de visibilité relative d'une radiation monochromatique est le rapport du facteur de visibilité de cette radiation à la valeur maximum du facteur de visibilité

The following translation of these definitions has been proposed by the National Illumination Committee of Great Britain * —

(7) **Brightness.** The brightness in a given direction of a surface emitting light is the quotient of the luminous intensity measured in that direction by the area of this surface projected on a plane perpendicular to the direction considered

(8) *The unit of brightness is the candle per unit area of surface*

(9) **The Transmission Factor** of a body is the ratio of the flux transmitted by the body to the flux incident upon it.

(10) **The Absorption Factor** of a body is the ratio of the flux absorbed by the body to the flux incident upon it

* Report for *1924* Illum. Eng., 18, *1925*, p. 40.

(11) **The Reflection Factor** of a body is the ratio of the flux reflected by the body to the flux incident upon it.

The flux reflected according to the laws of specular reflection is called specularly reflected flux, and the corresponding reflection factor is called the factor of specular reflection The flux diffused, *i e*, that sent out in directions other than that of specular reflection, gives the diffuse reflection factor. The total reflection factor is obtained by considering the whole of the flux reflected by the body

(12) **The Total Flux of a source** is the flux emitted by that source in all directions

(13) **The Upper Hemispherical Flux** of a source is the flux emitted by that source above the horizontal plane passing through its centre

(14) **The Lower Hemispherical Flux** of a source is the flux emitted by that source below the horizontal plane passing through its centre.

(15) **The Mean Spherical Intensity** of a source is the average value of the intensity of that source in all directions in space

(16) **The Mean Upper Hemispherical Intensity** of a source is the average value of the intensity of that source in all directions above the horizontal plane passing through its centre

(17) **The Mean Lower Hemispherical Intensity** of a source is the average value of the intensity of that source in all directions below the horizontal plane passing through its centre

(18) **The Mean Horizontal Intensity** of a source is the average value of the intensity of that source in all directions in the horizontal plane passing through its centre.

(19) **The Reduction Factor** of the mean spherical intensity of a source is the ratio of the mean spherical intensity to the mean horizontal intensity

(20) **The Efficiency of a source** is the ratio of the total luminous flux emitted to the total power consumed. In the case of an electric lamp it is expressed in lumens per watt. In the case of a source depending upon combustion it may be expressed in lumens per thermal unit per unit of time.

(21) **The Visibility Factor** for monochromatic radiation is the ratio of the luminous flux to the corresponding energy flux.

The relative visibility factor of a monochromatic radiation is the ratio of the visibility factor of that radiation to the maximum value of the visibility factor.

The following additional definitions have been adopted by the Illuminating Engineering Society of New York, and by the American Engineering Standards Committee * .—

LUMINOUS FLUX

(22) **Light.** The term light is used in various ways

(*a*) To express the *visual sensation* produced normally when radiant flux (*q v*) within the proper limits of wave-length, of sufficient intensity and of sufficient duration, impinges on the retina

(*b*) To denote the *luminous flux* (*q v*) which produces the visual sensation

(*c*) By extension, even to denote *radiant flux* of wave-lengths outside of the visible spectrum (*e g*, ultra-violet light)

(23) **Radiant Flux** is the rate of energy radiation, and is expressed in *ergs per second* or in *watts*

(24) **Quantity of Light** is the product of the luminous flux by the time it is maintained It is the time integral of luminous flux

(25) The **Lumen-Hour** is the unit of quantity of light It is equal to a flux of 1 lumen continued for one hour

(26) The **Lambert** is the average brightness of any surface emitting or

* Illum Eng. Soc N Y, Trans., 20, *1925*, p 629.

reflecting 1 lumen per square centimetre, or the uniform brightness of a perfectly diffusing surface emitting or reflecting 1 lumen per square centimetre

For most purposes the **millilambert,** 0·001 lambert, is the preferable practical unit

Brightness expressed in candles per square centimetre may be reduced to lamberts by multiplying by π

Brightness expressed in candles per square inch may be reduced to lamberts by multiplying by $\pi/6\ 45 = 0{\cdot}487$

(27) The **Foot-Lambert** is the average brightness of any surface emitting or reflecting 1 lumen per square foot, or the uniform brightness of a perfectly diffusing surface emitting or reflecting 1 lumen per square foot

A completely reflecting surface under an illumination of 1 foot-candle, therefore, has an average brightness of 1 foot-lambert, the average brightness of any surface in foot-lamberts is the product of the illumination in foot-candles by the reflection factor of the surface

One foot-lambert is equal to 1 076 millilamberts.

One foot-lambert is equal to $1/144\ \pi$ candles per square inch, brightness expressed as candles per square inch may, therefore, be reduced to foot-lamberts by multiplying by 452

(28) The **Mechanical Equivalent of Light** is the ratio of radiant flux to luminous flux for the wave-length of maximum visibility, and is expressed in *ergs per second per lumen*, or in *watts per lumen* It is the reciprocal of the maximum absolute visibility.

As a standard value for the mechanical equivalent of light, the figure 0·0015 watt per lumen is recommended.

This term has been used in a variety of senses. As here defined it refers only to the minimum mechanical equivalent of light and corresponds to monochromatic light of maximum visibility The reciprocal of this quantity is sometimes called the luminous equivalent of radiation.

(29) A **Luminosity Curve** of a source of light is a curve showing for each wave-length the luminous flux per element of wave-length Therefore it gives, wave-length by wave-length, the product of the radiant flux and the visibility

(30) The **Luminous Efficiency** of any source is the ratio of the luminous flux to the radiant flux from the source For practical purposes it is usually expressed in *lumens per watt radiated* (This is not to be confused with the efficiency of a source See definition No 20)

Surfaces and Media Modifying Luminous Flux

(31) **Diffusing** surfaces and media are those which break up the incident flux and distribute it more or less in accordance with the cosine law, as for example, white plaster and opal glass

(32) **Redirecting** surfaces and media are those which change the direction of the luminous flux in a definite manner, as for example, a mirror or a lens

(33) **Scattering** surfaces and media are those which redirect the luminous flux and break it up into a multiplicity of separate pencils, as for example, rippled glass

Illumination

(34) **Unidirectional** illumination on a surface is that produced by a single light source of relatively small dimensions. It is characterised by the fact that a small opaque object placed near the illuminated surface casts a sharp shadow

(35) **Multidirectional** illumination on a surface is that produced by several separated light sources of relatively small area. It is characterised by the fact that a small opaque object placed near the illuminated surface casts several shadows.

(36) **Diffused** illumination is that produced either by primary or secondary light sources having dimensions relatively large with respect to the distance from the point illuminated, and scattering light in all directions It is characterised by relative lack of shadow Diffused illumination may be derived principally from a single direction, as in the light from a skylight window, or from all directions, as in the open air. Perfectly diffused illumination on a surface is shadowless

In any practical case of illumination on a surface there is usually a mixture of the above types

(37) **Coefficient of Utilisation** of an illumination installation on a given plane is the total flux received by that plane divided by the total flux from the lamps illuminating it When not otherwise specified, the plane of reference is assumed to be a horizontal plane 30 inches (76 cm.) from the floor.

(38) **Variation Factor** of an illumination installation is the ratio of either the maximum or minimum illumination on a given plane to the average illumination on that plane

(39) **Variation Range** of illumination on a given plane is the ratio of the maximum illumination to the minimum illumination on that plane.

Photometry

(40) A **Primary Luminous Standard** is one by which the unit of light is established, and from which the values of other standards are derived A satisfactory primary standard must be reproducible from specifications.

(41) A **Secondary Standard** is one calibrated by comparison with a primary standard The use of the term may also be extended to include standards which have not been directly measured against the primary standards, but derive their assigned values indirectly from the primary standards

Because of the lack of a satisfactory primary standard of light the unit is actually maintained in most laboratories by electric incandescent lamps serving as reference standards The values assigned to these standards were originally agreed upon as representing the average value of the accepted primary standard as nearly as this could be determined This procedure is formally recognised in France and the United States

(42) A **Working Standard** is any standardised luminous source for daily use in photometry.

(43) A **Comparison Lamp** is a lamp of constant, but not necessarily known, candle-power against which a working standard and test lamps are successively compared in a photometer

(44) A **Test Lamp,** in a photometer, is a lamp to be tested.

(45) A **Performance Curve** is a curve representing the behaviour of a lamp in any particular (candle-power, consumption, *etc*) at different periods during its life

(46) A **Characteristic Curve** is a curve expressing a relation between two variable properties of a luminous source, as candle-power and volts, candle-power and rate of fuel consumption, *etc*

(47) A **Horizontal Distribution Curve** is a polar curve representing the luminous intensity of a lamp, or lighting unit, in a plane perpendicular to the axis of the unit, and with the unit at the origin

(48) A **Vertical Distribution Curve** is a polar curve representing the luminous intensity of a lamp, or lighting unit, in a plane passing through the axis of the unit and with the unit at the origin. Unless otherwise specified, a vertical distribution curve is assumed to be an average vertical distribution curve, such as may in many cases be obtained by rotating the unit about its axis and measuring the average intensities at the different elevations It is recommended that in vertical distribution curves angles of elevation shall be counted positively from the nadir as zero to the zenith as 180° In the case

of incandescent lamps, it is assumed that the vertical distribution curve is taken with the tip downward.

(49) The **Apparent Candle-power** of an extended source of light is the candle-power of a point source of light which would produce the same illumination at the distance employed

COLOUR NOMENCLATURE

(50) **Quality of Luminous Flux** is that property of luminous flux determined by its spectral distribution

(51) **Colour of Luminous Flux** is the subjective evaluation by the eye of the quality of luminous flux Any colour can be expressed in terms of its hue and saturation

(52) **Hue** is that property of colour by which the various spectral regions are characteristically distinguished All colours except purples and white may be matched in hue with spectral colours In the case of a purple, the spectral hue which is complementary to the hue of the purple is ordinarily used for scientific designation

(53) **Two hues are complementary** if they may be mixed to produce white. White may be considered as a colour having no hue By the mixture of luminous fluxes of two or more hues, properly chosen both as to hue and intensity, a resultant luminous flux may be obtained which has the colour white Whenever luminous fluxes of two or more hues are mixed, the resultant luminous flux, though it may have some dominant hue, will ordinarily be evaluated subjectively as having an admixture of white

(54) **Saturation of a Colour** is its degree of freedom from admixture with white Monochromatic spectral light may, for purposes of measurement, be considered as having a saturation of 100 per cent As white light is added the saturation decreases until, when the hue entirely disappears, the saturation is zero White, therefore, is the limiting colour, having no hue and zero saturation

The terms "**purity**" and "**chroma**" have also been used to denote the quantity above defined under the name "saturation" (Report of Committee on Colorimetry, Opt Soc Am, J, 6, *1922*, p 535) The name "chroma" has also been used for what is above called "quality" (Report of Committee on Spectrophotometry, Opt Soc Am, J, 10, *1925*, p 179)

The term "**dominant wave-length**" is sometimes used to denote the wave-length of the homogeneous (monochromatic) light which, when mixed in the correct proportion with white, gives a match with the light under consideration (Report of Committee on Spectrophotometry, *loc cit*, *supra*)

No general agreement has yet been reached on the subject of colour nomenclature (See, *e g.*, L C Martin, Nature, 114, *1924*, p 790)

A number of terms have been proposed for use in connection with the transmission of light through a plate of given thickness, composed either of a transparent solid or of a clear solution (Report of Committee on Spectrophotometry, Opt Soc Am, J, 10, *1925*, p 177)

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*P G Nutting Opt Soc. Am, J, 4, *1920*, p 230

*C.I E, Proc, 5, *1921*, pp 40, 58, 74, 98 and 103, Illum. Eng, 14, *1921*, p 171, Illum Eng. Soc N. Y, Trans, 16, *1921*, p 246, Gas Inst, Proc, *1920–21*, p 636, Gas J, 156, *1921*, p 226, Rev Gén de l'El, 10, *1921*, p 81; Licht. u Lampe, *1921*, p 578, E u M (Lichttechnik), 42, *1924*, p. 244.

E. Alberts. J G W., 64, *1921*, p. 155

*L Bloch. Licht u Lampe, *1922*, p 515

D H. Tuck El World, 80, *1922*, p 719

P. G. Nutting Opt Soc. Am, J, 7, *1923*, p. 407.

*Committee on Nomenclature and Standards Illum Eng Soc N Y, Trans, 18, *1923*, p 46, Licht u Lampe, *1923*, p 82

E. Karrer Opt Soc Am, J., 8, *1924*, p 30

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*A Blondel Rev Gén de l'El, 16, *1924*, pp 771 and 896, Licht u Lampe, *1925*, p 361

*Osterreich. Lichttechnische Gesell E u M (Lichttechnik), 43, *1925*, p 40

* In papers marked with an asterisk there are given suggested symbols for the various photometric quantities defined On the difficulty resulting from the use of special symbols different from the characters included in ordinary type, see El World, 58, *1911*, p 787

APPENDIX III

CONVERSION FACTORS

(I)

1 Hefner candle = 0·9000 international candles, hence
1 Hefner lumen = 0·9000 international lumens, and
To convert values in Hefner units to international units multiply by 0 9000,
To convert values in international units to Hefner units multiply by 1 1111.
The above rule applies to values of luminous intensity (candle-power), luminous flux, illumination, and brightness

(II)

TABLE OF CONVERSION FACTORS FOR ILLUMINATION UNITS.

	Lux	Milliphots	Foot-candles
Lux	1	0 1	0·092903
Milliphots	10	1	0 92903
Foot-candles	10 764	1 0764	1

(Value in unit in left-hand column) × (conversion factor) = (value in unit shown at top of column)

TABLE OF CONVERSION FACTORS FOR BRIGHTNESS UNITS

	c /cm 2	c /m 2	c /in 2	c /ft 2	Lamberts	Milli-lamberts	Foot-Lamberts
Candles per sq cm	1	10,000	6 452	920 0	3 1416	3,141 6	2,919
Candles per sq metre	0 0001	1	0 0006452	0 9290	0 00031416	0 31416	0 2919
Candles per sq inch	0 1550	1,550	1	144	0 4869	486 9	452 4
Candles per sq foot	0 0010764	10 764	0 006944	1	0 0003382	3 382	3 1416
Lamberts (Apparent lumens per sq cm)	0 3183	3,183	2 054	295 7	1	1,000	929 0
Millilamberts	0 0003183	3 183	0 002054	0 2957	0 001	1	0 9290
Foot-Lamberts (Equivalent foot-candles) (Apparent lumens per sq foot)	0 0003426	3 426	0 00214	0 3183	0 0010764	1 0764	1

(Value in unit in left-hand column) × (conversion factor) = (value in unit at top of column)

APPENDIX IV

THE LUMINOSITY (VISIBILITY) FACTOR

mμ	0	1	2	3	4	5	6	7	8	9
400	0004	0005	0006	0006	0007	0008	0009	0009	0010	0011
10	0012	0013	0015	0017	0019	0022	0025	0028	0031	0035
20	0040	0046	0052	0058	0065	0073	0081	0089	0098	0107
30	0116	0126	0136	0146	0157	0168	0180	0192	0204	0217
40	023	024	026	027	028	029	031	032	034	036
450	038	040	042	044	046	048	050	053	055	058
60	060	063	065	068	071	074	077	080	084	087
70	091	095	099	103	108	112	117	122	128	133
80	139	144	150	156	163	169	176	183	191	199
90	208	217	227	237	247	258	270	282	295	309
500	323	339	355	372	389	407	426	445	464	483
10	503	524	545	566	587	608	629	650	670	690
20	710	728	745	762	778	793	808	822	836	849
30	862	874	885	896	906	915	924	932	940	947
40	954	960	966	971	975	980	984	987	990	993
550	995	997	998	999	1	1	1	999	998	997
60	995	993	990	987	983	979	974	969	964	958
70	952	945	938	931	923	915	907	898	889	880
80	870	860	849	839	828	817	805	793	781	769
90	757	745	732	720	707	695	682	670	657	644
600	631	618	605	593	580	567	554	541	528	515
10	503	490	478	465	453	441	429	417	405	393
20	381	369	357	345	333	321	309	298	287	276
30	265	255	245	235	226	217	208	199	191	183
40	175	167	160	152	145	138	132	125	119	113
650	107	101	096	091	086	081	077	073	069	065
60	061	057	054	051	048	045	042	039	037	034
70	032	030	028	026	025	023	022	020	019	018
80	017	0159	0148	0138	0128	0119	0111	0103	0095	0088
90	0082	0076	0071	0066	0061	0057	0053	0050	0047	0044
700	0041	0038	0036	0033	0031	0029	0028	0026	0024	0022
10	0021	00196	00183	00171	00159	00148	00138	00129	00120	00112
20	00105	00098	00091	00085	00079	00074	00069	00064	00060	00056
30	00052	00048	00045	00042	00039	00036	00033	00031	00029	00027
40	00025	00023	00022	00020	00019	00017	00016	00015	00014	00013
750	00012	00011	00010	00009	00009	00008	00008	00007	00007	00006
60	00006									

Note —The above table has been constructed by interpolation between the internationally adopted values given in the second column.*

* C I E , Proc , 6, *1924*, p 67.

APPENDIX V

SPECTRAL ENERGY DISTRIBUTION FOR A CAVITY RADIATOR (BLACK BODY)

TABLES have been prepared to enable the spectral distribution of energy from a cavity radiator at any given temperature to be calculated with a minimum of labour * For radiation of any given frequency the rate of change of energy emission with change of temperature is so rapid that tables of energy distribution, unless very extensive, are almost useless for purposes of interpolation †

The most concise method of expressing the results over a wide temperature range is by means of a nomograph such as that shown in Fig. 301. For all practical purposes (*i.e*, to an accuracy of about 1 per cent) Wien's formula (see p 135) may be used instead of Planck's throughout the visible spectrum up to a temperature of 4,000° K Hence

$$\log (E_\nu/C_1) - 3 \log \nu + C_2\nu/T = 0,$$

or

$$\log (E_\lambda/C_1) + 5 \log \lambda + C_2/\lambda T = 0$$

Each of these equations leads at once to a simple nomograph ‡ in which the left-hand vertical line (T) is a reciprocal scale, and the right-hand one (E_ν or E_λ) a logarithmic The λ or ν scale is obtained graphically, using a table of calculated values of E such as that given in the "Report of the Committee on Colorimetry of the Optical Soc of America" (J, 6, *1922*, pp 558–9) The nomograph for E_λ is shown in Fig 301 Any straight line drawn across the diagram intersects the three scales at points showing corresponding values of T, λ and $E_\lambda/d\lambda$. In order to obtain a higher accuracy of reading on the E scale, two scales of λ have been drawn When the *upper* one of these is used, the appropriate figuring of the E scale is that shown at the *left* of the E line, while when the lower λ scale is employed the figuring on the *right* of the E scale must be used

The nomograph has been drawn for $C_2 = 1{\cdot}4330$ cm degs and $C_1 = 1$ If it be desired to use any other value of C_2, the point taken on the left-hand scale should be 1 4330 T/C_2 instead of T Similarly, the values of E read on the right-hand scale may be multiplied by any desired value of C_1

It is clear that a similar nomograph may be constructed to give E_ν. The spectral light distribution may be shown by a diagram constructed on the same plan, for, since the candle-power in any given wave-length interval, I_λ, is equal to $E_\lambda K_\lambda$, it follows that

$$\log (I_\lambda/C_1) + \log (\lambda^5/K_\lambda) + C_2/\lambda T = 0.$$

* W. W Coblentz. Bureau of Standards, Bull 15, *1920*, p 617 A series of graphs, has been published by M. K Frehafer and C. L. Snow in Bureau of Standards Miscellaneous Publ., No. 56, *1925*

† See, *e g*, Report of Committee on Colorimetry, Opt. Soc Am, J, 6, *1922*, pp 558–9

‡ See, *e.g*, S Brodetsky, "First Course in Nomography" (Bell, *1920*), p 91, or Article "Nomography" in Dict Appl Phys, vol 3, p 640

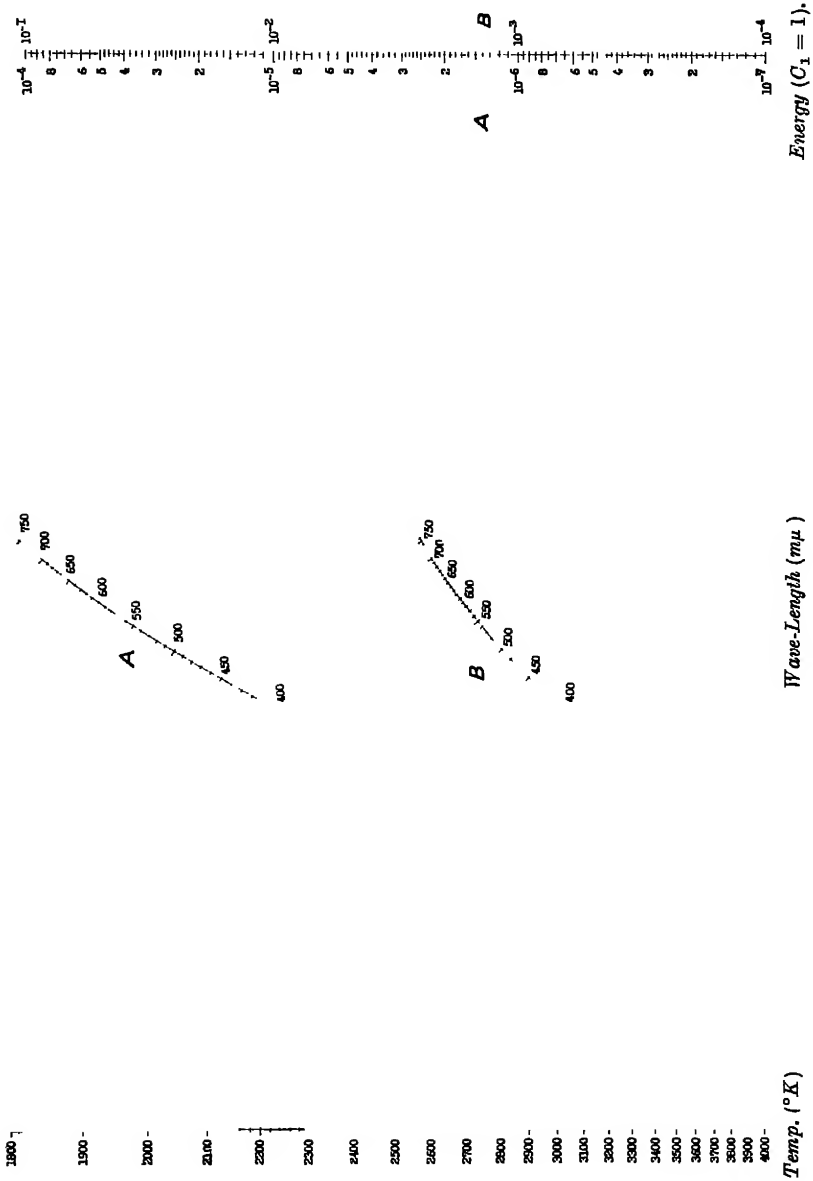

FIG 301 —Nomograph for finding the Spectral Energy Distribution for a Cavity Radiator

APPENDIX VI

THE CALCULATION OF CANDLE-POWER IN BENCH PHOTOMETRY

WHEN the "fixed distance" method (see Chapter VI, p 162) is used, the calculations involved in candle-power photometry on the bench are very simple, necessitating only the use of a table of squares or a slide rule

When this method cannot be used the calculations are more complicated, involving the introduction of the function $d^2/(d-x)^2$, for if d be the distance between the sub-standard (candle-power I_S) and the test lamp (candle-power I_T), then, if the test lamp be at the bench zero and the distance of the photometer head from it be d_T,

$$I_T = I_S d_T^2/(d-d_T)^2 = (I_S/d^2)\left(\frac{1}{d_T} - \frac{1}{d}\right)^{-2}$$

Alternatively, if the sub-standard be at the bench zero, $I_T = I_S d^2\left(\frac{1}{d_S} - \frac{1}{d}\right)^2$.

If, now, d be so chosen in relation to I_S that, in the first case, I_S/d^2, or, in the second case, $I_S d^2$ is a multiple of ten, I_T can be very simply calculated, the second case being slightly easier and involving only three operations, *viz.*:

(i.) Extraction from the tables of $1/d_S$

(ii.) Subtraction of a constant ($1/d$)

(iii.) Squaring the result

Tables have been prepared to enable the value of $d_T^2(d-d_T)^2$ to be obtained directly. These tables suffer from the disadvantage that the value of d is fixed, generally at an arbitrary value which is suitable only for a comparatively small number of cases met with in practice.*

A graph of the function $y = d_T^2/(d-d_T)^2$ becomes very unsuitable for accurate work at values of y above about 2·5 A more suitable form of graph is that shown in Fig 302, where log $\{d_T/(d-d_T)\}$ is plotted against d_T † If carefully drawn on accurate logarithmic paper, this curve is quite suitable for work to an accuracy of 0·5 per cent. Separate curves may be prepared for d = 1,000, 2,000, 3,000, 5,000 and 7,000 The one shown in the figure is for d = 3,000 Alternatively, a nomograph may be used ‡

* J. Castell-Evans "Physico-Chemical Tables" (Griffin, *1902*), vol 1, p 26 (d = 100)

E. Liebenthal. "Prakt Phot," p 432 (d = 2,500)

G Schuchardt "Prakt Anleitung zum Photometrieren," p 46 (d = 2,300)

W. Bertelsmann "Rechentafeln f. Beleuchtungstechniker" (F. Enke, Stuttgart, *1910*) (d = 2,500).

L. Ubbelohde "Tabelle f. Lichtstärkemessungen" (Fr. Schmidt u Haensch, Berlin) (d = 2,500)

† N A Halbertsma. Archiv f Elektrot., 1, *1912*, p 136

‡ L. Block. E T.Z., 43, *1922*, p 73

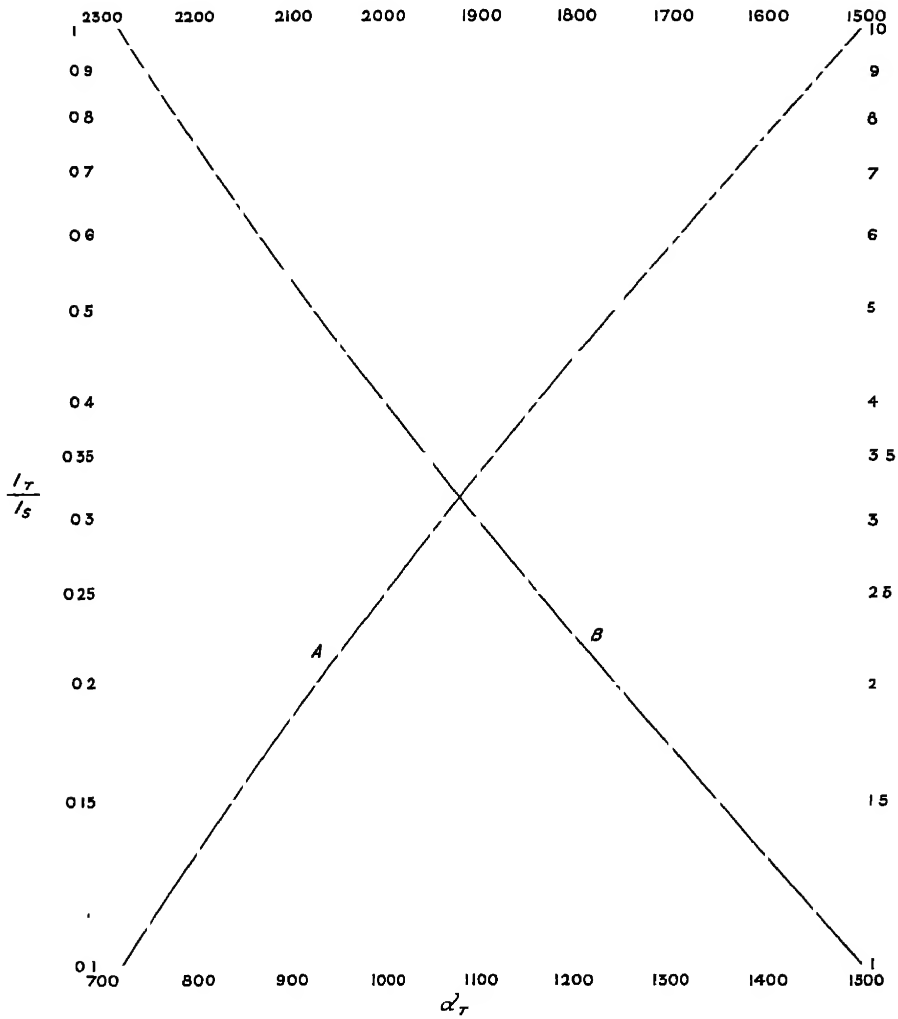

Fig 302 —Curves for Candle-power Calculation ($d = 3{,}000$)

For Curve A use left-hand and bottom scales

For Curve B use right-hand and top scales

APPENDIX VII

REFLECTION AND TRANSMISSION FACTORS OF COMMON AND IMPORTANT MATERIALS

Material	Reflection Factor	
	D'ffused Light	Direct Light *
Magnesium oxide .	0 96 (1)	0·92 (7)
Magnesium carbonate (block)	0 98 (2)	0·93 (7)
Plaster of Paris	0·91 (2)	0·87 (7)
Matt white celluloid	0 80–0 85 (7)	0·75–0 80 (7)
White blotting paper	0 80–0 85 (7)	0·75–0 80 (7)
Calcium carbonate .	0·96 (3)	0 92 (7)
Depolished opal glass	0·75–0·85 (7)	0 75–0·80 (7)
Polished opal glass	0 76 (4)	—
Polished silver ($\lambda = 0{\cdot}50\ \mu$)	—	0 91 (5)
,, ,, ($\lambda = 0\ 60\ \mu$)	—	$0\ 92_5$
,, ,, ($\lambda = 0\ 70\ \mu$)	—	$0\ 94_5$
Back-silvered glass ($\lambda = 0{\cdot}50\ \mu$)	—	0·81–0 87 (6)
,, ,, ($\lambda = 0\ 60\ \mu$)	—	0 82–0 88
,, ,, ($\lambda = 0\ 70\ \mu$)	—	0·84–0 90

* Normal incidence In the case of diffusing surfaces the angle of view is 30° and the unit is the brightness of a perfectly diffusing surface of 100 per cent reflection factor

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See also F. A Benford, Gen. El Rev, 23, *1920*, p 72

A. H. Taylor. Opt Soc Am, J, 4, *1920*, p 9, Illum Eng Soc N Y, Trans, 15, *1920*, p 811, El World, 76, *1920*, p 467, Bureau of Standards, Bull, 16, *1920*, p 431, and 17, *1921*, p 5

E Karrer. Opt Soc. Am, J., 5, *1921*, p 96, Bureau of Standards, Bull, 17, *1921*, p 203.

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(4) A H Taylor Illum Eng Soc N Y, Trans, 15, *1920*, p 815, Bureau of Standards, Bull., 17, *1921*, p 5

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(7) Measurements made at the National Physical Laboratory.

Other references are :—

(*a*) Various Diffusing Surfaces W E Sumpner Phys Soc, Proc, 12, *1893*, p. 10, Phil Mag, 35, *1893*, p 81 See note (38), p 232.

W W. Coblentz Bureau of Standards, Bull, 9, *1913*, p. 283

(*b*) Pigments, etc . L Bell. Illum Eng. Soc. N. Y, Trans, 2, *1907*, p 653; Illum. Eng 1, *1908*, p 72, Science Abs, 11, *1908* (B), p. 200

L Bell. El World, 65, *1915*, p. 211.

* H A Gardner Frank Inst, J, 181, *1916*, p 99 (The values given in this paper should be increased by about 10 per cent)
* W Harrison and E A Anderson Illum Eng Soc N Y, Trans, 15, *1920*, p 97
* F E Cady and H B Dates "Illuminating Engineering" (Chapman and Hall, *1925*), p 306
(*c*) Papers and inks Committee on Glare, Illum Eng Soc N Y., Trans, 10, *1915*, p. 379
(*d*) Surfaces met with out-of-doors H. H Kimball. Illum. Eng Soc N Y., Trans, 19, *1924*, p 217
W W Coblentz Bureau of Standards, Bull, 9, *1913*, p 283
(*e*) Simple chemical compounds (white) P G Nutting, L A Jones and F. A Elliott Illum Eng Soc N Y, Trans, 9, *1914*, p 593 (The values given in this paper should be increased by about 10 per cent)
(*f*) Various metals and alloys "Recueil de Constantes Physiques" (Soc Franç de Phys, *1913*), pp 546 *et seq*
Landolt-Bornstein "Physikalisch-Chemische Tabellen" (Springer, Berlin, *1923*), vol 2, pp 903 *et seq*

* In these papers actual specimens of coloured surfaces are given with their reflection factors

Material	Transmission Factor *	
	Diffused Light	Direct Light †
Sand-blasted clear glass	0 70 (1)	0 37 (2)
Flashed opal glass (0·1 mm thick)	—	0·39 (2)
Pot opal glass (polished) (2 mm thick)	—	0·13 (2)
Pot opal glass (depolished)	—	0·30 (3)

* The light lost by reflection has been included, so that the above figures represent $(1 - \rho - a)$
† Normal incidence angle of view 30° The unit is the brightness of a perfectly diffusing surface of 100 per cent. transmission factor
N B—Values of reflection or transmission factors are included in many of the papers referred to in notes (4) to (10) on p 372

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(2) Report of Committee on Glare Illum Eng Soc N Y, Trans, 11, *1916*, p 367.
(3) E J Edwards Illum Eng Soc N Y, Trans, 9, *1914*, p 1014

APPENDIX VIII

BRIGHTNESS, BRIGHTNESS TEMPERATURE AND COLOUR TEMPERATURE OF COMMON SOURCES OF LIGHT

Source	Brightness (c/mm2)	Brightness Temp Degrees K ($\lambda = 0\ 665\ \mu$)	Colour Temp Degrees K
Candle	0·004 to 0·006 (1)	—	1,930
Hefner (whole flame)	—	—	1,880
Pentane (10 c p)	—	—	1,920
Paraffin (flat wick)	0 0125	1,500	2,055
,, (round wick)	0·015	1,530	1,920
Gas (bat's-wing flame) (600 B Th U)	0 004 (2)	—	2,160
Acetylene (Kodak burner)	0 108	1,730	2,360
Welsbach mantle (upright)	0 048 (2)	—	—
,, ,, (inverted)	0 058 (3)	-	—
,, ,, (high pressure)	0 25 (3)	—	—
Carbon fil lamp (untreated) (2·6 l p w)	0 55 †	2,030	2,080
Gem lamp (4 l p.w)	0·78	2,130	2,195
* Tungsten fil lamp (vacuum, 7 9 l p w)	1 25 †	2,150	2,400
,, ,, (gas-filled, 12·9 l p w)	5 97 (4)	—	2,740 (4)
,, ,, (gas-filled, 15·2 l p w)	7 72 (4)	-	2,810 (4)
,, ,, (gas-filled, 18·1 l p w)	10 00 (4)	—	2,920 (4)
,, ,, (gas-filled, 21·2 l p w)	13·25 (4)	—	3,000 (4)
Mercury vapour (glass)	0·023 (2)	—	—
Arc crater (solid carbon)	172 (5)	3,700 (6) (λ=0 65 μ)	3,780 (7)
Clear blue sky	0 004 (8)	—	12,000 to 24,000 (9)
Zenith sun (at earth's surface)	1,650	—	5,400 (10)

* See also chapter IX , pp 272 *et seq*

† It has been found that both carbon and tungsten show deviations from the cosine law of emission See A. G Worthing, Astrophys. J , 36, *1912*, p 345 , El Rev and W Elect , 62, *1913*, p 706 , Nela Bull , 1, *1917*, p. 148

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(8) H E. Ives and M Luckiesh Illum Eng Soc N. Y, Trans, 6, *1911*, p. 700, P J Waldram, Illum Eng, 1, *1908*, p 814, and 7, *1914*, p 17 See also references in note (69), p 376
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APPENDIX IX

COLOUR OF COMMON LIGHT SOURCES

Source	Hue	Satn	Ref	Sensation Values (%)			Ref
	($m\mu$)	(%)		R	G	B	
Candle .	593	87	(1)	—	—	—	—
Hefner	593	86	(1)	54 1	38 9	7 0	(3)
Pentane . .	592	85	(1)	—	—	—	—
Acetylene (flat flame)	585·5	64	(1)	48 6	40 7	10 7	(2)
Welsbach mantle (¾ per cent. ceria)	582 4	71	(3)	46 8	40 1	13 1	(3)
Carbon fil lamp (2 6 l p w)	590	84	(3)	52 2	39 0	8 8	(3)
,, ,, (2 7 l p w)	591 5	75	(1)	—	—	—	—
Tungsten fil lamp (vacuum, 7 7 l p w)	588 4	65 5	(3)	49 3	38 9	11 8	(3)
,, ,, (gas-filled, 12 l p w)	586	66	(1)	—	—	—	—
,, ,, (gas-filled, 16 l p w) .	586 9	56	(3)	44 8	38 5	16 7	(3)
,, ,, (gas-filled, 24 l p w.)	584 5	55	(1)	—	—	—	—
Mercury vapour .	495 6	25	(3)	23 5	29 2	47 3	(3)
Arc crater (solid carbon)	584 1	22	(3)	39 6	35 6	24 8	(3)
Zenith sun (at earth's surface)	—	0	(1)	33 3	33 3	33 3	(1)
Black body at 5,000° K	—	0	(2 & 3)	33 3	33 3	33 3	(2 & 3)

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APPENDIX X

CHARACTERISTIC EQUATIONS FOR ELECTRIC INCANDESCENT LAMPS

THE relations between the candle-power (or flux) (F), potential (E) and current (I) for electric lamps are often expressed by empirical equations of the general form $A = mB^n$ or $\frac{B}{A}\frac{dA}{dB} = n$ [1]. The values of n for modern lamps are as follows:—

Type of Lamp	$\frac{E}{F}\frac{dF}{dE}$	$\frac{I}{F}\frac{dF}{dI}$	$\frac{E}{I}\frac{dI}{dE}$
Carbon fil. (untreated) . .	6·6	5·1	1·3
„ „ (Gem) . . .	4·8	3·6	1·3
Tungsten fil (vacuum) . .	3·59	6·16	0·58
„ „ (gas-filled) . .	Variable [2].	—	—

The above values refer to lamps operating at about the normal working efficiency for each type. It has been found that, although n is constant for small variations of the quantities concerned, it alters gradually with the efficiency of the lamp. A more complicated expression than that given above is, therefore, required for interpolation over wide ranges [3], and for tungsten filament vacuum lamps of normal type the following equations have been found [4] to represent the observed values over a range extending from 3·1 to 14 lumens per watt:—

If x = log (voltage ratio)
y_1 = log (actual lumens per watt)
y_2 = log (lumens ratio)
y_3 = log (wattage ratio)
y_4 = log (current ratio),

$$y_1 = -0{\cdot}918\,x^2 + 2{\cdot}009\,x + 0{\cdot}91514, \qquad y_2 = -0{\cdot}946\,x^2 + 3{\cdot}592\,x,$$
$$y_3 = -0{\cdot}028\,x^2 + 1{\cdot}583\,x, \qquad y_4 = -0{\cdot}028\,x^2 + 0{\cdot}583\,x$$

Tables have been prepared to facilitate calculation by means of the equations [4]

To a degree of accuracy often sufficient for practical purposes these equations may be used in the form of a graph such as that shown in Fig 303. The scale of abscissæ is a logarithmic scale of efficiencies (y_1), while the ordinate scale is a logarithmic scale of voltage, lumen, *etc.*, ratios (x, y_2, etc.) The curves representing the characteristic equations are drawn so that the ordinate 100 on each curve corresponds with the efficiency value chosen as a basis, *viz*, 8·225 l p w (1·2 w.p m.h.c.).

The method of using these curves will be clear from the following examples :—

(I.) Relation between Potential and Efficiency

Problem.—The efficiency of a lamp is 7 lumens per watt at 170 volts. Find (*a*) the efficiency at 210 volts, and
(*b*) the voltage at which it has an efficiency of 11·5 l p w.

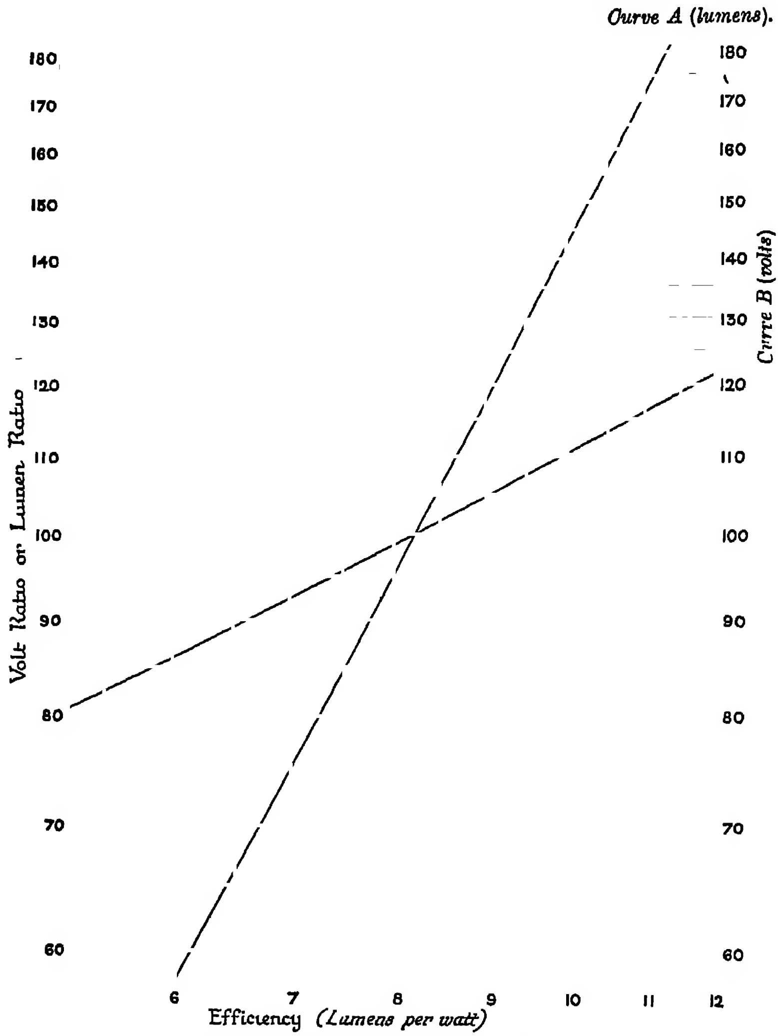

Fig. 303 —Characteristic Equations for Tungsten Filament Vacuum Lamps

Solution.—The volt ratio at 7 l p w is 92·5 per cent (curve *B*)

(*a*) The volt ratio at the required efficiency is therefore (210/170) × 92·5 = 114·2 per cent

Corresponding with this volt ratio (curve *B*), the required efficiency is found as 10·7 l p w

(*b*) The volt ratio at 11·5 l.p.w. is 119 per cent (curve *B*) Hence the required voltage is (119/92·5) × 170 = 219 volts

(II) Relation between Flux (Candle-power) and Efficiency

This problem is dealt with exactly as in (I), except that curve *A* is used instead of curve *B*.

(III) Relation between Flux or Candle-power and Potential

(*N.B.*—Either flux or potential must be given at some stated efficiency.)

Problem.—The candle-power of a lamp is 53 5 at 117 volts, the corresponding efficiency being 9 5 l p w

Find (*a*) the candle-power at 125 volts, and

(*b*) the voltage at which the candle-power is 37·5

Solution.—At the efficiency 9·5 l p w. the volt ratio is 107·6 per cent, and the c p. ratio 129 3 per cent

(*a*) At a volt ratio of $(125/117) \times 107\ 6 = 115$ per cent the efficiency is 10·7 l p w (actually this number need not be noted), and the corresponding c p. ratio is 165·5 per cent.

The required candle-power is, therefore, $(165{\cdot}5/129{\cdot}3) \times 53{\cdot}5 = 68{\cdot}5$ candles

(*b*) At a c p. ratio of $(37\ 5/53{\cdot}5) \times 129\ 3 = 90\ 7$ per cent the corresponding volt ratio (*i e*, the volt ratio at the same efficiency) is 97·5 per cent

The required voltage is, therefore, $(97\ 5/107{\cdot}6) \times 117 = 106$ volts.

It is worth noticing that if the scale of ordinates be of the same size as that of a standard slide rule, the multiplications and divisions involved in the above work may be avoided For instance, in Problem I. (*a*) the distance between the graduations 170 and 210 on the slide rule is marked off vertically above the point 92 5 on the ordinate scale. This gives the 114·2 mark as the ordinate corresponding with the new efficiency The scale in Fig 303 is the same as the bottom scale of a so-called 10-inch rule, *viz*, 25 cm between 1 and 10

The following table gives the characteristics (uncorrected for end losses) of tungsten filament vacuum lamps at various efficiencies ([5]) :—

Characteristics of Tungsten Filament Lamps (Uncorrected)

Lum effy (l p w)	Colour Temp Degs K	Brightness (c /mm²)	Per cent Volts	Per cent Watts	Per cent. Lumens	$n = \frac{EdF}{FdE}$
2 50	2,021	0 200	55 0	38 7	10 1	4 1
3·65	2,128	0·356	63 6	48 7	18 6	3 9
5 00	2,231	0 613	73 3	61·1	32 2	3 8
6 70	2,335	1 005	83 5	75 2	52 2	3 6
8 60	2,440	1 57	95·0	92·1	82 9	3 5
9 80	2,493	1 93	100 0	100 0	100 0	3 4
10 8	2,516	2 375	105·9	109 6	122 0	3 4
13 2	2,652	3 47	118 5	130 6	179 3	3 3
15 8	2,758	4 98	132 3	155 6	257 0	3 2

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The numbers refer to pages. Numbers in brackets indicate reference notes.

VOCABULARY

To facilitate reference to foreign literature, the French and German equivalents of certain photometric technical terms are given after the index entry of those terms. Equivalents which are almost identical with the English word are omitted.

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